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Entanglement dynamics in two-component Bose–Einstein condensates*

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Based on the exact solution of the time-dependent Schrödinger equation for two-species Bose–Einstein condensates (BECs) consisting of two hyperfine states of the atoms coupled by a tuned adiabatic and time-varying Raman coupling, we obtain analytically the entanglement dynamics of the system with various initial states, particularly the SU(2) coherent state, for both of cases with and without the nonlinear interactions. It is shown that the effect of nonlinear interaction on the entanglement appears only in a longer time period depending on the BEC parameters.

Keywords: entanglement dynamics, gauge transformation, spin coherent state
PACC: 0365, 7450

1. Introduction

The experimental realization of Bose–Einstein condensates (BECs) of trapped alkali atomic clouds[1] has stimulated the active study in the new regime. Since then, BEC has been a popular platform for the study of various quantum phenomena experimentally and theoretically.[2−9] Particularly, it has been shown that the coherent collisional interactions in a BEC allow the generation of substantial many-particle entanglement in the two-component condensate composed of atoms with two internal states during the free evolution of the condensates.[10] With the capability of extremely long coherence time in BECs[4] engineering many-particle entanglement, the study of the entanglement characteristics of BECs is of significant interest in quantum information and computation. The entanglement dynamics[11,12] of two-species BECs has been investigated intensively.

In this paper, we study the entanglement dynamics of the BEC of trapped atomic gas in a single trap consisting of atoms with two internal states that are coupled by a Raman laser or microwave field, which is a bipartite system. For the convenience of computation, the system is described by a giant or mesoscopic pseudospin using Schwinger realization of angular momentum operators in terms of two-mode bosons as shown in Refs. [11,13]. With the exact solution of the model the time evolution of entanglement of this two-mode BEC system is evaluated analytically. We investigate the time evolution of entanglement with various parameters of the scattering lengths of interatom collisions, Josephson coupling strength, and initial states for the time-dependent system. The entanglement dynamics of two-species BECs in an $SU(2)$ coherent state[14] (also referred to as the generalized spin coherent state) also is evaluated particularly. In addition, Berry phase emerges naturally in the mesoscopic pseudospin model because the coupling between two components varies with time periodically.

The rest of the paper is organized as follows. Section 2 is devoted to discussing our model and formulating it in pseudospin operators. In Section 3 we present the entanglement formulation of the two-species BECs in pseudo-angular-momentum representation. In Section 4 we evaluate the time-evolution of the entanglement with various initial states. Finally, a brief summary is given in Section 5.

2. The model

The phenomenon where many interacting bosons are restricted to occupy the same two-dimensional

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single-particle Hilbert space is known as the Josephson effect. When the two single-particle states, or modes, are separated spatially, it is described as external Josephson effect. We consider a pair of tunnel-like-coupled BECs confined in one trap consisting of atoms with two hyperfine states $|A\rangle$ and $|B\rangle$, which is described as internal Josephson effect. In the one-mode approximation of condensation, the system is described by the Hamiltonian of second quantization (natural unit $\hbar = 1$ is used throughout the paper)[3]

$$\hat{H} = \sum_{l=a,b} \hat{H}_l + \hat{H}_{\rm int} + \hat{H}_f, \tag{1}$$

where

$$\hat{H}_a = \omega_a \hat{a}^+ \hat{a} + \frac{\eta_a}{2} \hat{a}^+ \hat{a}^+ \hat{a} \hat{a}, \tag{2}$$

$$\hat{H}_b = \omega_b \hat{b}^+ \hat{b} + \frac{\eta_b}{2} \hat{b}^+ \hat{b}^+ \hat{b} \hat{b} \tag{3}$$

with

$$\omega_l = \int {\rm d}^3 r \phi_l^* (\ r) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_l (r) \right] \phi_l (\ r), l = a, b.$$

The interaction term between two species of atoms is

$$\hat{H}_{\rm int} = \chi \hat{a}^+ \hat{a} \hat{b}^+ \hat{b}, \tag{4}$$

and the transition term is

$$\hat{H}_f = G \left(\hat{a}^+ \hat{b} {\rm e}^{{\rm i}\varphi(t)} + \hat{b}^+ \hat{a} {\rm e}^{-{\rm i}\varphi(t)} \right). \tag{5}$$

Here η_a and η_b are the interaction constants between the same species of atoms, respectively, and χ is the interspecies interaction constant. The transition between different species in the rotating wave approximation with a small detuning is denoted by the Josephson coupling strength G and the phase $\varphi(t) = \Delta t$, where Δ is the detuning between the frequencies of the laser and transition from the state $|A\rangle$ to the state $|B\rangle$. We note in particular that $\hat{H}_f$ describes Josephson-like coupling which interchanges internal atomic states in a coherent manner. The study of dynamics of the system would be greatly simplified by introducing the Schwinger pseudo-angular-momentum $[SU(2)]$ operators defined as $\hat{J}_x = \frac{1}{2} \left(\hat{a}^+ \hat{b} + \hat{b}^+ \hat{a} \right)$, $\hat{J}_y = \frac{1}{2i} \left(\hat{a}^+ \hat{b} - \hat{b}^+ \hat{a} \right)$, $\hat{J}_z = \frac{1}{2} \left(\hat{a}^+ \hat{a} - \hat{b}^+ \hat{b} \right)$, and the Casimir invariant $\hat{J}^2 = \frac{\hat{N}}{2} \left(\frac{\hat{N}}{2} + 1 \right)$, where $\hat{N} = \hat{a}^+ \hat{a} + \hat{b}^+ \hat{b}$ is the total boson number operator, a conserved quantity here. Thus the Hamiltonian can be written as

$$\hat{H} = \hat{H}_0 + G \left(\hat{J}_+ {\rm e}^{{\rm i}\varphi(t)} + \hat{J}_- {\rm e}^{-{\rm i}\varphi(t)} \right) \tag{6}$$

with $\hat{H}_0 = \omega_0 \hat{J}_z + q \hat{J}_z^2$ and $\hat{J}_\pm = \hat{J}_x \pm {\rm i} \hat{J}_y$, where $\omega_0 = \omega_a - \omega_b + (N-1)(\eta_a - \eta_b)/2$, and $q = (\eta_a + \eta_b)/2 - \chi$. The pseudo-angular-momentum operators $\hat{J}_+$, $\hat{J}_-$ and $\hat{J}_z$ satisfy the usual angular momentum communication relations $\left[\hat{J}_z, \hat{J}_\pm \right] = \pm \hat{J}_\pm$, and $\left[\hat{J}_+, \hat{J}_- \right] = 2 \hat{J}_z$.

The simultaneous eigenstates $|j, m\rangle$ of the operators $\hat{J}^2$ and $\hat{J}_z$ have eigenvalues j and m, which are related to the total number of atoms in BECs and the imbalance number between the mode a and the mode b by $j = N/2$ and $m = \{-N/2, -N/2+1, \cdots, N/2\}$. The quantum state $|\psi(t)\rangle$ can be expressed as

$$|\psi(t)\rangle = \sum_{m=-j}^{j} C_m (t) |j, m\rangle, \tag{7}$$

where $C_m(t)$ are complex coefficients. The state $|j, m\rangle$ can be obtained from the vacuum by boson creation operators, i.e.

$$|j, m\rangle = \frac{1}{\sqrt{(j+m)!(j-m)!}} \left(\hat{a}^+ \right)^{j+m} \left(\hat{b}^+ \right)^{j-m} |0\rangle. \tag{8}$$

3. Entanglement between the two modes

In this section we study the entanglement of two-species BECs in the pseudo-angular-momentum representation. In this bipartite system the two species a and b of atoms are entangled, and thus the entanglement between them is accessible and useful from the viewpoint of quantum information or communication. Following the standard measure [15,16] of entanglement of pure states of bipartite system, the entropy of entanglement is defined as the von Neumann entropy of the reduced density operator of either of the subsystems $E(\hat{\rho}) = S(\hat{\rho}_a) = S(\hat{\rho}_b)$. The reduced density operator of a subsystem is found by tracing out the other subsystem via the partial trace. The reduced density operator for subsystem a is determined by

$$\hat{\rho}_a = {\rm Tr}_b (\hat{\rho}), \tag{9}$$

with $\hat{\rho} = |\psi(t)\rangle \langle \psi(t)|$ being the density operator of the whole system, where ${\rm Tr}_b$ is the partial trace over the subsystem b. The entropy of entanglement is then given by

$$E(\hat{\rho}) = -{\rm Tr} (\hat{\rho}_a \ln \hat{\rho}_a) = -\sum_k \lambda_k \ln (\lambda_k), \tag{10}$$

where $\{\lambda_k\}$ are the set of eigenvalues of the reduced density operator $\hat{\rho}_a$.

It is interesting to note that the eigenstates of pseudo-angular-momentum can be decomposed into

$$|j,m\rangle = |N/2+m\rangle_a \, |N/2-m\rangle_b \tag{11}$$

with $|N/2+m\rangle_a$ and $|N/2-m\rangle_b$ being the eigenstates of atom number operators of species a and b, respectively. So in the $\{|j,m\rangle\}$ subspace, the density operator describing a general state of the system is given by

$$\begin{aligned}\hat{\rho} &= |\psi\rangle\langle\psi| \\ &= \sum_{m,n=-j}^{j} C_m C_n^* \, |j+m\rangle_a \, |j-m\rangle_{ab} \, \langle j+n|_b \, \langle j-n| .\end{aligned}$$

Taking the partial trace with respect to mode b yields the reduced density operator for mode a,

$$\hat{\rho}_a = \mathrm{Tr}_b(\hat{\rho}) = \sum_{m=-j}^{j} |C_m|^2 \, |j+m\rangle_{aa} \langle j+m| .$$

We can see that the reduced density operator in this case is diagonal and the eigenvalues are simply $\lambda_i = |C_i|^2$. Thus, the entropy of entanglement between the two modes of the coupled BECs is given by

$$E(\rho) = -\sum_{m=-j}^{j} |C_m|^2 \ln\left(|C_m|^2\right), \tag{12}$$

which means that the entanglement between the two-species can be calculated from the coefficients C_m in the angular momentum basis. The maximal entanglement is obviously $E_{\max}(\rho) = \ln(2j+1)$.

4. Entanglement dynamics

In practical experiment, the difference between the scattering lengths for the internal states of same atoms is very small so that the condition $\eta_a \simeq \eta_b \simeq \chi$ can be satisfied. Therefore q is very small generally. We study the case of $q = 0$ for simplicity, which is exactly solvable.

4.1. The case of $q = 0$

We define a generalized gauge transformation in terms of the unitary operator [17]

$$\hat{R}(t) = \exp\left[\lambda/2\left(\hat{J}_- e^{i\varphi(t)} - \hat{J}_+ e^{-i\varphi(t)}\right)\right], \tag{13}$$

which has the same form as the generator of $SU(2)$ coherent states Eq.(A1), where λ is the auxiliary parameter. The time-dependent Schrödinger equation is covariant under the gauge transformation such that

$$i\frac{d}{dt}|\psi'(t)\rangle = \hat{H}'|\psi'(t)\rangle ,$$

where

$$\hat{H}' = \hat{R}\hat{H}\hat{R}^+ - i\hat{R}\frac{\partial}{\partial t}\hat{R}^+, \quad |\psi'(t)\rangle = \hat{R}|\psi(t)\rangle ,$$

and the state $|\psi(t)\rangle$ is assumed to be the solution of original Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle .$$

We obtain the Hamiltonian operator

$$\hat{H}' = \alpha \hat{J}_z, \quad \alpha = \sqrt{(\omega_0+\Delta)^2 + 4G^2},$$

with auxiliary parameter λ chosen as

$$\sin\lambda = -\frac{2G}{\omega_0+\Delta}\cos\lambda$$

and

$$\cos\lambda = \frac{\omega_0+\Delta}{\alpha}.$$

Thus the exact general solution of the original Schrödinger equation is

$$\begin{aligned}|\psi(t)\rangle &= \sum_{m=-j}^{j} c_m e^{i\varepsilon_m(t)} \, |j,m(t)\rangle , \\ |j,m(t)\rangle &= \hat{R}(t)\,|j,m\rangle ,\end{aligned} \tag{14}$$

with $|j,m\rangle$ being the pseudo-angular-momentum eigenstate. Here the total phase is

$$\begin{aligned}\varepsilon_m(t) = &\int_0^t dt' \left\langle j,m \left| i\hat{R}^+(t')\frac{\partial}{\partial t'}\hat{R}(t') \right| j,m \right\rangle \\ &- \int_0^t dt' \left\langle j,m \left| \hat{R}^+(t')\hat{H}(t')\hat{R}(t') \right| j,m \right\rangle .\end{aligned}$$

As defined by Berry, the second term in the above equation is the usual dynamical phase while the first term is the geometric phase, i.e. Berry phase, which, for one period T, i.e. $\varphi(T) - \varphi(0) = 2\pi$, is found to be

$$\begin{aligned}\gamma_m(T) =& m(\cos\lambda - 1)\oint d\varphi \\ =& m\left(\frac{\omega_0+\Delta}{\alpha} - 1\right)2\pi.\end{aligned} \tag{15}$$

The Berry phase does not depend on an explicit form of the function $\varphi(t)$ and is simply m times as large as a solid angle with the polar angle λ, in agreement with

the result in Ref.[13]. Below we investigate the entanglement dynamics with the state $|\psi(t)\rangle$. The general solution of Eq.(14) for the initial state $|\psi(0)\rangle = |j, m_0\rangle$ can be expressed as

$$\begin{aligned}|\psi(t)\rangle =& \hat{R}^{+}(t)\,\mathrm{e}^{-\mathrm{i}\hat{H}'t}\hat{R}(0)\,|j,m_0\rangle \\ =& \sum_{m''=-j}^{j} C_{m''}(t)\,|j,m''\rangle,\end{aligned}$$

where

$$C_{m''}(t) = \sum_{m'=-j}^{j} \mathrm{e}^{-\mathrm{i}\alpha t m'} c_{m'}(t)\, c_{m''}(t) \tag{16}$$

with

$$\begin{aligned}c_{m'} =& \sum_{m=m_0}^{j} (-1)^{m-m_0} \frac{\left(1+\tau^2\right)^{-m} \tau^{\left(2m-m_0-m'\right)}}{(m-m_0)!\,(m-m')!} \\ &\times \frac{(j+m)!}{(j-m)!}\sqrt{\frac{(j-m_0)!\,(j-m')!}{(j+m_0)!\,(j+m')!}},\end{aligned}$$

with

$$m' \le m_0,$$

$$\begin{aligned}c_{m'} =& \sum_{m=m'}^{j} (-1)^{m-m_0} \frac{\left(1+\tau^2\right)^{-m} \tau^{\left(2m-m_0-m'\right)}}{(m-m_0)!\,(m-m')!} \\ &\times \frac{(j+m)!}{(j-m)!}\sqrt{\frac{(j-m_0)!\,(j-m')!}{(j+m_0)!\,(j+m')!}},\end{aligned} \tag{17}$$

with

$$m' > m_0,$$

and

$$\begin{aligned}c_{m''} =& \sum_{m=m'}^{j} (-1)^{m-m''}\, \mathrm{e}^{\mathrm{i}\left(m'-m''\right)\varphi(t)} \\ &\times \frac{\left(1+\tau^2\right)^{-m} \tau^{\left(2m-m'-m''\right)}}{(m-m')!\,(m-m'')!} \\ &\times \frac{(j+m)!}{(j-m)!}\sqrt{\frac{(j-m')!\,(j-m'')!}{(j+m')!\,(j+m'')!}},\end{aligned}$$

with

$$m'' \le m',$$

$$\begin{aligned}c_{m''} =& \sum_{m=m''}^{j} (-1)^{m-m''}\, \mathrm{e}^{\mathrm{i}\left(m'-m''\right)\varphi(t)} \\ &\times \frac{\left(1+\tau^2\right)^{-m} \tau^{\left(2m-m'-m''\right)}}{(m-m')!\,(m-m'')!} \\ &\times \frac{(j+m)!}{(j-m)!}\sqrt{\frac{(j-m')!\,(j-m'')!}{(j+m')!\,(j+m'')!}},\end{aligned} \tag{18}$$

with

$$m'' > m'.$$

Here

$$\tau = \sqrt{\frac{1-\omega_0/\alpha}{1+\omega_0/\alpha}}. \tag{19}$$

Then we can obtain the entanglement according to Eq.(12). To make the physical meaning clearer, we return to Fock state space with the expression (11) here. In Fig.1(a) and Fig.1(b), we show the time evolution of entanglement for $N = 10$ and $\omega_0 + \Delta = 0$ with initial states having no imbalance $|5\rangle_a\,|5\rangle_b$ and the largest imbalance $|10\rangle_a\,|0\rangle_b$ at $t = 0$, which correspond to the pseudo-angular-momentum states $|5, 0\rangle$ and $|5, 5\rangle$, respectively. In this case, the Berry phase is $\gamma_m(T) = -2\pi m$. The entropy of entanglement oscillates periodically in both cases because of the Raman coupling. Also, it is obvious that the period of time evolution, which is equal to $\dfrac{\pi}{2G}$ here, decreases with the increase of the coupling drive between two species. Notice the difference between them; there are small fluctuations near the maximal entanglement when the initial particle imbalance number is zero, but the time evolution is smooth when the initial imbalance is the maximum. The corresponding case for $\omega_0 + \Delta = 0.01$ is displayed in Fig. 2. In this situation two phenomena that we can observe in Fig.1 can be compared with the case of $\omega_0 + \Delta = 0$: (i) the evolution period, which is equal to $\dfrac{\pi}{G}$ here, doubles that in the former case with a valley appearing at half of the period; (ii) the valley at half of the period becomes deeper and the bottom of valley tends to zero finally as the coupling drive G increases. The atom number dependences of entanglement for $\omega_0 + \Delta = 0$ and $\omega_0 + \Delta = 0.01$ are plotted in Fig.3. It is shown that when the atom number increases, the entanglement increases and the valley at half of the period becomes shallow.

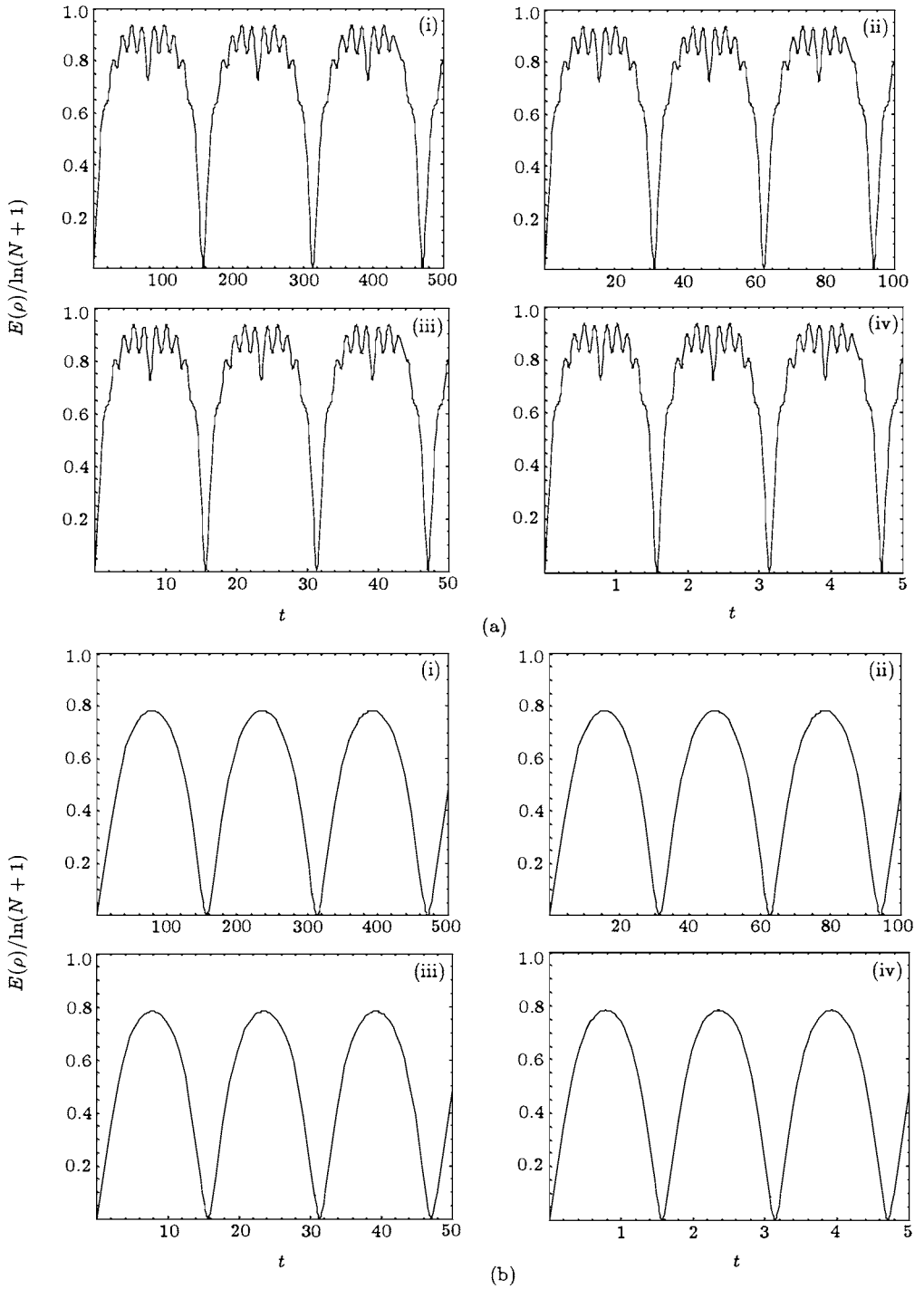


Fig.1. Time evolution of entanglement for $\omega_0 + \Delta = q = 0$ and $N = 10$. (a) $|\psi(0)\rangle = |N/2\rangle_a |N/2\rangle_b$ and $G = 0.01$ (i), 0.05 (ii), 0.1 (iii), 1 (iv). (b) $|\psi(0)\rangle = |N\rangle_a |0\rangle_b$ and $G = 0.01$ (i), 0.05 (ii), 0.1 (iii), 1 (iv).

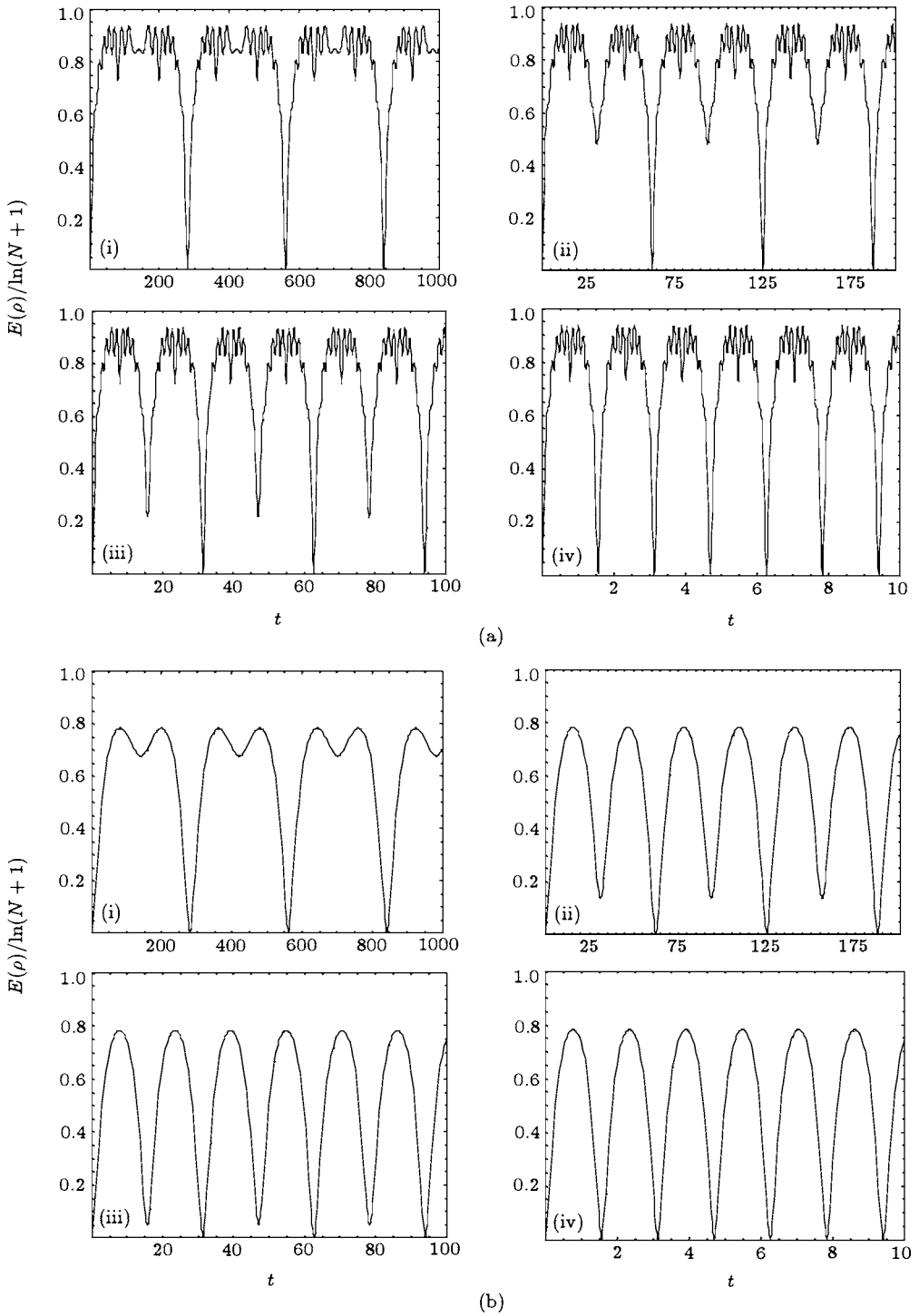


Fig.2. Time evolution of entanglement for $\omega_0 + \Delta = 0.01$, $q = 0$ and $N = 10$. (a) $|\psi(0)\rangle = |N/2\rangle_a |N/2\rangle_b$ and $G = 0.01$ (i), 0.05 (ii), 0.1 (iii), 1 (iv). (b) $|\psi(0)\rangle = |N\rangle_a |0\rangle_b$ and $G = 0.01$ (i), 0.05 (ii), 0.1 (iii), 1 (iv).

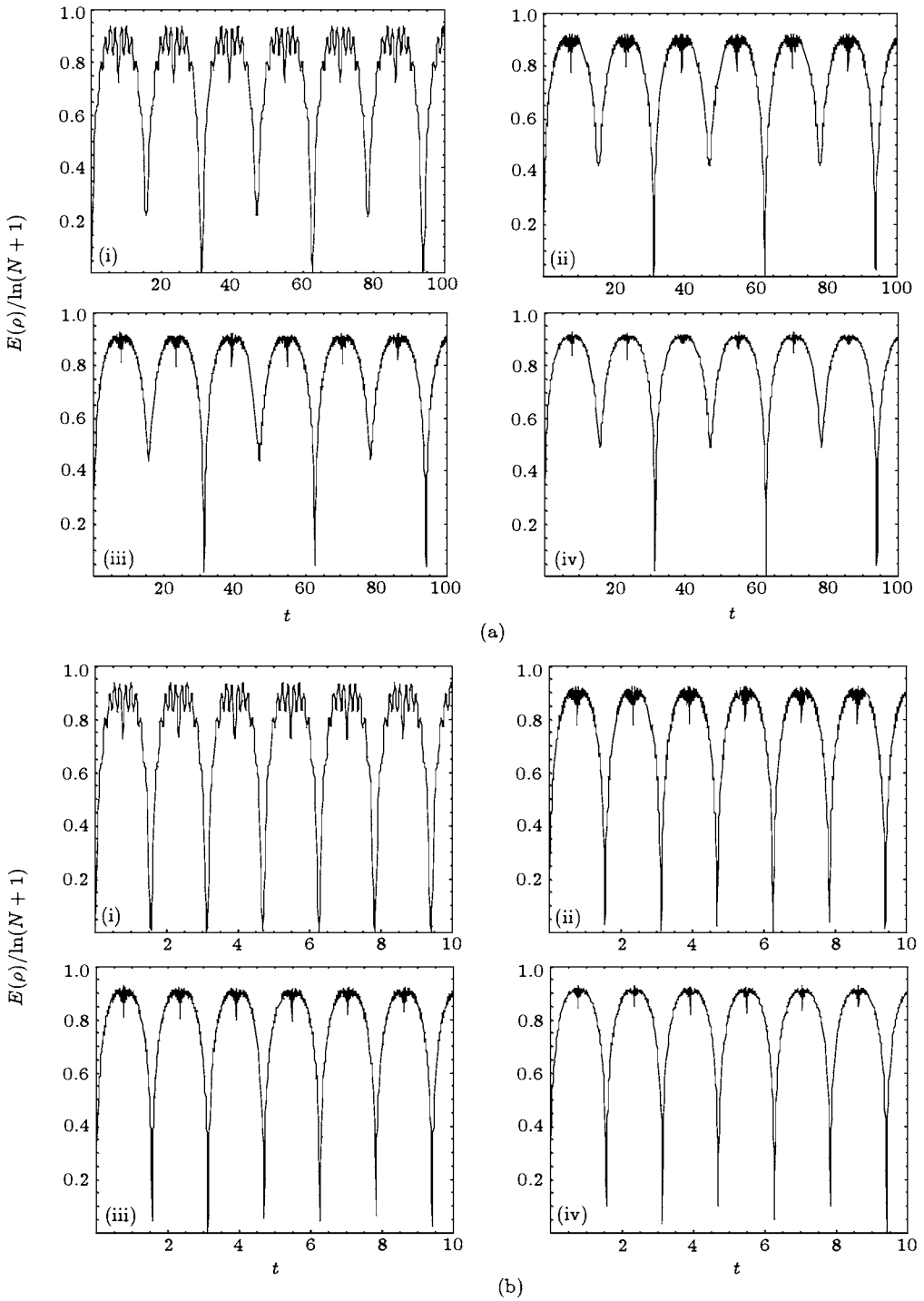


Fig.3. Time evolution of entanglement for $\omega_0 + \Delta = 0.01$, $q = 0$ and $|\psi(0)\rangle = |N/2\rangle_a |N/2\rangle_b$. (a) $G = 0.1$ and $N = 10$ (i), 30 (ii), 50 (iii), 100 (iv). (b) $G = 1$ and $N = 10$ (i), 30 (ii), 50 (iii), 100 (iv).

4.2. The case with nonlinear interaction

In the two-component BEC composed of different hyperfine states of the same atoms, q is very small in fact so that $q \ll G$ can be satisfied well. Also, the condition of $\omega_0 + \Delta = 0$ is reasonable for a single trap. By using the unitary transformation as shown in the above Eq.(13) and the relation

$$\begin{aligned}\hat{R}\hat{J}_z^2\hat{R}^+ =& \hat{J}_z^2\cos^2\lambda + \frac{1}{4}(\hat{J}_+^2 e^{i2\varphi(t)} \\ &+ \hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+ + \hat{J}_-^2 e^{-i2\varphi(t)})\sin^2\lambda \\ &+ \frac{1}{2}\left[\left(\hat{J}_z\hat{J}_+ + \hat{J}_+\hat{J}_z\right)e^{i\varphi(t)}\right]\cos\lambda\sin\lambda \\ &+ \frac{1}{2}\left[\left(\hat{J}_z\hat{J}_- + \hat{J}_-\hat{J}_z\right)e^{-i\varphi(t)}\right]\cos\lambda\sin,\end{aligned}$$

we obtain

$$\hat{H}' = -2G\hat{J}_z - \frac{1}{2}q\hat{J}_z^2,$$

where the terms of $\hat{J}_+^2 \mathrm{e}^{\mathrm{i}2\varphi(t)}$ and $\hat{J}_-^2 \mathrm{e}^{-\mathrm{i}2\varphi(t)}$ have been neglected as a reasonable approximation on condition that $q \ll G$. It is easy to obtain the exact general solution of the original Schrödinger equation as

$$\begin{aligned}|\psi(t)\rangle =& \hat{R}^+(t)\,\mathrm{e}^{-\mathrm{i}\hat{H}'t}\hat{R}(0)\,|j, m_0\rangle \\ =& \sum_{m''=-j}^{j} C_{m''}(t)\,|j, m''\rangle\end{aligned}$$

with

$$C_{m''}(t) = \sum_{m'=-j}^{j} \mathrm{e}^{\mathrm{i}\left(2Gm' + \frac{1}{2}qm'^2\right)t} c_{m'} c_{m''}, \qquad (20)$$

where $c_{m'}$ and $c_{m''}$ have the same forms as Eq.(17) and Eq.(18) respectively. Then the entanglement dynamics can be obtained with Eq.(12). For the same reason as the above, we return to Fock state space. In Fig.4 we plot the time evolution of entanglement with $N = 10$ and $|\psi(0)\rangle = |10\rangle_a |0\rangle_b$. It is interesting to observe that the entanglement quickly approaches the maximum and the system remains in the most entangled state. However the evolution of the entanglement is not very smooth, but has the most pronounced decrease occurring with period π/q. This is indeed the case even for different G and N, which is evident also in Fig.5. The periodically pronounced decreases become less when there are more atoms in the system.

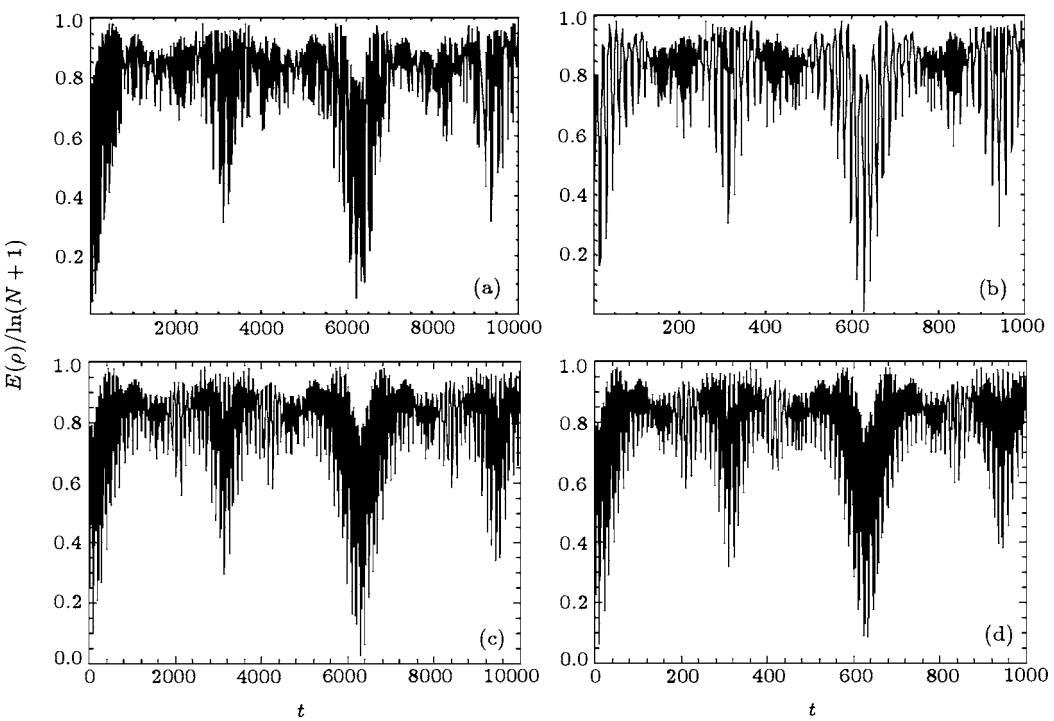


Fig.4. Time evolution of entanglement with nonlinear interaction. (a) $G = 0.1$ and $q = 0.001$, (b) $G = 0.1$ and $q = 0.01$, (c) $G = 1$ and $q = 0.001$, (d) $G = 1$ and $q = 0.01$ with $|\psi(0)\rangle = |N\rangle_a |0\rangle_b$ and $N = 10$.

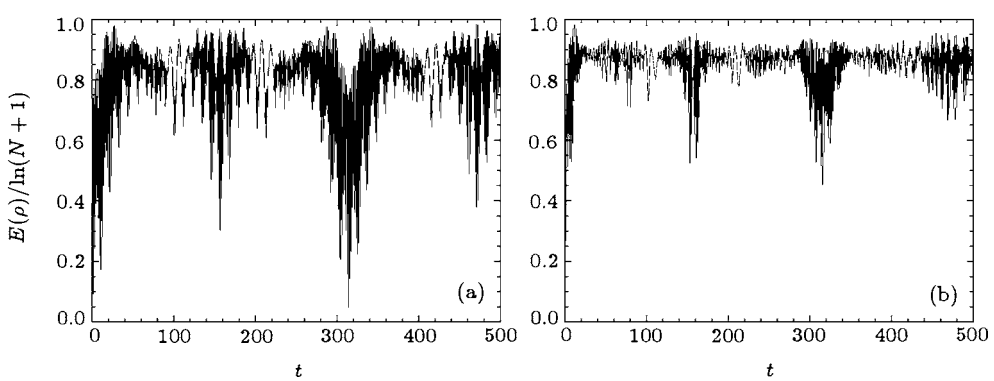


Fig.5. Time evolution of entanglement for different particle number: (a) $N = 10$ and (b) $N = 50$ with $q = 0.02$, $G = 1$ with $|\psi(0)\rangle = |N\rangle_a |0\rangle_b$.

4.3. Initial pseudo-spin coherent states

Since the two-species BECs are experimentally created by coupling two hyperfine states with radiation field, it has been shown that the prepared initial state can be a particular case of the $SU(2)$ coherent state that describes a state with well-defined relative phase between the two species. When the initial state is a general $SU(2)$ coherent state $|\psi(0)\rangle = |\mathbf{n}\rangle$, the state at time t seems to be

$$|\psi(t)\rangle = \sum_{m''=-j}^{j} |j, m''\rangle\, C_{m''}(t) \left[\begin{array}{c} 2j \\ j+m'' \end{array}\right]^{1/2} \times \left(\cos\frac{1}{2}\theta\right)^{j-m''} \left[\exp(-\mathrm{i}\phi)\sin\frac{1}{2}\theta\right]^{j+m''} \qquad (21)$$

(for the definition of $SU(2)$ coherent state see Appendix). The coefficients $C_{m''}(t)$ are given in Eq.(16) for the case $q = 0$ and in Eq.(20) for the case $q \neq 0$. The entanglement dynamics is shown in Fig.6 for the former case with various values of the coupling drive G and angle θ (for simplicity we set $\varphi(t) = \phi = 0$). It shows the same dynamic behaviour as that for the initial state $|\psi(0)\rangle = |N\rangle_a |0\rangle_b$ except for the oscillation amplitude. In fact it is easy to prove that when the initial state is the maximal imbalance state $|N\rangle_a |0\rangle_b$, $|\psi(t)\rangle$ will be an $SU(2)$ coherent state. When $\omega_0 + \Delta = 0.01$, the evolution period is doubled compared with the case of $\omega_0 + \Delta = 0$. The valleys appearing at half of the period also become deeper and the bottoms of them tend to zero with the increase of coupling drive. The evolution portraits of entanglement with $q \neq 0$ are shown in Fig.7. As the former, the most important effect of the nonlinear interaction is that the entanglement approaches the maximum quickly and the most pronounced decreases occur with period π/q. The particle number dependences of entanglement are shown in Fig.7(c) and Fig.7(d) for fixed q and G. The periodic pronounced decreases become less when there are more atoms in the system.

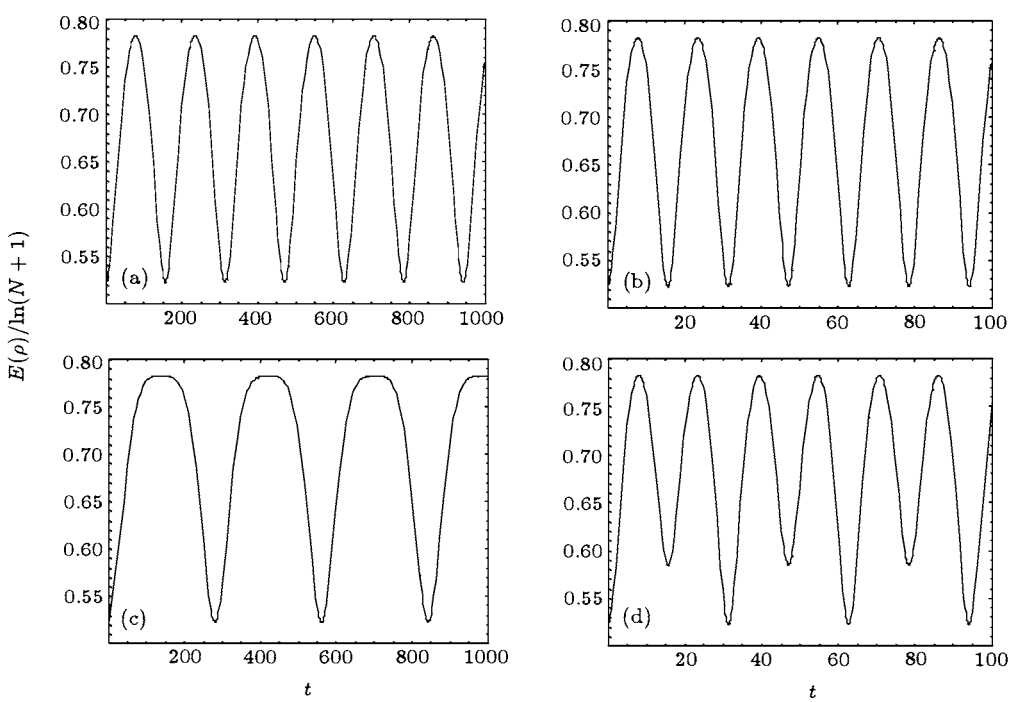


Fig.6. Time evolution of entanglement for the initial state $|\psi(0)\rangle = |n\rangle$, $\varphi(t) = \phi = 0$ and $\theta = 4\pi/5$. Top: $G = 0.01$ (left) and $G = 0.1$ (right) with $q = 0$, $\omega_0 + \Delta = 0$ and $N = 10$. Bottom: $G = 0.01$ (left) and $G = 0.1$ (right) with $q = 0$, $\omega_0 + \Delta = 0.01$.

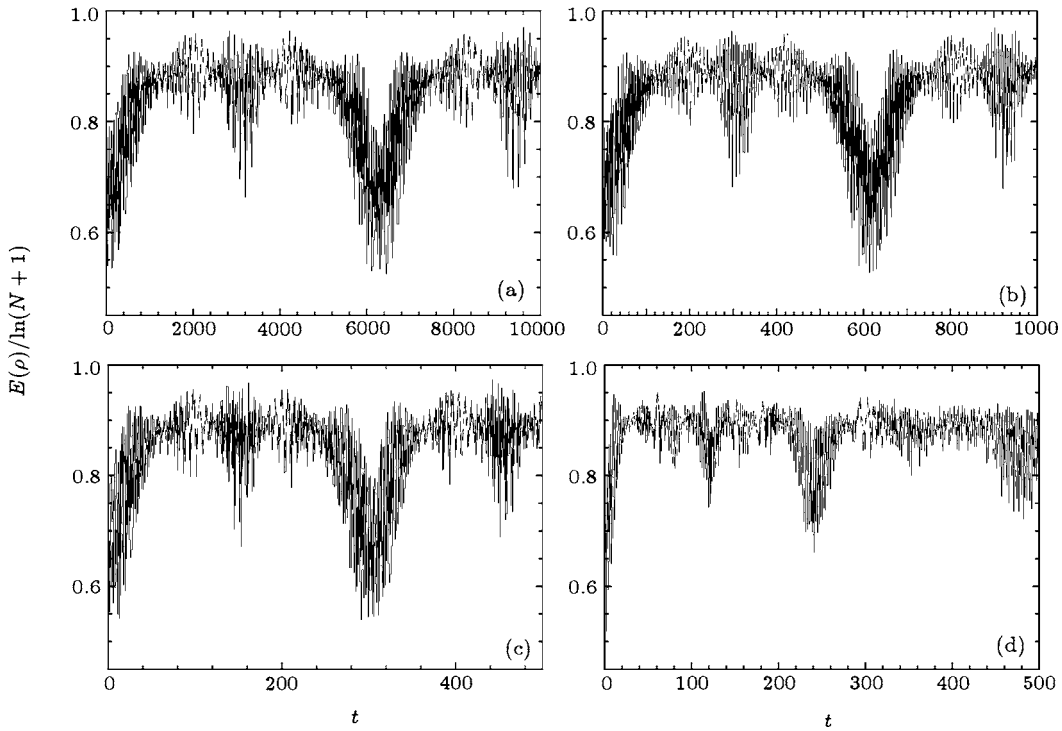


Fig.7. Time evolution of entanglement with nonlinear interaction for the initial state $|\psi(0)\rangle = |\mathbf{n}\rangle$, $\varphi(t) = \phi = 0$ and $\theta = 4\pi/5$. (a) $q = 0.001$ and $N = 10$, (b) $q = 0.01$ and $N = 10$, (c) $q = 0.02$ and $N = 10$, (d) $q = 0.02$ and $N = 50$ with $G = 1$.

5. Summary

We have investigated the entanglement dynamics of BECs consisting of N atoms in two hyperfine states coupled by a Raman laser with exact solution of time-dependent Schrödinger equation. Various behaviours of the entanglement are shown with different parameters, as well as the initial states, particularly when the initial state is an $SU(2)$ coherent state. We mainly conclude that: (i) in the case of $q = 0$, the entanglement oscillates with a period depending on the coupling drive G; (ii) in the case of $q \neq 0$, for the evolution in a short period, which is in the order of $\frac{\pi}{G}$, the behaviour of entanglement is similar to that in the case of $q = 0$, but for a longer period in the order of $\frac{\pi}{q}$, obviously different behaviour appears; (iii) in both cases, the entanglement increases with the increase of the atom number.

Appendix A: $SU(2)$ coherent state

The $SU(2)$ coherent state that we used is defined as [14]

$$|\boldsymbol{n}\rangle = \hat{\Omega}(\zeta)\,|j,-j\rangle\,,$$

where

$$\hat{\Omega}(\zeta) = \exp\left(\zeta \hat{J}_+ - \zeta^* \hat{J}_-\right). \tag{A1}$$

This is called the coherent state in south pole gauge, as compared with the generation of the coherent state from the extreme state $|j,j\rangle$ where it is called the state in the north pole gauge. The coherent state can be expanded in terms of $|j,m\rangle$ as follows:

$$|\boldsymbol{n}\rangle = \sum_{m=-j}^{j} |j,m\rangle \left[\begin{array}{c} 2j \\ j+m \end{array}\right]^{1/2} \left(\cos\frac{1}{2}\theta\right)^{j-m} \times \left[\exp(-\mathrm{i}\phi)\sin\frac{1}{2}\theta\right]^{j+m}$$

with $\zeta = \frac{\theta}{2}\mathrm{e}^{-\mathrm{i}\phi}$.

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References

[1] Anderson M H, Ensher J R, Matthews M R, Wieman C E and Cornell E A 1995 *Science* **269** 198
Davis K B, Mewes M O, Andrews M R, van Druten N J, Durfee D S, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969
Bradley C C, Sackett C A, Tollett J J and Hulet R G 1995 *Phys. Rev. Lett.* **75** 1687

[2] Parkins A S and Walls D F 1998 *Phys. Rep.* **303** 1
Dalfovo F, Giorgini S, Pitaevskii L and Stringari S 1999 *Rev. Mod. Phys* **71** 463

[3] Leggett A J 2001 *Rev. Mod. Phys.* **73** 307

[4] Hall D S, Matthews M R, Wieman C E and Cornell E A 1998 *Phys. Rev. Lett.* **81** 1543
Matthews M R, Anderson B P, Haljan P C, Hall D S, Holland M J, Williams J E, Wieman C E and Cornell E A 1999 *Phys. Rev. Lett.* **83** 3358
Stamper-Kurn D M, Andrews M R, Chikkatur A P, Inouye S, Miesner H J, Stenger J and Ketterle W 1998 *Phys. Rev. Lett.* **80** 2027

[5] Andrews M R, Townsend C G, Miesner H J, Durfee D S, Kurn D M and Ketterle W 1997 *Science* **275** 637

[6] Shin Y, Saba M, Pasquini T A, Ketterle W, Pritchard D E and Leanhardt A E 2004 *Phys. Rev. Lett.* **92** 050405

[7] Milburn G J, Corney J, Wright E M and Walls D F 1997 *Phys. Rev.* A **55** 4318

[8] Smerzi A, Fantoni S, Giovanazzi S and Shenoy S R 1997 *Phys. Rev. Lett.* **79** 4950

[9] Chen Z D, Liang J Q, Shen S Q and Xie W F 2004 *Phys. Rev.* A **69** 023611

[10] Sørensen A, Duan L M, Cirac J I and Zoller P 2001 *Nature* **409** 63

[11] Hines A P, McKenzie R H and Milburn G J 2003 *Phys. Rev.* A **67** 013609

[12] Sanz L, Angelo R M and Furuya K 2003 *J. Phys. A: Math. Gen.* **36** 9737

[13] Fuentes-Guridi I, Pachos J, Bose S, Vedral V and Choi S 2002 *Phys. Rev.* A **66** 022102

[14] Zhang W M, Feng D H and Gilmore R 1990 *Rev. Mod. Phys.* **62** 867

[15] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev.* A **53** 2046

[16] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[17] Lai Y Z, Liang J Q, Müller-Kirsten H J W and Zhou J G 1996 *Phys. Rev.* A **53** 3691