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Head-on collision of ring dark solitons in Bose–Einstein condensates*

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The ring dark solitons and their head-on collisions in a Bose–Einstein condensates with thin disc-shaped potential are studied. It is shown that the system admits a solution with two concentric ring solitons, one moving inwards and the other moving outwards, which in small-amplitude limit, are described by the two cylindrical KdV equations in the respective reference frames. By using the extended Poincaré–Lighthill–Kuo perturbation method, the analytical phase shifts following the head-on collisions between two ring dark solitary waves are derived. It is shown that the phase shifts decrease with the radial coordinate r according to the $r^{-1/3}$ law and depend on the initial soliton amplitude and radius.

Keywords: ring dark solitons, Bose-Einstein condensates, head-on collision
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In recent years, experimental and theoretical investigations on nonlinear properties of solitons and vortices in Bose-Einstein condensates (BECs) have been received much interest. In a BEC, in addition to the atom-atom interactions, all atoms move in an external trap potential, i.e., the system is inhomogeneous, and the behaviours of different solitons are very diverse. Solitons in BECs have been observed by many experiments.[1−6] Theoretical studies on solitary waves in BECs have been traditionally discussed in terms of the classical Gross–Pitaevskii (GP) equation for a weakly correlated system. Based on the GP equation, most of investigations on soliton dynamics in BECs have been focused on one-dimensional (1D) solitons propagating in elongated cigar-shaped and quasi-1D periodic traps,[7−18] or plane-solitons (decaying only in one spatial direction and hence essentially 1D) in a 3D trap.[2,4,11,19] The head-on collision of two dark solitons in a cigar-shaped BEC was studied in Ref.[12] and a constant phase shift of each soliton due to the collision was obtained analytically. However, the actual stability of the theoretically predicted 1D solitary waves is an open question within the proper 3D environment of realistic traps. Experimental observation[11,20] and numerical calculation of the 3D GP equation[19] show that a soliton initially created in a finite trap is unstable and its amplitude, width, and shape vary as it propagates in the trap and eventually decays into vortex rings. In a recent theoretical work,[21] the 2D soliton waves created in a disc-shaped BEC[20] have been considered by using a multiple-scales perturbation method. The results show that the propagation of lump-like 2D solitons are possible in the disc-shaped BEC. However, these lump-like solitons are obtained in the 2D Cartesian coordinates, i.e. in an unbounded planar geometry. The numerical calculation of solitary waves propagating in a cigar-shaped cylindrical BEC shows that the relevant solitary waves are found to be hybrids of quasi-1D solitons and 3D vortex rings.[22,23]

It is well known that, in higher dimensions, dark stripe solitons can form and become unstable against transverse snaking.[11] However, the instability of the dark stripe solitons, characterized by a maximum perturbation wave number $Q_{\max}$, may be suppressed by bending a dark stripe to close it into an annulus of length $L < 2\pi/Q_{\max}$.[24] On the other hand, the soliton is expected to have a circular symmetry and the

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stable ring dark soliton can be observed. In recent work,[25,26] the concept of ring dark solitons in BEC was first introduced and the ring dark solitons in a cylindrical BEC with disc-shaped trap was studied, and the existence of both oscillatory and stationary solitons was predicted. The numerical results showed that instabilities gradually set in and, as a result, shallow ring dark solitons decay slowly, while the deeper ones develop the snake instability. It is the aim of the present paper to discuss the propagation and head-on collision of two ring dark solitons in a bounded cylindrical BEC with disc-shaped trap. By using a multiple-scales perturbation method, we show that, asymptotically, the ring dark solitons in the BEC evolve according to the cylindrical Korteweg–de Vries (KdV) equation and the amplitude of the solitons changes with the radial coordinate r. Then we pay attention to the head-on collision between two solitons initially travelling inwards and outwards separately. The phase shifts following the head-on collision are obtained analytically and shown that, asymptotically, they decrease with the radial coordinate r according to the $r^{-1/3}$ law, and depend on the initial soliton configuration (amplitudes, initial position, etc.).

We consider the axially symmetric solitary waves travelling along the radial direction in a BEC with disc-shaped trap. The evolution of the weakly coupled BEC at low temperatures is governed by the time-dependent cylindrical Gross–Pitaevskii (GP) equation with external trapping potential $V(r)$,

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = \left[-\frac{1}{2}\nabla^2 + V(r) + Q|\Psi|^2 \right] \Psi, \qquad (1)$$

and the disc-shaped trap has the form

$$V(r) = \frac{1}{2}\left(\frac{\omega_r}{\omega_z}\right)^2 r^2 + \frac{1}{2}z^2, \qquad (2)$$

where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$. ω_r and ω_z are frequencies of the trap in the radial r and axial z directions, respectively. Wave function Ψ, time t, and variables (r, z) are normalized by $(N/a_z^3)^{1/2}$, ω_z^{-1}, and a_z, respectively. Here $a_z = [\hbar/(m\omega_z)]^{1/2}, Q = 4\pi N a_s/a_z$ with a_s being the s-wave scattering length; m is the mass of the atom and N the number of atoms in BEC.

We consider the excitations created in the BEC with very thin disc-shaped trap: i.e., the case in which the trapping potential in the r direction is much weaker than that in the z direction, mathematically, $\omega_r/\omega_z << 1$. This means that the motion of atoms in the z direction is essentially frozen and is governed by the ground-state wave function of the corresponding harmonic oscillator. Hence, the excitations can propagate only in r direction. According to the above assumption, we have[20]

$$\Psi(r, z, t) = G_0(z)A(r, t)\exp[-\mathrm{i}\mu t + \mathrm{i}\phi(r, t)], \qquad (3)$$

where $G_0(z) = \exp(-z^2/2)$ is the ground-state wave function of the 1D harmonic oscillator with the potential $z^2/2$ in the z direction, μ is the chemical potential of the condensate and ϕ is a phase function related to the excitation. In order to get insight of the ring dark solitons and their head-on collisions analytically, we can neglect the slowly varying radial trapping potential because ω_r/ω_z is very small. Then, substituting Eq.(3) into Eq.(1), we obtain

$$\frac{\partial A}{\partial t} + \frac{\partial A}{\partial r}\frac{\partial \phi}{\partial r} + \frac{A}{2}\left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r}\right) = 0, \qquad (4)$$

$$-\frac{1}{2}\left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) - \left(\mu - \frac{1}{2}\right)A + \left[\frac{\partial \phi}{\partial t} + \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2\right]A + Q'A^3 = 0, \qquad (5)$$

where $Q' = I_0 Q$ is an effective interaction constant with $I_0 = \int_{-\infty}^{\infty} \mathrm{d}z G_0^4(z) / \int_{-\infty}^{\infty} \mathrm{d}z G_0^2(z) = 1/\sqrt{2}$. Because the contribution of the higher-order eigenmodes of the harmonic oscillator in the z direction is very small and can be safely neglected, so, in deducing Eq.(5), we have multiplied Eq.(1) by $G_0^*(z)$ and then integrated once with respect to z to eliminate the dependence on z.[7,21]

In order to obtain the nonlinear evolution of the ring dark solitons, we introduce the multiple-scale variables $\xi = \epsilon(r - ct)$ and $R = \epsilon^3 r$, and the following scalings:

$$A = u_0 + \epsilon^2(a_0 + \epsilon^2 a_1 + \cdots), \qquad (6)$$

$$\phi = \epsilon(\phi_0 + \epsilon^2 \phi_1 + \cdots), \qquad (7)$$

where ϵ is a parameter characterizing the relative amplitude of the excitation. For small ϵ, the excitation of ring dark soliton is shallow and the shallow dark ring soliton is long-lived object.[25] For larger ϵ, however, the deep ring dark soliton will not only emit strong radiation fields composed of many secondary ring solitons,[27] but also will be subjected to snake instability and eventually will split into vortexes,[25] so the study of ring dark solitons collision is meaningless. When $\epsilon \to 0$, Eqs.(6) and (7) indicate that the

condensate background, i.e. without perturbation, is recovered.

Then, substituting the above expansions into Eqs.(4) and (5) and collecting the terms in the different powers of ϵ, we can obtain several nth-order reduced equations. To the leading order, we have

$$-c\frac{\partial a_0}{\partial \xi}+\frac{1}{2}u_0\frac{\partial^2\phi_0}{\partial\xi^2}=0, \tag{8}$$

$$-2Q'u_0^2a_0+cu_0\frac{\partial\phi_0}{\partial\xi}=0, \tag{9}$$

which results in

$$a_0=\frac{u_0}{2c}\frac{\partial\phi_0}{\partial\xi}; \tag{10}$$

and $c^2=Q'u_0^2=\mu-1/2$ is also obtained from the solvability condition. For the next order, we have

$$-c\frac{\partial a_1}{\partial\xi}+\frac{1}{2}u_0\frac{\partial^2\phi_1}{\partial\xi^2}=-\frac{3c}{u_0}a_0\frac{\partial a_0}{\partial\xi}-2c\frac{\partial a_0}{\partial R}-\frac{c}{R}a_0, \tag{11}$$

$$-2c^2a_1+cu_0\frac{\partial\phi_1}{\partial\xi}=-\frac{1}{2}\frac{\partial^2a_0}{\partial\xi^2}+\frac{3c^2}{u_0}a_0^2. \tag{12}$$

In deducing Eqs.(11) and (12), Eq.(10) has been used. The solvability condition for Eqs.(11) and (12) reads

$$\frac{\partial a_0}{\partial R}+\alpha a_0\frac{\partial a_0}{\partial\xi}-\beta\frac{\partial^3a_0}{\partial\xi^3}+\frac{1}{2R}a_0=0, \tag{13}$$

where $\alpha=3/u_0,\beta=1/(8c^2)$. Equation(13) is the cylindrical KdV equation describing the small-amplitude ring dark solitons in the BEC.

Now we consider the soliton solution to Eq.(13). Because we are interested in the asymptotic properties of the collision process as discussed below, we use the asymptotic solutions of Eq.(13), rather than their exact solutions, which involve the Airy function.[28] The quasi-solitary wave solution of Eq.(13) for larger R is[29]

$$a_0=-M\left(\frac{R_0}{R}\right)^{2/3}\mathrm{sech}^2\left\{\left(\frac{\alpha M}{12\beta}\right)^{1/2}\left(\frac{R_0}{R}\right)^{1/3}\times\left[\xi+\frac{\alpha M}{3}\left(\frac{R_0}{R}\right)^{2/3}R\right]\right\}, \tag{14}$$

where $R_0=\epsilon^3r_0$, M is the amplitude of the cylindrical solitary wave with initial radius $r=r_0$. It is clear that Eq.(14) describes a ring dark soliton and its amplitude varies according to $R^{-2/3}$ law. That is, the solitons shape, amplitude, and phase are varying when the solitons propagate in BEC, which is confirmed by the experiments mentioned before.

Now we turn to discuss the head-on collision of two ring dark solitons. We assume that in the BEC, the two solitons, A and B, initially have radius $r=r_A$ and $r=r_B<r_A$. The two solitons are assumed to be far apart in the initial state and the soliton $A(B)$ is travelling inwards (outwards) with respect to BEC. After some time they interact, collide, and then depart from each other. We expect that the collision will be quasi-elastic, so that it will only cause shifts of the post-collision trajectories (phase shift). In order to analyse the effect of collision, we employ an extended Poincaré–Lighthill–Kuo (PLK) perturbation method,[30,31] according to which we introduce the stretched coordinates,

$$\xi=\epsilon(r-ct-r_A)+\epsilon^2P_0(\eta,R)+\epsilon^4P_1(\xi,\eta,R)+\cdots, \tag{15}$$

$$\eta=\epsilon(r+ct-r_b)+\epsilon^2Q_0(\xi,R)+\epsilon^4Q_1(\xi,\eta,R)+\cdots, \tag{16}$$

$$R=\epsilon^3r, \tag{17}$$

where ξ and η denote the trajectories of the two solitons travelling inwards and outwards separately. The wave velocity c and the variables P_j and Q_j are to be determined; ϵ is a small and ordering parameter for the series expansion. From Eqs.(15)–(17) we have

$$\frac{\partial}{\partial r}=\epsilon\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+\epsilon^3\left(\frac{\partial}{\partial R}+P_{0\eta}\frac{\partial}{\partial\xi}+Q_{0\xi}\frac{\partial}{\partial\eta}\right)+\cdots, \tag{18}$$

$$\frac{\partial}{\partial t}=\epsilon c\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+\epsilon^3c\left(P_{0\eta}\frac{\partial}{\partial\xi}-Q_{0\xi}\frac{\partial}{\partial\eta}\right)+\cdots \tag{19}$$

Introducing the asymptotic expansion

$$A=u_0+\epsilon^2(a_0+\epsilon^2a_1+\cdots), \tag{20}$$

$$\phi=\epsilon(\phi_0+\epsilon^2\phi_1+\cdots), \tag{21}$$

then, substituting Eqs.(18)–(21) into Eqs.(4) and (5) and equating the same powers of ϵ, we obtain a hierarchy of coupled equations in different order of ϵ. To the leading order, we have

$$c\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)a_0+\frac{1}{2}u_0\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)^2\phi_0=0, \tag{22}$$

$$2Q'u_0a_0+c\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_0=0. \tag{23}$$

The solutions of Eqs.(22) and (23) read

$$a_0=f_1(\xi,R)+f_2(\eta,R), \tag{24}$$

$$\phi_0=\frac{2c}{u_0}\left[\int_{\xi|_{t=0}}^{\xi}f_1(\xi,R)\mathrm{d}\xi-\int_{\eta|_{t=0}}^{\eta}f_2(\eta,R)\mathrm{d}\eta\right], \tag{25}$$

and under the solvability condition, the wave velocity $c^2 = Q'u_0^2$ is also obtained. The unknown functions of f_1 and f_2 will be determined using the higher-order equations. Equations (24) and (25) imply that, for the leading order of ϵ, we have two waves, one of which, $f_1(\xi, R)$, travels outwards, and the other, $f_2(\eta, R)$, inwards.

For the next higher orders of ϵ, we can deduce

$$\begin{aligned}\frac{u_0}{c}\frac{\partial^2\phi_1}{\partial\xi\partial\eta} =& -\left(\frac{\partial f_1}{\partial R} + \alpha f_1\frac{\partial f_1}{\partial\xi} - \beta\frac{\partial^3 f_1}{\partial\xi^3} + \frac{1}{2R}f_1\right)\\ &+\left(\frac{\partial f_2}{\partial R} + \alpha f_2\frac{\partial f_2}{\partial\eta} - \beta\frac{\partial^3 f_2}{\partial\eta^3} + \frac{1}{2R}f_2\right)\\ &-2\left(\frac{\partial P_0}{\partial\eta} - \frac{1}{2u_0}f_2\right)\frac{\partial f_1}{\partial\xi}\\ &+2\left(\frac{\partial Q_0}{\partial\xi} - \frac{1}{2u_0}f_1\right)\frac{\partial f_2}{\partial\eta}, \end{aligned} \tag{26}$$

where α and β are the same as defined earlier.

When Eq.(26) is integrated about ξ and η on both sides, the first (second) term on the right-hand side of Eq.(26) will be proportional to $\eta(\xi)$ because the integrated function is independent of $\eta(\xi)$. Thus the first two terms of Eq.(26) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence we have

$$\frac{\partial f_1}{\partial R} + \alpha f_1\frac{\partial f_1}{\partial\xi} - \beta\frac{\partial^3 f_1}{\partial\xi^3} + \frac{1}{2R}f_1 = 0, \tag{27}$$

$$\frac{\partial f_2}{\partial R} + \alpha f_2\frac{\partial f_2}{\partial\eta} - \beta\frac{\partial^3 f_2}{\partial\eta^3} + \frac{1}{2R}f_2 = 0. \tag{28}$$

The third and fourth terms in Eq.(26) are not secular terms in this order, but they will become secular in the next order.[30,31] Hence we have

$$\frac{\partial P_0}{\partial\eta} = \frac{1}{2u_0}f_2, \tag{29}$$

$$\frac{\partial Q_0}{\partial\xi} = \frac{1}{2u_0}f_1. \tag{30}$$

Equations (27) and (28) are inward and outward travelling wave KdV equations in the reference frames of ξ and η, respectively. As discussed in previous section, their corresponding asymptotic solutions for large R are

$$\begin{aligned} f_1 =& -M_A\left(\frac{R_A}{R}\right)^{2/3}\mathrm{sech}^2\left\{\left(\frac{\alpha M_A}{12\beta}\right)^{1/2}\left(\frac{R_A}{R}\right)^{1/3}\right.\\ &\left.\times\left[\xi + \frac{\alpha M_A}{3}\left(\frac{R_A}{R}\right)^{2/3}R\right]\right\}, \end{aligned} \tag{31}$$

$$\begin{aligned} f_2 =& -M_B\left(\frac{R_B}{R}\right)^{2/3}\mathrm{sech}^2\left\{\left(\frac{\alpha M_B}{12\beta}\right)^{1/2}\left(\frac{R_B}{R}\right)^{1/3}\right.\\ &\left.\times\left[\eta + \frac{\alpha M_B}{3}\left(\frac{R_B}{R}\right)^{2/3}R\right]\right\}, \end{aligned} \tag{32}$$

where $R_A = \epsilon^3 r_A$ and $R_B = \epsilon^3 r_B$, $M_A(M_B)$ is the amplitude of soliton $A(B)$ with initial radius $r = r_A (r = r_B < r_A)$.

The leading phase changes due to the collision can be calculated from Eqs.(29) and (30):

$$\begin{aligned} P_0 =& \frac{1}{2u_0}\left(\frac{12\beta M_B}{\alpha}\right)^{1/2}\left(\frac{R_B}{R}\right)^{1/3}\left\{\tanh\left\{\left(\frac{\alpha M_B}{12\beta}\right)^{1/2}\left(\frac{R_B}{R}\right)^{1/3}\left[\eta + \frac{\alpha M_B}{3}\left(\frac{R_B}{R}\right)^{2/3}R\right]\right\}\right.\\ &\left.-\tanh\left\{\left(\frac{\alpha M_B}{12\beta}\right)^{1/2}\left(\frac{R_B}{R}\right)^{1/3}\left[\eta|_{t=0} + \frac{\alpha M_B}{3}\left(\frac{R_B}{R}\right)^{2/3}R\right]\right\}\right\}, \end{aligned} \tag{33}$$

$$\begin{aligned} Q_0 =& \frac{1}{2u_0}\left(\frac{12\beta M_A}{\alpha}\right)^{1/2}\left(\frac{R_A}{R}\right)^{1/3}\left\{\tanh\left\{\left(\frac{\alpha M_A}{12\beta}\right)^{1/2}\left(\frac{R_A}{R}\right)^{1/3}\left[\xi + \frac{\alpha M_A}{3}\left(\frac{R_A}{R}\right)^{2/3}R\right]\right\}\right.\\ &\left.-\tanh\left\{\left(\frac{\alpha M_A}{12\beta}\right)^{1/2}\left(\frac{R_A}{R}\right)^{1/3}\left[\xi|_{t=0} + \frac{\alpha M_A}{3}\left(\frac{R_A}{R}\right)^{2/3}R\right]\right\}\right\}, \end{aligned} \tag{34}$$

where $\xi|_{t=0} = -\eta|_{t=0} = \epsilon(r_A - r_B)$. Using Eqs.(15), (16) and (33), (34) we can estimate the corresponding phase shifts Δ_A and Δ_B as follows:

$$\begin{aligned} \Delta_A =& -\epsilon^2\frac{1}{2u_0}\left(\frac{12\beta M_B}{\alpha}\right)^{1/2}\left(\frac{r_B}{r}\right)^{1/3}\left\{\tanh\left\{\left(\frac{\alpha M_B}{12\beta}\right)^{1/2}\left(\frac{r_B}{r}\right)^{1/3}\left[\epsilon(-2ct + r_A - r_B) + \epsilon^3\frac{\alpha M_B}{3}\left(\frac{r_B}{r}\right)^{2/3}r\right]\right\}\right.\\ &\left.-\tanh\left\{\left(\frac{\alpha M_B}{12\beta}\right)^{1/2}\left(\frac{r_B}{r}\right)^{1/3}\left[\epsilon(r_A - r_B) + \epsilon^3\frac{\alpha M_B}{3}\left(\frac{r_B}{r}\right)^{2/3}r\right]\right\}\right\}, \end{aligned} \tag{35}$$

$$\begin{aligned} \Delta_B =& -\epsilon^2\frac{1}{2u_0}\left(\frac{12\beta M_A}{\alpha}\right)^{1/2}\left(\frac{r_A}{r}\right)^{1/3}\left\{\tanh\left\{\left(\frac{\alpha M_A}{12\beta}\right)^{1/2}\left(\frac{r_A}{r}\right)^{1/3}\left[\epsilon(2ct + r_B - r_A) + \epsilon^3\frac{\alpha M_A}{3}\left(\frac{r_A}{r}\right)^{2/3}r\right]\right\}\right.\\ &\left.-\tanh\left\{\left(\frac{\alpha M_A}{12\beta}\right)^{1/2}\left(\frac{r_A}{r}\right)^{1/3}\left[\epsilon(r_B - r_A) + \epsilon^3\frac{\alpha M_A}{3}\left(\frac{r_A}{r}\right)^{2/3}r\right]\right\}\right\}, \end{aligned} \tag{36}$$

when returning to the original variables. If the initial separation between the two solitons is large enough, i.e. $r_A - r_B >> 1$, and the observation time $t >> t_c = (r_A - r_B)/2$ (t_c is the collision time), from Eqs.(35) and (36) we obtain

$$\Delta_A = \epsilon^2 \frac{1}{u_0} \left(\frac{12\beta M_B}{\alpha} \right)^{1/2} \left(\frac{r_B}{r} \right)^{1/3}, \qquad (37)$$

$$\Delta_B = -\epsilon^2 \frac{1}{u_0} \left(\frac{12\beta M_A}{\alpha} \right)^{1/2} \left(\frac{r_A}{r} \right)^{1/3}. \qquad (38)$$

Equation (37) and (38) indicate that the phase shifts of the two ring solitons due to the head-on collisions are not constant but proportional to $r^{-1/3}$ and depend on their initial radii and amplitudes.

In summary, the propagation of ring dark solitons and their head-on collisions in a BEC with thin disc-shaped potential has been studied by applying an extended PLK perturbation method to the GP equation, which governs the evolution of ring dark solitons in BEC. It is shown that the GP equation indeed admits a solution with two concentric ring solitons, one moving inwards and the other moving outwards, which in small-amplitude limit, can be described by the two cylindrical KdV equations in the respective reference frames. The reduction to the cylindrical KdV equation may be useful to understand the ring dark soliton dynamics. The analytical leading-order phase shifts due to head-on collision between the two ring dark solitary waves in a BEC are derived. It is demonstrated that, approximately, the shifts with the radial coordinate r vary according to the $r^{-1/3}$ law and depend on the initial soliton configuration (amplitude, radius, etc.). Higher-order corrections, not considered here, may give some secondary structures in the collision event. This work is in progress.

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