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New exact solitary wave solutions to generalized mKdV equation and generalized Zakharov–Kuzentsov equation*

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In this paper, based on hyperbolic tanh-function method and homogeneous balance method, and auxiliary equation method, some new exact solitary solutions to the generalized mKdV equation and generalized Zakharov–Kuzentsov equation are constructed by the method of auxiliary equation with function transformation with aid of symbolic computation system Mathematica. The method is of important significance in seeking new exact solutions to the evolution equation with arbitrary nonlinear term.

Keywords: generalized mKdV equation, generalized Zakharov–Kuzentsov equation, nonlinear evolution equation, auxiliary equation, exact solitary wave solutions

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1. Introduction

In soliton theory, some direct methods are proposed by use of the computer algebraic systems, such as, hyperbolic tanh-function method, homogeneous balance method, Jacobian elliptic function expansion method, auxiliary equation method and so on.[1−11] In Refs.[12] and [13], some new exact solutions to the generalized mKdV equation (GmKdV) and generalized Zakharov–Kuzentsov equation (GZK) are obtained by the use of the trial function method and projective Riccati equation method. The generalized mKdV equation contains the classical KdV equation and mKdV equation in a solitary phenomenon, and the generalized Zakharov–Kuzentsov equation contains the ZK equation and mZK equation. Hence it is of important significance in discussing the exact solitary wave solutions to the GmKdV equation and GZK equation in theory and application. In this paper, an auxiliary equation and a function transformation are introduced to obtain the new exact solitary wave solutions to the GmKdV equation and GZK equation with the help of symbolic computation system Mathematica.

2. Exact solitary wave solutions to generalized mKdV equation

The generalized mKdV equation is expressed as

$$u_t + \alpha u^\gamma u_x + \beta u_{xxx} = 0, \tag{1}$$

where α, β, γ are constants.

Substituting

$$u(x,t) = u(\xi), \quad \xi = x + \omega t,$$

with constant ω to be determined, into Eq.(1), then integrating once with respect to ξ and setting the integration constant to zero, we obtain the following ordinary differential equation:

$$\omega u(\xi) + \frac{\alpha}{1+\gamma} u^{1+\gamma}(\xi) + \beta u''(\xi) = 0. \tag{2}$$

In Eq.(2), we assume $u(\xi)$ to be in the following form:

$$u(\xi) = [v(\xi)]^{1/\gamma}. \tag{3}$$

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Substituting Eq.(3) into Eq.(2), we obtain

$$\gamma^2(1+\gamma)\omega v^2(\xi)+\gamma^2\alpha v^3(\xi)-\beta(-1+\gamma^2)(v')^2 + \beta\gamma(1+\gamma)v(\xi)v''(\xi)=0. \quad (4)$$

In Eq.(4), we assume $v(\xi)$ to be in the following form:

$$v(\xi)=g_0+g_1z(\xi)+g_2z^2(\xi), \quad (5)$$

where $z(\xi)$ is determined by the following auxiliary equation:

$$\Big(\frac{\mathrm{d}\,z(\xi)}{\mathrm{d}\,\xi}\Big)^2=cz^2(\xi)+bz^4(\xi)+az^6(\xi). \quad (6)$$

We obtain the following solutions of Eq.(6):

$$z(\xi)=\Big(\frac{c(-1+(\coth 2\sqrt{c}\xi\pm\mathrm{csch}2\sqrt{c}\xi)^2)}{b+2\sqrt{ac}(\coth 2\sqrt{c}\xi\pm\mathrm{csch}2\sqrt{c}\xi)}\Big)^{1/2}; \quad (7)$$

$$z(\xi)=2\Big(\frac{c(\cosh 2\sqrt{c}\xi+\sinh 2\sqrt{c}\xi)}{4ac-(b+\cosh 2\sqrt{c}\xi+\sinh 2\sqrt{c}\xi)^2}\Big)^{1/2}; \quad (8)$$

$$z(\xi)=\Big(\frac{2c\,\mathrm{sech}\,2\sqrt{c}\xi}{\sqrt{b^2-4ac}-b\,\mathrm{sech}\,2\sqrt{c}\xi}\Big)^{1/2}; \quad (9)$$

$$z(\xi)=\Big(\frac{-bc\,\mathrm{sech}^2\sqrt{c}\xi}{b^2-ac(1-\tanh\sqrt{c}\xi)^2}\Big)^{1/2}; \quad (10)$$

$$z(\xi)=2\{2c^2\,\mathrm{sech}\,2\sqrt{c}\xi/[b^2+4c(c-a)-4bc\,\mathrm{sech}\,2\sqrt{c}\xi + (b^2-4c(a+c))\tanh 2\sqrt{c}\xi]\}^{1/2}; \quad (11)$$

$$z(\xi)=\Big(\frac{c(-1+(\tanh 2\sqrt{c}\xi\pm \mathrm{i}\,\mathrm{sech}\,2\sqrt{c}\xi)^2)}{b+2\sqrt{ac}(\tanh 2\sqrt{c}\xi\pm \mathrm{i}\,\mathrm{sech}\,2\sqrt{c}\xi)}\Big)^{1/2}; \quad (12)$$

$$z(\xi)=\Big(\frac{c\,\mathrm{csch}\,\frac{1}{2}\sqrt{c}\xi\,\mathrm{sech}\,\frac{1}{2}\sqrt{c}\xi}{4\sqrt{ac}\cosh\sqrt{c}\xi+2b\sinh\sqrt{c}\xi}\Big)^{1/2}; \quad (13)$$

$$z(\xi)=\Big(\frac{-c+(\frac{\sqrt{c(P^2+R^2)}-\sqrt{c}P\cosh 2\sqrt{c}\xi}{R+P\sinh 2\sqrt{c}\xi})^2}{b-\frac{2\sqrt{a}(\sqrt{c(P^2+R^2)}-\sqrt{c}P\cosh 2\sqrt{c}\xi)}{R+P\sinh 2\sqrt{c}\xi}}\Big)^{1/2}; \quad (14)$$

$$z(\xi)=\Big(\frac{-c\,\mathrm{sech}\,\frac{1}{2}\sqrt{c}\xi}{b\cosh\sqrt{c}\xi+2\sqrt{ac}\sinh\sqrt{c}\xi}\Big)^{1/2}; \quad (15)$$

$$z(\xi)=\Big(\frac{-c+(\frac{\sqrt{c(-P^2+R^2)}+\sqrt{c}P\sinh 2\sqrt{c}\xi}{R+P\cosh 2\sqrt{c}\xi})^2}{b+\frac{2\sqrt{a}(\sqrt{c(-P^2+R^2)}+\sqrt{c}P\sinh 2\sqrt{c}\xi)}{R+P\cosh 2\sqrt{c}\xi}}\Big)^{1/2}; \quad (16)$$

$$z(\xi)=\Big(\frac{c(-1+(\frac{c\cosh 2\sqrt{c}\xi}{\sqrt{c}\sinh 2\sqrt{c}\xi\pm \mathrm{i}\sqrt{c}})^2)}{b+\frac{2c\sqrt{a}\cosh 2\sqrt{c}\xi}{\sqrt{c}\sinh 2\sqrt{c}\xi\pm i\sqrt{c}}}\Big)^{1/2}; \quad (17)$$

$$z(\xi)=\Big(\frac{c(-1+(\frac{c\sinh 2\sqrt{c}\xi}{\sqrt{c}\cosh 2\sqrt{c}\xi\pm\sqrt{c}})^2)}{b+\frac{2c\sqrt{a}\sinh 2\sqrt{c}\xi}{\sqrt{c}\cosh 2\sqrt{c}\xi\pm\sqrt{c}}}\Big)^{1/2}; \quad (18)$$

where $a>0, c>0$, $R^2-P^2>0$, R, and P are arbitrary constants and $R\neq 0$, $P\neq 0$.

$$z(\xi)=\Big(\frac{c\sec^2\sqrt{-c}\xi}{-b+2\sqrt{-ac}\tan\sqrt{-c}\xi}\Big)^{1/2}; \quad (19)$$

$$z(\xi)=\Big(\frac{c\,\mathrm{csc}^2\sqrt{-c}\xi}{b+2\sqrt{-ac}\cot\sqrt{-c}\xi}\Big)^{1/2}; \quad (20)$$

$$z(\xi)=\Big(\frac{-c(1+(\tan 2\sqrt{-c}\xi\pm\sec 2\sqrt{-c}\xi)^2)}{b-2\sqrt{-ac}(\tan 2\sqrt{-c}\xi\pm\sec 2\sqrt{-c}\xi)}\Big)^{1/2}; \quad (21)$$

$$z(\xi)=\Big(\frac{c\,\mathrm{csc}\,\frac{1}{2}\sqrt{-c}\xi\sec\frac{1}{2}\sqrt{-c}\xi}{-4\sqrt{-ac}\cos\sqrt{-c}\xi-2b\sin\sqrt{-c}\xi}\Big)^{1/2}; \quad (22)$$

$$z(\xi)=\Big(\frac{-c+(\frac{\pm\sqrt{c(-P^2+R^2)}-\sqrt{-c}P\cos 2\sqrt{-c}\xi}{R+P\sin 2\sqrt{-c}\xi})^2}{b-\frac{2\sqrt{a}(\pm\sqrt{c(-P^2+R^2)}-\sqrt{-c}P\cos 2\sqrt{-c}\xi)}{R+P\sin 2\sqrt{-c}\xi}}\Big)^{1/2}; \quad (23)$$

$$z(\xi)=\Big(\frac{c\sec\sqrt{-c}\xi}{-b\cos\sqrt{-c}\xi+2\sqrt{-ac}\sin\sqrt{-c}\xi}\Big)^{1/2}; \quad (24)$$

$$z(\xi)=\Big(\frac{-c+(\frac{\pm\sqrt{c(-P^2+R^2)}+\sqrt{-c}P\sin 2\sqrt{-c}\xi}{R+P\cos 2\sqrt{-c}\xi})^2}{b-\frac{2\sqrt{a}(\pm\sqrt{c(-P^2+R^2)}+\sqrt{-c}P\sin 2\sqrt{-c}\xi)}{R+P\cos 2\sqrt{-c}\xi}}\Big)^{1/2}; \quad (25)$$

$$z(\xi)=\Big(\frac{c(-1+(\frac{c\cos 2\sqrt{-c}\xi}{\sqrt{-c}\sin 2\sqrt{-c}\xi\pm\sqrt{-c}})^2)}{b-\frac{2c\sqrt{a}\cos 2\sqrt{-c}\xi}{\sqrt{-c}\sin 2\sqrt{-c}\xi\pm\sqrt{-c}}}\Big)^{1/2}; \quad (26)$$

$$z(\xi)=\Big(\frac{c(-1+(\frac{c\sin 2\sqrt{-c}\xi}{\sqrt{-c}\cos 2\sqrt{-c}\xi\pm\sqrt{-c}})^2)}{b+\frac{2c\sqrt{a}\sin 2\sqrt{-c}\xi}{\sqrt{-c}\cos 2\sqrt{-c}\xi\pm\sqrt{-c}}}\Big)^{1/2}, \quad (27)$$

where $a>0$, $c<0$, $R^2-P^2>0$, R, and P are arbitrary constants and $R\neq 0$, $P\neq 0$.

Substituting Eqs.(5) with Eq.(6) into Eq.(4) and setting the coefficients of $z^j(\xi)$ $(j=0,1,2,\cdots,8)$ to zero, we obtain the following nonlinear algebraic equations:

$$\gamma^2(\omega+\gamma\omega+\alpha g_0)g_0^2=0, \qquad \gamma[c\beta(1+\gamma)+\gamma(2(1+\gamma)\omega+3\alpha g_0)]g_1g_0=0,$$
$$c\beta(1+\gamma)(g_1^2+4\gamma g_0g_2)+\gamma^2[3\alpha g_0(g_1^2+g_0g_2)(1+\gamma)\omega(g_1^2+6g_0g_2)]=0,$$
$$g_1[2b\beta\gamma(1+\gamma)g_0+(1+\gamma)(c\beta(4+\gamma)+2\gamma^2\omega)g_2+\alpha\gamma^2(g_1^2+g_0g_2)]=0,$$
$$b\beta(1+\gamma)[(1+\gamma)g_1^2+6\gamma g_0g_2]+g_2[(1+\gamma)(4c\beta+\gamma^2\omega)g_2+3\alpha\gamma^2(g_1^2+g_0g_2)]=0,$$
$$g_1[3a\beta\gamma(1+\gamma)g_0+g_2(4b\beta(1+\gamma)^2+3\alpha\gamma^2g_2)]=0,$$
$$g_2^2[2b\beta(2+3\gamma+\gamma^2)+\alpha\gamma^2g_2]+a\beta(1+\gamma)[(1+2\gamma)g_1^2+8\gamma g_0g_2]=0,$$
$$a\beta(1+\gamma)^2g_2^2=0, \qquad a\beta(4+11\gamma+7\gamma^2)g_1g_2=0\,.$$

Solving this set of equations with the aid of Mathematica, we obtain the solutions as follows:

$$a=0,\quad c=-\frac{\gamma^2\omega}{4\beta},\quad g_0=g_1=0,\quad g_2=-\frac{2b\beta\Delta}{\alpha\gamma^2}; \tag{28}$$

$$a=0,\quad c=-\frac{4\omega}{\beta},\quad g_0=g_1=0,\quad g_2=-\frac{3b\beta}{4\alpha},\quad \gamma=-4; \tag{29}$$

$$a=0,\quad g_0=-\frac{6c\beta}{\alpha},\quad g_1=0,\quad g_2=-\frac{6b\beta}{\alpha},\quad \omega=2c\beta,\quad \gamma=2; \tag{30}$$

$$a=0,\quad g_0=-\frac{8c\beta}{\alpha},\quad g_1=0,\quad g_2=-\frac{12b\beta}{\alpha},\quad \omega=4c\beta,\quad \gamma=1, \tag{31}$$

where $\Delta=2+3\gamma+\gamma^2$.

Substituting Eq.(28) with Eqs.(7)–(27), respectively, into Eqs.(5) and (3), we obtain the new exact solitary wave solutions of Eq.(1) as follows:

$$u_1(x,t)=\Big[\frac{\omega\Delta}{2\alpha}[-1+(\coth K\xi\pm\operatorname{csch}K\xi)^2]\Big]^{1/\gamma};\qquad u_2(x,t)=\Big[\frac{-2b\omega\Delta}{\alpha}\frac{\cosh K\xi+\sinh K\xi}{(b+\cosh k\xi+\sinh k\xi)^2}\Big]^{1/\gamma};$$

$$u_3(x,t)=\Big[\frac{b\Delta\omega}{\alpha(-b+\sqrt{b^2}\cosh K\xi)}\Big]^{1/\gamma};\qquad u_4(x,t)=\Big[\frac{-\omega\Delta}{2\alpha}\operatorname{sech}^2K\xi\Big]^{1/\gamma};$$

$$u_5(x,t)=\Big[\frac{-4b\beta\gamma^2\Delta\omega^2\operatorname{sech}K\xi}{\alpha[\Omega+4b\beta\gamma^2\omega\operatorname{sech}K\xi+(4b^2\beta^2-\gamma^4\omega^2)\tanh K\xi]}\Big]^{1/\gamma};$$

$$u_6(x,t)=\Big[\frac{\omega\Delta}{2\alpha}[-1+(\tanh K\xi\pm\mathrm{i}\operatorname{sech}K\xi)^2]\Big]^{1/\gamma};\qquad u_7(x,t)=\Big[\frac{\omega\Delta}{8\alpha}\sinh^2\frac14K\xi\operatorname{csch}^2\frac14K\xi\Big]^{1/\gamma};$$

$$u_8(x,t)=\Big[\frac{-\beta\Delta}{2\alpha\gamma^2}\Big[\frac{\gamma^2\omega}{\beta}+\Big(\frac{K\sqrt{P^2+R^2}-PK\cosh K\xi}{R+P\sinh K\xi}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_9(x,t)=\Big[\frac{-\beta\Delta}{2\alpha\gamma^2}\Big[\frac{\gamma^2\omega}{\beta}+\Big(\frac{K\sqrt{P^2-R^2}+PK\sinh K\xi}{R+P\cosh K\xi}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{10}(x,t)=\Big[\frac{\omega\Delta}{2\alpha}\Big[-1-\frac{\gamma^2\omega}{\beta}\Big(\frac{\cosh K\xi}{K\sinh K\xi\pm\mathrm{i}K}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{11}(x,t)=\Big[\frac{\omega\Delta}{2\alpha}\Big[-1-\frac{\gamma^2\omega}{\beta}\Big(\frac{\sinh K\xi}{K\cosh K\xi\pm K}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{12}(x,t)=\Big[\frac{-\omega\Delta}{2\alpha}\sec^2\frac12E\xi\Big]^{1/\gamma};\qquad u_{13}(x,t)=\Big[\frac{-\omega\Delta}{2\alpha}\csc^2\frac12E\xi\Big]^{1/\gamma};$$

$$u_{14}(x,t)=\Big[\frac{-\omega\Delta}{2\alpha}[1+(\tan E\xi\pm\sec E\xi)^2]\Big]^{1/\gamma};\quad u_{15}(x,t)=\Big[\frac{-\omega\Delta}{8\alpha}\sec^2\frac14E\xi\csc^2\frac14E\xi\Big]^{1/\gamma};$$

$$u_{16}(x,t)=\Big[\frac{-\beta\Delta}{2\alpha\gamma^2}\Big[\frac{\gamma^2\omega}{\beta}+\Big(\frac{\pm E\sqrt{P^2-R^2}-PE\cos E\xi}{R+P\sin E\xi}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{17}(x,t)=\Big[\frac{-\beta\Delta}{2\alpha\gamma^2}\Big[\frac{\gamma^2\omega}{\beta}+\Big(\frac{\pm E\sqrt{P^2-R^2}+PE\sin E\xi}{R+P\cos E\xi}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{18}(x,t)=\Big[\frac{\omega\Delta}{2\alpha}\Big[-1-\frac{\gamma^2\omega}{\beta}\Big(\frac{\cos E\xi}{E\sin E\xi\pm E}\Big)^2\Big]\Big]^{1/\gamma};$$

$$u_{19}(x,t)=\Big[\frac{\omega\Delta}{2\alpha}\Big[-1-\frac{\gamma^2\omega}{\beta}\Big(\frac{\sin E\xi}{E\cos E\xi\pm E}\Big)^2\Big]\Big]^{1/\gamma};\qquad u_{20}(x,t)=\Big[\frac{-\omega\Delta}{2\alpha}\sec^2\frac12E\xi\Big]^{1/\gamma},$$

where $K=\sqrt{-\gamma^2\omega/\beta}>0$, $E=\sqrt{\gamma^2\omega/\beta}>0$, $\Omega=4b^2\beta+\gamma^4\omega^2$, $\xi=x+\omega t$.

3. Exact solitary wave solutions to the generalized Zakharov–Kuzentsov equation

The equation is expressed as

$$u_t+\beta u^{2p}u_x+\delta u_{xxx}+su_{xyy}=0. \tag{32}$$

Making transformation $u(x,y,t)=u(\xi)$, $\xi=x+ky+\omega t$ in Eq.(32), with constants k and ω to be determined, and integrating once with respect to ξ and setting the integration constant to zero, we obtain the following ordinary differential equation:

$$\omega u(\xi)+\frac{\beta}{1+2p}u^{1+2p}(\xi)+(\delta+sk^2)u''(\xi)=0. \tag{33}$$

In Eq.(33), we assume $u(\xi)$ to be in the following form:

$$u(\xi)=[v(\xi)]^{1/p}. \tag{34}$$

Substituting Eq.(34) into Eq.(33), we obtain the following equation:

$$p^2(1+2p)\omega v^2(\xi)+p^2\beta v^4(\xi)-(-1-p+2p^2)(\delta+sk^2)[v'(\xi)]^2+p(1+2p)(sk^2+\delta)v(\xi)v''(\xi)=0, \tag{35}$$

In Eq.(35), we assume $v(\xi)$ to be in the following form:

$$v(\xi)=g_0+g_1z(\xi)+g_2z^2(\xi), \tag{36}$$

where $z(\xi)$ is determined by the auxiliary equation(6).

Substituting Eq.(36) with Eq.(6) into Eq.(35) and setting the coefficients of $z^j(\xi)$ $(j=0,1,2,\cdots,8)$ to zero, we obtain the following nonlinear algebraic equations:

$$\begin{aligned}
&p^2g_0^2(\omega+2p\omega+\beta g_0^2)=0, \qquad pg_0[c(1+2p)(sk^2+\delta)+2p(\omega+2p\omega+2\beta g_0^2)]g_1=0,\\
&c(1+2p)(sk^2+\delta)(g_1^2+4pg_0g_2)+p^2[(1+2p)\omega(g_1^2+2g_0g_2)+2\beta g_0^2(3g_1^2+2g_0g_2)]=0,\\
&g_1\Big[2bp(1+2p)(sk^2+\delta)g_0+4c(sk^2+\delta)g_2+9cp(sk^2+\delta)g_2+4p^3\omega g_2\\
&\quad+2p^2[(csk^2+c\delta+\omega)g_2+2\beta g_0(g_1^2+3g_0g_2)]\Big]=0,\\
&4c(sk^2+\delta)g_2^2+8cp(sk^2+\delta)g_2^2+2p^3\omega g_2^2+b(1+2p)(sk^2+\delta)[(1+p)g_1^2+6pg_0g_2]\\
&\quad+p^2[\omega g_2^2+\beta(g_1^4+12g_0g_1^2g_2+6g_0^2g_2^2)]=0,\\
&g_1\Big[3ap(1+2p)(sk^2+\delta)g_0+4g_2[b(1+3p+2p^2)(sk^2+\delta)+p^2\beta(g_1^2+3g_0g_2)]\Big]=0,\\
&a(1+2p)(sk^2+\delta)[(1+2p)g_1^2+8pg_0g_2]+2g_2^2[b(2+5p+2p^2)(sk^2+\delta)+p^2\beta(3g_1^2+2g_0g_2)]=0,\\
&g_1g_2[a(4+15p+14p^2)(sk^2+\delta)+4p^2\beta g_2^2]=0, \qquad g_2^2[4a(1+3p+2p^2)(sk^2+\delta)+p^2\beta g_2^2]=0.
\end{aligned}$$

Solving this set of equations with the aid of Mathematica, we obtain the solutions as follows:

$$\omega=-\frac{4c(sk^2+\delta)}{p^2}, \quad a=-\frac{p^2\beta g_2^2}{4(1+3p+2p^2)(sk^2+\delta)}, \quad b=0, \quad g_0=g_1=0; \tag{37}$$

$$\omega=-\frac{c(sk^2+\delta)}{p^2}, \quad a=0, \quad b=-\frac{p^2\beta g_1^2}{(1+3p+2p^2)(sk^2+\delta)}, \quad g_0=g_2=0; \tag{38}$$

$$\omega=\frac{\beta g_0^2}{3}, \quad c=-\frac{\beta g_0^2}{6(sk^2+\delta)}, \quad a=-\frac{\beta g_2^2}{24(sk^2+\delta)}, \quad b=-\frac{\beta g_2g_0}{6(sk^2+\delta)}, \quad g_1=0, \quad p=1; \tag{39}$$

$$\omega=\frac{\beta g_0^2}{5}, \quad c=-\frac{\beta g_0^2}{5(sk^2+\delta)}, \quad a=-\frac{\beta g_2^2}{15(sk^2+\delta)}, \quad b=-\frac{4\beta g_2g_0}{15(sk^2+\delta)}, \quad g_1=0, \quad p=2. \tag{40}$$

Substituting Eqs.(37), (38) with Eqs.(7)–(18), respectively, into Eqs.(36) and (34), we gain the exact solitary solutions to Eq.(32) in the following form (the exact solutions in Eqs.(19)–(27) are not given):

$$u_{21}^{\pm}(x,y,t)=\Big(\frac{\sqrt{c}[-1+(\coth 2\sqrt{c}\xi_1\pm\operatorname{csch}2\sqrt{c}\xi_1)^2]}{H(\coth 2\sqrt{c}\xi_1\pm\operatorname{csch}2\sqrt{c}\xi_1)}\Big)^{1/p};$$

$$u_{22}^{\pm}(x,y,t)=\Big(\frac{c}{H^2}[-1+(\coth 2\sqrt{c}\xi_2\pm\operatorname{csch}2\sqrt{c}\xi_2)^2]\Big)^{\frac{1}{2p}};$$

$$u_{23}(x,y,t)=\Big(\frac{4cg_2(\cosh 2\sqrt{c}\xi_1+\sinh 2\sqrt{c}\xi_1)}{cg_2^2H^2-(\cosh 2\sqrt{c}\xi_1+\sinh 2\sqrt{c}\xi_1)^2}\Big)^{1/p};$$

$$u_{24}(x,y,t)=\Big[2g_1\Big(-\frac{c(\cosh 2\sqrt{c}\xi_2+\sinh 2\sqrt{c}\xi_2)}{(g_1^2H^2+\cosh 2\sqrt{c}\xi_2+\sinh 2\sqrt{c}\xi_2)^2}\Big)^{1/2}\Big]^{1/p};$$

$$u_{25}(x,y,t)=\Big[\frac{2\sqrt{c}\operatorname{sech}2\sqrt{c}\xi_1}{B}\Big]^{1/p};\quad u_{26}(x,y,t)=\Big[\frac{2c\operatorname{sech}2\sqrt{c}\xi_2}{-H^2(1+\operatorname{sech}2\sqrt{c}\xi_2)}\Big]^{\frac{1}{2p}};$$

$$u_{27}(x,y,t)=\Big[\frac{\sqrt{c}\operatorname{sech}\sqrt{c}\xi_2}{B}\Big]^{1/p};\quad u_{28}(x,y,t)=\Big[\frac{8cAg_2\operatorname{sech}2\sqrt{c}\xi_1}{4cA+p^2\beta g_2^2-(4cA-p^2\beta g_2^2)\tanh 2\sqrt{c}\xi_1}\Big]^{1/p};$$

$$u_{29}(x,y,t)=\Big[2g_1\big(\frac{2c^2\operatorname{sech}2\sqrt{c}\xi_2}{4c^2+H^4g_1^4-H^2cg_1^2\operatorname{sech}2\sqrt{c}\xi_2+(-4c^2+H^4g_1^4)\tanh 2\sqrt{c}\xi_2}\big)^{1/2}\Big]^{1/p};$$

$$u_{30}^{\pm}(x,y,t)=\Big(\frac{\sqrt{c}[-1+(\tanh 2\sqrt{c}\xi_1\pm\mathrm{i}\operatorname{csch}2\sqrt{c}\xi_1)^2]}{H(\tanh 2\sqrt{c}\xi_1\pm\mathrm{i}\operatorname{csch}2\sqrt{c}\xi_1)}\Big)^{1/p};$$

$$u_{31}^{\pm}(x,y,t)=\Big(\frac{c[-1+(\tanh 2\sqrt{c}\xi_2\pm\mathrm{i}\operatorname{csch}2\sqrt{c}\xi_2)^2]}{H}\Big)^{\frac{1}{2p}};\quad u_{32}(x,y,t)=\Big(\frac{\sqrt{c}c\operatorname{csch}\frac{\sqrt{c}}{2}\xi_1\operatorname{sech}\frac{\sqrt{c}}{2}\xi_1\operatorname{sech}\sqrt{c}\xi_1}{2H}\Big)^{1/p};$$

$$u_{33}(x,y,t)=\Big(\frac{c\operatorname{csch}\frac{\sqrt{c}}{2}\xi_2\operatorname{sech}\frac{\sqrt{c}}{2}\xi_2}{2H}\Big)^{1/p};\quad u_{34}(x,y,t)=\Big(\frac{(R+P\sinh 2\sqrt{c}\xi_1)}{H(\sqrt{c(P^2+R^2)}-P\sqrt{c}\cosh 2\sqrt{c}\xi_1)}\Psi(\xi_1)\Big)^{1/p},$$

where

$$\Psi(\xi_1)=-c+\frac{(\sqrt{c(P^2+R^2)}-P\sqrt{c}\cosh 2\sqrt{c}\xi_1)^2}{(R+P\sinh 2\sqrt{c}\xi_1)^2}.$$

$$u_{35}(x,y,t)=\Big(\frac{1}{H}\Big(-c+\frac{(\sqrt{c(P^2+R^2)}-P\sqrt{c}\cosh 2\sqrt{c}\xi_2)^2}{(R+P\sinh 2\sqrt{c}\xi_2)^2}\Big)^{1/2}\Big)^{1/p};$$

$$u_{36}(x,y,t)=\Big(\frac{(R+P\cosh 2\sqrt{c}\xi_1)}{H(\sqrt{c(-P^2+R^2)}+P\sqrt{c}\sinh 2\sqrt{c}\xi_1)}\Phi(\xi_1)\Big)^{1/p};$$

where

$$\Phi(\xi_1)=-c+\frac{(\sqrt{c(-P^2+R^2)}+P\sqrt{c}\sinh 2\sqrt{c}\xi_1)^2}{(R+P\cosh 2\sqrt{c}\xi_1)^2}.$$

$$u_{37}(x,y,t)=\Big(\frac{1}{H}\Big(-c+\frac{(\sqrt{c(-P^2+R^2)}+P\sqrt{c}\sinh 2\sqrt{c}\xi_2)^2}{(R+P\cosh 2\sqrt{c}\xi_2)^2}\Big)^{1/2}\Big)^{1/p};$$

$$u_{38}^{\pm}(x,y,t)=\Big(\frac{[\sqrt{c}\sinh 2\sqrt{c}\xi_1\pm\mathrm{i}\sqrt{c}]\operatorname{sech}2\sqrt{c}\xi_1}{H}\Upsilon(\xi_1)\Big)^{1/p};$$

where

$$\Upsilon(\xi_1)=-1+\frac{4c\cosh^2 2\sqrt{c}\xi_1}{(2\sqrt{c}\sinh 2\sqrt{c}\xi_1\pm 2\mathrm{i}\sqrt{c})^2}.$$

$$u_{39}^{\pm}(x,y,t)=\Big(\frac{c}{H}\Big[-1+\frac{4c\cosh^2 2\sqrt{c}\xi_2}{(2\sqrt{c}\sinh 2\sqrt{c}\xi_2\pm 2\mathrm{i}\sqrt{c})^2}\Big]^{1/2}\Big)^{1/p};$$

$$u_{40}^{\pm}(x,y,t)=\Big(\frac{[\sqrt{c}\cosh 2\sqrt{c}\xi_1\pm\sqrt{c}]\operatorname{csch}2\sqrt{c}\xi_1}{H}\Gamma(\xi_1)\Big)^{1/p};$$

where

$$\Gamma(\xi_1)=-1+\frac{4c\sinh^2 2\sqrt{c}\xi_1}{(2\sqrt{c}\cosh 2\sqrt{c}\xi_1\pm 2\sqrt{c})^2}.$$

$$u_{41}^{\pm}(x,y,t)=\Big(\frac{c}{H}\Big[-1+\frac{4c\sinh^2 2\sqrt{c}\xi_2}{(2\sqrt{c}\cosh 2\sqrt{c}\xi_2\pm 2\sqrt{c})^2}\Big]^{1/2}\Big)^{1/p};$$

$$u_{42}(x,y,t)=\Big(\frac{-\sqrt{c}\operatorname{csch}\sqrt{c}\xi_1\operatorname{sech}\sqrt{c}\xi_1}{H}\Big)^{1/p};\quad u_{43}(x,y,t)=\Big(\frac{\sqrt{c}\operatorname{sech}\sqrt{c}\xi_2}{B}\Big)^{1/p},$$

where

$$H = \left(\frac{-p^2\beta}{A}\right)^{1/2}, \quad p^2\beta A < 0, \quad c > 0, \quad A = (1 + 3p + 2p^2)(sk^2 + \delta),$$
$$B = \left(\frac{p^2\beta}{A}\right)^{1/2} > 0, \quad \xi_1 = x + ky - \frac{4c(sk^2+\delta)}{p^2}t, \quad \xi_2 = x + ky - \frac{c(sk^2+\delta)}{p^2}t,$$

g_1, and g_2 are arbitrary constants and $g_1 \neq 0$, $g_2 \neq 0$.

4. Conclusion and discussion

In this paper, we propose a method of auxiliary equation with function transformation, and obtain the new exact solitary solutions to the generalized mKdV generalized Zakharov–Kuzentsov equation, $u_1(x,t)$–$u_3(x,t)$, $u_5(x,t)$– $u_{20}(x,t)$, $u_{21}^{\pm}(x,y,t)$–$u_{24}(x,y,t)$, $u_{26}(x,y,t)$, $u_{28}(x,y,t)$–$u_{42}(x,y,t)$. We also gain the exact solitary wave solutions to the corresponding equation where $\gamma = 1$, $\gamma = 2$, $\gamma = -4$ and $p = 1$, $p = 2$, but these solutions are not discussed here. The approach given in this paper is applicable in seeking new exact solitary wave solutions to the two-dimensional extended Klein–Gordon equation[14]

$$u_{tt} - k(u_{xx} + u_{yy}) + \alpha u + \beta \mid u \mid^{2p} u + \gamma \mid u \mid^{4p} u = 0\,, \quad (p > 0),$$

and the Symmetric regularized long-wave equation[15]

$$u_{xxt} - u_t = (b_1\rho + b_2 u^{p+1} + b_3 u^{2p+1})_x,$$
$$b_i = \text{constant}, \quad p > 0, \quad (i = 1, 2, 3),$$
$$\rho_t + u_x = 0\,;$$

and some evolution equations. The above equations are with arbitrary nonlinear terms.

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