Vol 15 No 6, June 2006
1009-1963/2006/15(06)/1154-07

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Subwavelength rectangular cavity partially filled with left-handed materials*

Jiang Tian(姜 田), Chen Yan(陈 艳), and Feng Yi-Jun(冯一军)†

Department of Electronic Science and Engineering, Nanjing University, Nanjing 210093, *China*

(Received 9 January 2006; revised manuscript received 20 January 2006)

In this paper, we present the electromagnetic analysis of a rectangular cavity partially filled with a left-handed material slab. Our theoretical investigation shows that there exist novel resonant modes in the cavity, and such a cavity becomes a subwavelength cavity. The eigenvalue equation of the cavity is derived and the resonant frequencies of the novel modes are calculated by using numerical simulation. We also discuss the stability of the novel resonant modes and show the best condition under which a useful rectangular cavity of subwavelength dimensions with tolerable stability is obtained.

Keywords: left-handed material, rectangular cavity, subwavelength resonator, stability
PACC: 0350, 4110H, 7790

1. Introduction

In 1968, Veselago first studied the wave propagation in a hypothetical material with both a negative dielectric permittivity ε and a negative magnetic permeability μ.[1] Such materials are usually termed left-handed materials (LHMs), since the electric field, the magnetic field, and the wave vector of an electromagnetic wave propagating in such materials form a left-handed set. These materials exhibit some unique properties, which may lead to unconventional phenomena in guidance, radiation, and scattering of electromagnetic waves, such as backward wave, negative refraction,[2] inverse Doppler Effect,[3] and inverse Cerenkov radiation. Based on these phenomena a number of applications have been proposed: for example, Engheta has proposed an idea for a one-dimensional (1D) subwavelength cavity constructed by two parallel conducting planes filled with an LHM slab and a conventional dielectric slab, and has demonstrated theoretically that in such a 1D cavity a resonant mode can still exist even when the thicknesses of the two slabs are electrically very thin.[4] The stability and quality factor for the novel resonant mode have also been analysed, and it is shown that the resonant condition supporting such mode is unstable and gives zero resonant frequency in most cases.[5]

In this paper, we extend the analysis to a three-dimensional case. We theoretically investigate in detail a rectangular cavity partially filled with an LHM slab. We show that there exist resonant modes with resonant frequencies much lower than the cut-off frequency of the waveguide with the same cross section. Therefore the rectangular cavity resonant at these modes could have dimensions much less than the wavelength. We also analyse the stability of these resonant modes and their dependence on the material constitutive parameters and the geometric parameters of the cavity. Unlike the 1D case, we show that it is possible to construct a useful subwavelength rectangular cavity with tolerable stability.

2. Novel resonant modes

As shown in Fig.1, a rectangular cavity is partially filled with an LHM slab along the z-axis and the rest is filled with a conventional dielectric medium (denoted by RHM). The dimensions of the cavity along the x, y, and z-axes are denoted by a, b, and d, which satisfy the assumption $a \geq b$. The part I, which is filled with LHM, has a thickness of d_1 along the z-axis, and the part II, which is filled with RHM, has a thickness of

*Project supported by the State Key Development Program for Basic Research of China (Grant No 2004CB719800), the Doctoral Program Foundation of Institute of Higher Education of China (Grant No 20030284024).
†E-mail: yjfeng@nju.edu.cn

d_2. Their permittivity and permeability are denoted by ε_i and μ_i $(i = 1, 2)$ respectively. Subscripts 1 and 2 denote the quantities in part I and II, which are filled with the LHM slab and the conventional medium, respectively.

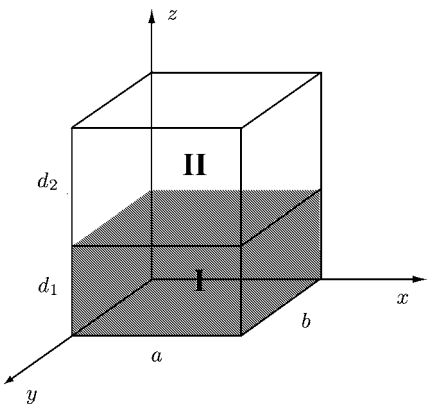


Fig.1. A rectangular cavity partially filled with a LHM slab.

From Maxwell's equations together with the boundary conditions for the field components, the cavity's eigenvalue equations of TE and TM modes can be derived as follows:[6]

$$\frac{\mu_1}{k_{z1}} \tan k_{z1} d_1 = -\frac{\mu_2}{k_{z2}} \tan k_{z2} d_1 \quad \text{(TE Mode)}, \quad (1)$$

$$\frac{k_{z1}}{\varepsilon_1} \tan k_{z1} d_1 = -\frac{k_{z2}}{\varepsilon_1} \tan k_{z2} d_2 \quad \text{(TM Mode)}, \quad (2)$$

where $k_{z1}^2 = \omega^2 \varepsilon_1 \mu_1 - k_{mn}^2$, and $k_{z2}^2 = \omega^2 \varepsilon_2 \mu_2 - k_{mn}^2$, which are the wave vectors along the z-direction in the two parts. And here $k_{mn}^2 = (m\pi/a)^2 + (n\pi/b)^2$ (for TE mode, $m, n = 0, 1, 2, \ldots$, except $m = n = 0$; and for TM mode, $m, n = 1, 2, 3, \ldots$). Equation (1) is very similar to the eigenvalue equation that Engheta considered in Ref.[4] except for the form of k.

Define $\omega_1^2 = k_{mn}^2/\varepsilon_1\mu_1$, and $\omega_2^2 = k_{mn}^2/\varepsilon_2\mu_2$, then the whole frequency range can be divided into three regions (Region A, B, and C, as shown in Fig.2) by ω_1 and ω_2. If we assume $\varepsilon_1\mu_1 > \varepsilon_2\mu_2$, then we have $\omega_1 < \omega_2$.

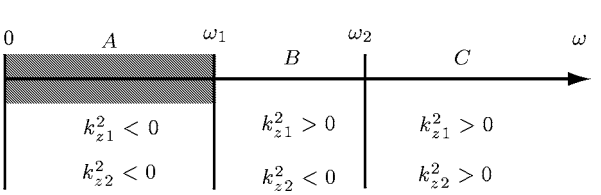


Fig.2. The frequency regions.

In the frequency region A, where $0 < \omega \leq \omega_1$, k_{z1} and k_{z2} are both imaginary, therefore the tangent functions in the eigenvalue equation, which include k_{zi}, are transformed to hyperbolic tangent functions. If the cavity is filled with two RHM media, which means $\varepsilon_1, \mu_1, \varepsilon_2, \mu_2 > 0$, the values on both sides of the eigenvalue equation have opposite signs, resulting in no solution to the eigenvalue equation. If the cavity is partially filled with an LHM medium with $\varepsilon_1, \mu_1 < 0$, the values on both sides of the eigenvalue equation have the same sign, therefore the eigenvalues probably exist. In Region B, where $\omega_1 < \omega \leq \omega_2$, k_{z1} is a real quantity while k_{z2} is an imaginary quantity. In Region C, where $\omega > \omega_2$, both k_{z1} and k_{z2} are real quantities, and the tangent functions in the eigenvalue equation remain unchanged. In the frequency regions B and C, the equation has solutions in spite of the sign of ε_1 and μ_1, and only the values of solutions are different. So the main difference between the cavities partially filled with an RHM or LHM medium is embodied within the frequency region A, where the partially LHM filled cavity has resonant modes, while the RHM filled cavity has no resonant modes. The novel resonant modes in the frequency region A could have resonant frequencies close to zero and their resonant wavelengths could be very large, which may lead to a subwavelength cavity. The wave vectors along the z-direction when the novel modes occur are imaginary quantities, and the field components of the dominant TE mode $(m = 1,\ n = 0)$ in Parts I and II can be written as follows:

$$\begin{aligned}
E_{y1} &= -\mathrm{j}\, k_x \sin k_x x \sinh(-\mathrm{j}\, k_{z1} z), \\
H_{x1} &= -\frac{k_x \beta_1}{\omega \mu_1} \sin k_x x \cosh(-\mathrm{j}\, k_{z1} z), \\
H_{z1} &= \mathrm{j}\, k_x \cos k_x x \sinh(-\mathrm{j}\, k_{z1} z), \\
E_{y2} &= -\mathrm{j}\, k_x A \sin k_x x \sinh(-\mathrm{j}\, k_{z2}(d - z)), \\
H_{x2} &= -\frac{k_x \beta_2}{\omega \mu_2} \sin k_x x \cosh(-\mathrm{j}\, k_{z2}(d - z)), \\
H_{z2} &= \mathrm{j}\, k_x A \cos k_x x \sinh(-\mathrm{j}\, k_{z2}(d - z)). \qquad (3)
\end{aligned}$$

From the field expressions, these novel resonant modes should have an evanescent field distribution along the z-axis on both sides of the interface of the two media. As examples, Figs.3 and 4 show the calculated electric and magnetic field distributions of the novel modes in such cavities. For convenience, all the field components are normalized to their maxima. It is explicit that the fields concentrate in the vicinity of the interface of two media and evanesce in the direction of leaving the interface. It is also shown that the

normal components of the EM fields on the two sides of the interface are anti-parallel.

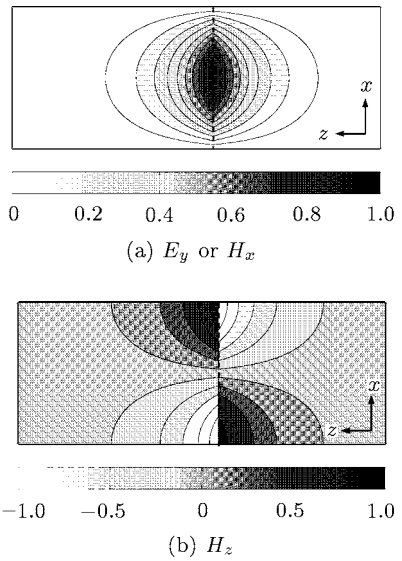


Fig.3. The evanescent field distribution of a TE mode in a cavity filled with LHM. The size of the cavity is = 1.5 cm, b = 1.5 cm, d = 3.9 cm. The dash line denotes the interface of the media, and the right part is LHM.

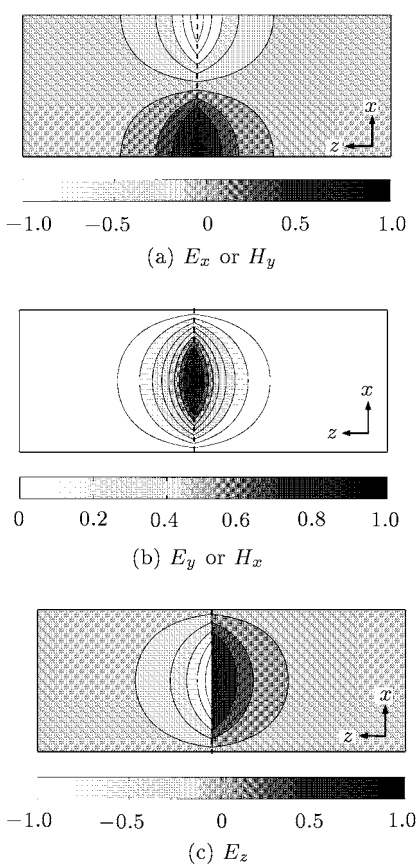


Fig.4. The evanescent field distribution of a TM mode in a cavity filled with LHM. The cavity has the same size as that in Fig.3 except for the location of the interface.

If $\varepsilon_1\mu_1 < \varepsilon_2\mu_2$, then $\omega_1 > \omega_2$. The lowest frequency region A will be separated by ω_2 while the other frequency regions B and C will be separated by ω_1 instead. Similar analysis shows that the novel resonant modes still exist in the frequency region A for a partially LHM filled cavity. To simplify the analysis, we assume $\varepsilon_1\mu_1 > \varepsilon_2\mu_2$ throughout the paper.

To perform analytical study on the novel resonant modes, we first introduce some normalized quantities for convenience, $\hat{\varepsilon} = -\varepsilon_1/\varepsilon_2$, $\hat{\mu} = -\mu_1/\mu_2$, $\alpha = d_2/d_1$, $\beta = d/\lambda_c$, $\lambda_c = 2\pi/k_{mn}$ and $\omega' = \omega/\omega_1$. Obviously, the normalized frequency $\omega' \leq 1$ is in the frequency region A. Substituting these normalized quantities into Eqs.(1) and (2), we obtain the following eigenvalue equations in Region A:

$$\frac{\hat{\mu}\tanh\left(\frac{2\pi\beta}{\alpha+1}\sqrt{1-\omega'^2}\right)}{\hat{n}\sqrt{1-\omega'^2}} = \frac{\tanh\left(\frac{2\pi\alpha\beta}{\hat{n}(\alpha+1)}\sqrt{\hat{n}^2-\omega'^2}\right)}{\sqrt{\hat{n}^2-\omega'^2}} \quad \text{(TE mode)}, \quad (4)$$

$$\frac{\tanh\left(\frac{2\pi\beta}{\alpha+1}\sqrt{1-\omega'^2}\right)}{\hat{\varepsilon}\sqrt{\hat{n}^2-\omega'^2}} = \frac{\tanh\left(\frac{2\pi\alpha\beta}{\hat{n}(\alpha+1)}\sqrt{\hat{n}^2-\omega'^2}\right)}{\hat{n}\sqrt{1-\omega'^2}} \quad \text{(TM mode)}, \quad (5)$$

where $\hat{n} = \sqrt{\hat{\varepsilon}\hat{\mu}}$.

Equations (4) and (5) are two transcendental equations, and the normalized resonant frequencies can be calculated by using numerical simulation. The numerical method used to solve the nonlinear equations in this paper is the Gauss-Newton method.[7,8] Equations (4) and (5) are solved numerically for the novel resonant modes in the cavity with different material parameters of the filling LHM and different geometric sizes of the cavity, which lead to different resonant frequencies. We find that only specified geometric sizes of the cavity could result in the novel TE or TM resonant modes for given material parameters $\hat{\varepsilon}$ and $\hat{\mu}$. Two examples are shown in Fig.5. Figures 5(a) and 5(b) show the regions of different geometric sizes of the cavity for two specified $\hat{\varepsilon}$ and $\hat{\mu}$ that support the novel TE or TM resonant modes. It is also shown in the figure that TE and TM modes do not occur simultaneously for a set of given material parameters and geometric sizes of the cavity. The value of α determines which mode will occur. If $\alpha > 1$, which means less than half of the cavity is filled by the LHM, TE

mode could occur; while if $\alpha < 1$, TM mode may occur. So by choosing the filling ratio, we can obtain the TE or TM resonant mode as required.

The occurrence of TE or TM mode is dependent on the geometric sizes of the cavity as well as the material parameters, therefore, as examples, in the following we choose $\hat{\varepsilon} = 1$, $\hat{\mu} = 2$ for TM mode analysis and $\hat{\varepsilon} = 2$, $\hat{\mu} = 1$ for TE mode analysis. In these cases the normalized resonant frequencies in Region A are calculated and shown in Fig.6. The low resonant frequencies occur in the vicinity of $\alpha = 1$ for either TE or TM modes, when the cavity is nearly half filled by the LHM. For the same filling ratio, the resonant frequency will increase as the dimension along the z-direction of the cavity decreasing.

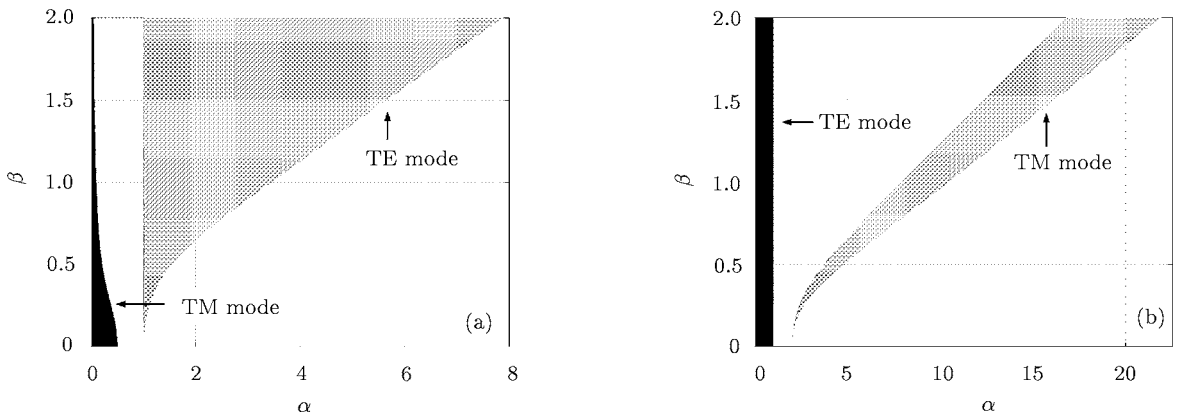


Fig.5. The regions of α (the filling ratio) and β in which the novel resonant mode exists for (a) $\hat{\mu} = 1$, $\hat{\varepsilon} = 2$, and (b) $\hat{\mu} = 2$, $\hat{\varepsilon} = 1$. The two separate regions belong to TE mode and TM mode. The region on the left is the region of TM mode. β changes from 0 to 2, and all the following simulations are in this interval of β.

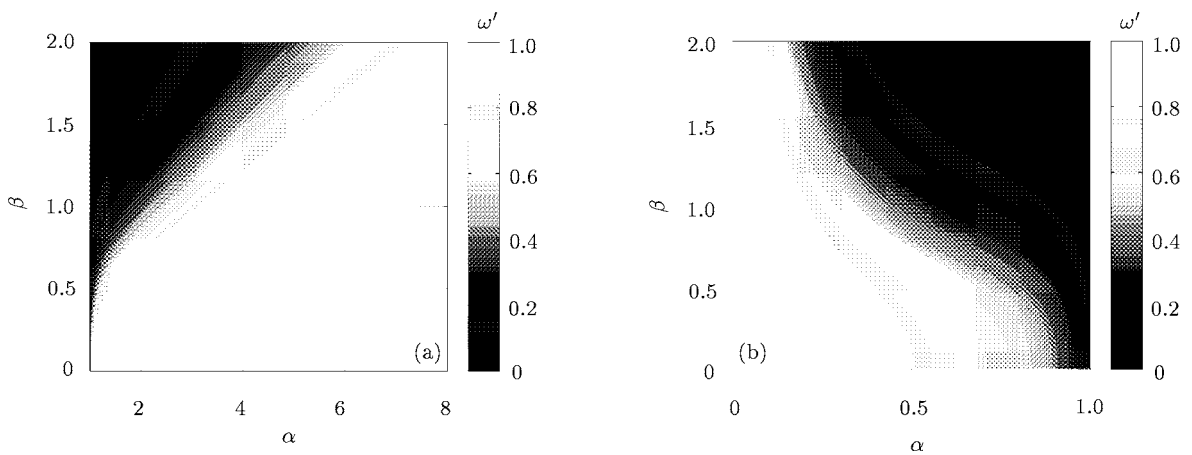


Fig.6. The simulation results of resonant frequency in Region A for (a) TE mode with $\hat{\mu} = 1$ and $\hat{\varepsilon} = 2$, (b) TM mode with $\hat{\mu} = 2$ and $\hat{\varepsilon} = 1$.

3. Subwavelength cavity

The above analysis conceptualizes an exciting possibility of designing a subwavelength rectangular cavity resonator. Subwavelength resonators can provide exciting future applications in design and miniaturization of compact passive devices and components in the microwave and optoelectronic circuits. For a rectangular cavity filled with a conventional medium, the largest resonant wavelength has the same order of magnitude as the dimensions of the cavity. But for the cavity filled with LHM, the eigenvalue in Region A is less than ω_1 with no lower limitation. Under certain conditions, the resonant frequencies could be very close to zero and the cavity can be electrically small. When the resonant frequency is low enough, the resonant wavelength will be much larger than the cavity's dimensions and the cavity becomes a subwavelength resonator. It is quite explicit that in some areas of Fig.6, the cavity becomes a subwavelength cavity. To

describe this phenomenon, we define a quantity

$$N \equiv \left| \frac{\lambda}{n_1 d} \right| = \frac{1}{\beta \omega'} , \tag{6}$$

where λ is the resonant wavelength in vacuum. We take $N = 10$ as the criterion of a subwavelength cavity in our analysis, which represents that the resonant wavelength in the LHM medium is 10 times larger than the cavity's dimension along the z-axis.

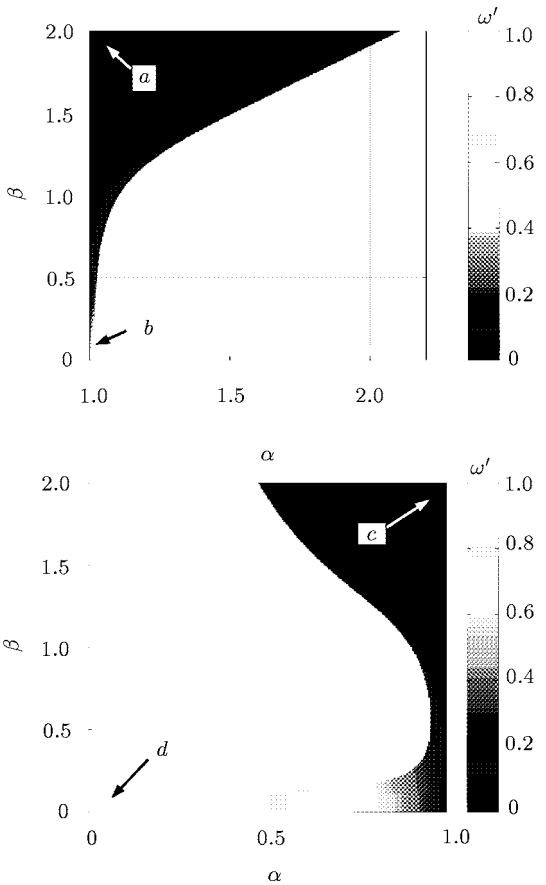


Fig.7. The resonant frequency of the subwavelength cavity in Region A for (a) TE mode, (b) TM mode. All the conditions are the same as those in Fig.6.

We re-calculate the filling ratio and the corresponding resonant frequency within the subwavelength limitation. In Fig.7, the grey colour zone represents the filling ratios that result in a subwavelength cavities and the corresponding resonant frequency. Two different types of subwavelength cavities are observed here, which are different from the 1D case analysed by others.[4−5] The first type is the cavity having a tiny resonant frequency ω (denoted by a and c in Fig.7), which corresponds to the cavity with a subwavelength dimensions of a, b, and d. The second type is a cavity whose resonant frequency is not very low, but has tiny β (denoted by b and d in Fig.7), which corresponds to the cavity with a subwavelength dimension of d, or with a subwavelength dimension along the z-axis. Both types result in subwavelength cavities which could not exist for rectangular cavities partially filled with conventional dielectrics.

4. Stability of the subwavelength cavity

The stability of the resonant mode to any parameter perturbation is very important for evaluating a resonator from a practical point of view. In one dimensional LHM filled cavity, it has been shown that the stability of the resonant condition supporting subwavelength mode depends strongly on the variation of filling ratio and the material parameters.[5] Therefore, in the three dimensional case, we have to study the stability of the subwavelength resonant modes to see whether or not we could construct a subwavelength resonator which has a tolerable stability.

Assume that a small perturbation of α, $\hat{\mu}$, or $\hat{\varepsilon}$ results in a shift of the resonant frequency, we denote the stability of the resonant mode by

$$\begin{aligned} s_\alpha &= \left| \frac{\mathrm{d}\omega'}{\mathrm{d}\alpha} \right| , \\ s_{\hat{\mu}} &= \left| \frac{\mathrm{d}\omega'}{\mathrm{d}\hat{\mu}} \right| , \\ s_{\hat{\varepsilon}} &= \left| \frac{\mathrm{d}\omega'}{\mathrm{d}\hat{\varepsilon}} \right| . \end{aligned} \tag{7}$$

Obviously, a smaller value of s represents a better stability. Equations (4) and (5) can be rewritten as $F(\omega, \alpha, \hat{\mu}, \hat{\varepsilon}) = 0$. From Eq.(7), one can easily obtain

$$\begin{aligned} s_x &= \left| \frac{\mathrm{d}\omega'}{\mathrm{d}x} \right| = \frac{|\mathrm{d}F/\mathrm{d}x|}{|\mathrm{d}F/\mathrm{d}\omega'|} , \\ &(x \text{ represents } \alpha,\ \hat{\mu} \text{ or } \hat{\varepsilon}) , \end{aligned} \tag{8}$$

The values of s calculated from Eq.(8) are plotted in Fig.8 for a particular subwavelength rectangular cavity partially filled with LHM. As shown in Figs. 8(a) and 8(b) and 8(c), for TE mode, the stability is strongly influenced by the permeability of the filling medium. The best stable area is located in the vicinity of the point $(\alpha, \beta) = (1, 0)$, but the stability with regard to filling ratio and permittivity becomes bad in that area. A compromise should be made between them. Comparing with Fig.7(a), the compactness of the cavity will be limited by this compromise, which is similar to the 1D case discussed in Ref.[5]. As shown in Figs. 8(d), 8(e) and 8(f), for TM mode, the stability is mainly influenced by the permittivity of the

filling media. The common stable area in the three figures is in the vicinity of $\beta = 0$ and $\alpha = 0$ to 0.5. Under that condition, the cavity has tiny electrical length along the z-direction. The resonant frequency is almost determined by ω_1. Comparing with the TE mode, the TM mode has better stability performance and is more practical. From the above discussion, we can see that it is possible to construct a subwavelength cavity with good stability.

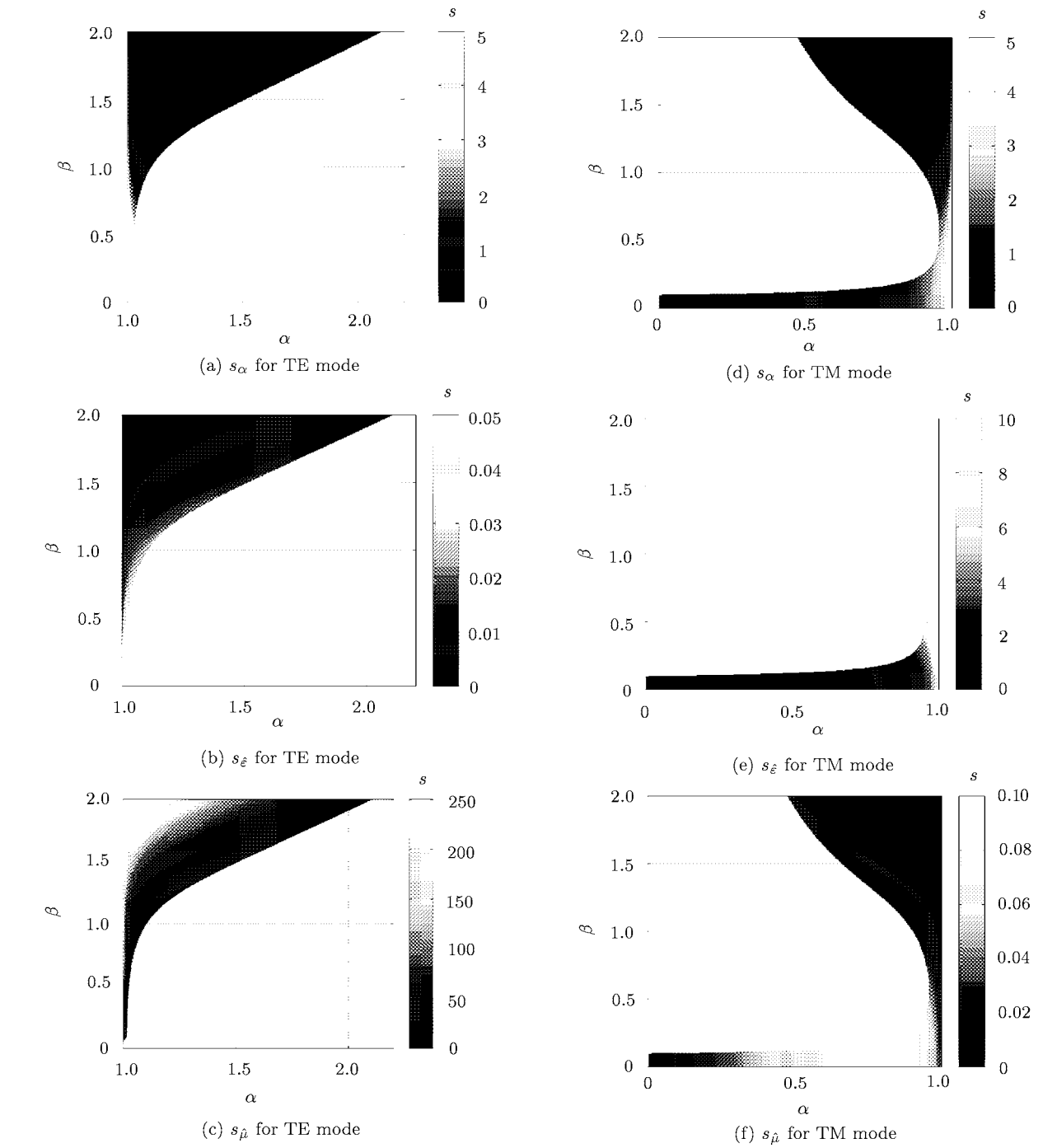


Fig.8. The stability calculated from Eq.(8). All the conditions are the same as those in Fig.6.

5. Conclusions

We have demonstrated by both theoretical analysis and numerical simulation that there exist novel resonant modes in a rectangular cavity partially filled with LHM which could lead to a subwavelength resonator. The field distribution in such a cavity is shown with an evanescent field distribution on the both sides of the interface of the media. Our analysis has shown that the filling ratio α determines whether the cavity is resonant at a TE or TM mode. The criterion for a subwavelength cavity has been described. Furthermore, considering the stability of the subwavelength cavity, we have found that a good stability with regard to all the parameters and high compactness can be achieved simultaneously for the TM mode, while the stability and the compactness will restrict each other for the TE mode. We can construct a subwavelength rectangular cavity having good stability with electrically small dimension along the z-direction for practical use. We believe that this kind of cavity can provide exciting future applications in design and miniaturization of compact devices and components in the microwave and optoelectronic circuits.

References

[1] Veselago V G 1968 *Sov. Phys. Usp.* **10** 509

[2] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77

[3] Seddon N and Bearpark T 2003 *Science* **302** 1537

[4] Engheta N 2002 *IEEE Antennas and Wireless Propagat. Lett.* **1** 10

[5] Shen L F, He S L and Xiao S S 2004 *Phys. Rev. B* **69** 115111

[6] Harrington R F 2001 *Time-Harmonic Electromagnetic Fields* (New York: Wiley/IEEE Press)

[7] Heath M T 2002 *Scientific Computing: An Introductory Survey, Second Edition* (New York: McGraw-Hill)

[8] Bertsekas D P 2000 *Nonlinear Programming:* Second Edition (Belmont: Athena Scientific)