Vol 15 No 5, May 2006
1009-1963/2006/15(05)/1035-06

Chinese Physics

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and IOP Publishing Ltd

Runaway electrons as a diagnostic of plasma internal magnetic fluctuations*

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(Received 10 May 2005; revised manuscript received 16 February 2006)

The transport of runaway electrons in a high-temperature plasma is relatively easy to measure in a steady state experiment and a perturbation experiment, which provides runaway electron diffusion coefficient $D_{\rm r}$. This diffusion coefficient is determined by internal magnetic fluctuations, so it can be interpreted in terms of a magnetic fluctuation level. The internal magnetic fluctuation level ($\tilde{b}_{\rm r}/B_{\rm T}$) is estimated to be about (2–4)$\times^{-4}$ in the HL-1M plasma. The results presented here demonstrate the effectiveness of using runaway electron transport techniques to determine internal magnetic fluctuations. A profile of magnetic fluctuation level in the HL-1M plasma can be estimated from $D_{\rm r}$.

Keywords: transport of runway electron, diffusivity, intrinsic magnetic fluctuations, HXR
PACC: 5225G, 5235R, 5270

1. Introduction

The observed loss rates of both particles and energy from confined toroidal plasmas are much higher than those predicted by neoclassical transport theory, which depend on mean plasma parameters and Coulomb collisions. Many types of plasma turbulence have been proposed to explain the anomalous plasma transport. The fluctuations of plasma parameters around their mean values can cause transport through electrostatic or magnetic fluctuations.[1] The fluctuation-driven radial particle flux is given by

$$\Gamma = \langle \tilde{E}_\theta \tilde{n} \rangle / B_{\rm t} + \langle \tilde{j}_{{\rm II}\varepsilon} \tilde{b}_{\rm r} \rangle / e B_{\rm t} \,, \qquad (1)$$

where $\tilde{E}_\theta$ is the fluctuating poloidal electric field, $\tilde{n}$ is the fluctuating electron density, $\tilde{b}_{\rm r}$ is the radial component of the fluctuating magnetic field, $\tilde{j}_{{\rm II}\varepsilon}$ is the fluctuating current parallel to the magnetic field, and $\langle \cdots \rangle$ denotes the time average. The fluctuation driven energy flux is given by

$$Q = \frac{3}{2} K_{\rm B} n \langle \tilde{E}_\theta \tilde{T} \rangle / B_{\rm t} + \frac{3}{2} K_{\rm B} T \langle \tilde{E}_\theta \tilde{n} \rangle / B_{\rm t} \,, \qquad (2)$$

where $K_{\rm B}$ is Boltzmann constant and $\tilde{T}$ is the fluctuating temperature. The fluctuation driven radial particle flux includes the electrostatic fluctuations and magnetic fluctuations. The fluctuation driven energy flux includes $\tilde{E}_\theta$ (the fluctuating poloidal electric field).

The edge turbulence is well characterized; it can explain the fluctuation-driven particle flux, but cannot well accounts for the energy fluxes. In the plasma interior, measurements of density fluctuations suggest that electrostatic fluctuation-driven transport cannot account for the observed fluxes, therefore the measurement of magnetic fluctuation is very important for a description of plasma confinement. There is still insufficient experimental data available to assess the importance of magnetic fluctuation for internal transport. In ohmically heated low β plasmas the measured magnetic fluctuation is too small to contribute to the thermal losses, but can explain the runaway electron transport. The difficulty in applying the magnetic probe measurements to the description of magnetic fluctuations in a plasma precludes a study of the magnetic fluctuation levels in the plasma interior, except in small tokamaks with short discharge durations. In this study, we demonstrate the use of a steady state approach and perturbation techniques to measure the diffusion of runaway electrons as a probe of internal magnetic fluctuation in plasma. Comparatively, the transport of runaway electrons in a plasma can be easily measured using a steady state approach for the runaway electron confinement time $\tau_{\rm r}$ that is deduced

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from hard x-ray (HXR) bremsstrahlung spectra, and using the perturbation techniques for τ_r that can be deduced from simultaneously recorded sawtooth oscillations of HXR flux, soft x ray (SXR) intensity and $\lambda = 3$ cm microwave radiation intensity (WUW), which provide local runaway electron diffusion coefficient D_r. Assuming that magnetic turbulence is responsible for the runaway electron transport, the diffusion coefficient can be interpreted in terms of a magnetic fluctuation level.

We find that the runway electron diffusion coefficient is determined by internal magnetic fluctuations rather than electrostatic fluctuations because of the high energy involved. Runaway electron behaviour can be used as a probe for internal magnetic fluctuations. The results presented here demonstrate the effectiveness of using runaway electron transport techniques to determine internal magnetic fluctuations. A profile of the magnetic fluctuation level in HL-1M plasma can be estimated from D_r.

2. Experiments

The experiments reported in this paper have been carried out on the HL-1M with $R = 1.02$ m, $a = 0.29$ m, a plasma current of around 120–150 kA, a toroidal field $B_t = 2$ T and a line averaged electron density of $1.0\times^{19} - 1.5\times10^{19}$ m^{-3}. The device and diagnostics are described in more detail in Ref.[2]. HXR flux was detected by 4 NaI (Ti) 7.6 cm × 7.6 cm scintillates working in the current mode or in pulse height analysis (PHA) mode in the measurements of the runaway electron energy and confinement time. We need to measure not only the energy spectra of the hard x-ray radiation in the PHA mode but also the evolution of the radiation of the hard x-ray in the current mode. The detectors housed separately in the tangential and perpendicular shields were used. An energy resolution of 10 keV/channel was obtained using a calibration with a ^{137}Cs source. Photons with energies lower than 50 keV were not detectable.

2.1. Runaway electron diffusion coefficient deduced from hard x-ray bremsstrahlung spectra

The runaway electron diffusion coefficient D_r is obtained using a steady state approach in which the runaway electron confinement time can be deduced from HXR bremsstrahlung spectra. The runaway electron confinement time τ_r and diffusion coefficient in an ohmically and additionally heated tokamak plasma reveal an anomalous behaviour as predicted in neoclassical models.

2.1.1. Scaling low for runway electrons from 1-D model

A one-dimensional numerical model[3] including generation and loss effects of runaway electrons is used to deduce the dependence of runaway electron energy ε_r on the runaway electron confinement time. The simulation results are presented in the form of a scaling law for ε_r against plasma parameters. The scaling of ε_r, therefore, the runaway electron confinement time τ_r, has been studied in the HL-1M tokamak by measuring hard x-ray spectra under different experimental conditions. The runaway electron confinement time is deduced in the range of plasma parameters available in the HL-1M. A tentative explanation for the scaling of the obtained data based on effects from magnetic turbulence is presented below.

The runaway electron confinement time τ_r is deduced by using a simple one-dimensional model for generation, acceleration and loss of runaway electrons, and by fitting the calculations of the bremsstrahlung radiation caused by collisions of runaway electrons with the background plasma and limiter to the HXR spectra through simulation. For this purpose, the electron distribution function obtained numerically is in a simple form of distorted Maxwellian distribution function. We can obtain a slide-away distribution function. The kinetic model that is applied to obtain the runaway electron energy has been described in Ref.[3], and it considers the electron acceleration in a parallel electric field $E_{||}$ and drag forces due to ion-electron and electron–electron collisions as well as electron–impurity collisions. The input parameters of the model are n_e, T_e, Z_{eff} and $E_{||}$ for the source and τ_r for the loss. The output value is the inverse slope of the simulated HXR spectra, which is called the "runaway average energy" ε_r. By using more than 80 different simulations, the results can be approximated by the following expression:

$$\varepsilon_r = (\bar{n}_e^{0.15} T_e^{-0.06} Z_{eff}^{-0.03})(V_P^{1.02}\tau_r^{1.1}), \qquad (3)$$

The expression (3) is referred to as scaling law for ε_r against plasma parameters and the runaway electron confinement time τ_r, where ε_r and T_e are in keV, n_e is in cm^{-3}, V_p in volt and τ_r in ms. The variation ranges of n_e, Z_{eff}, T_e, and V_p, used for these simulations, are respectively $0.4\times10^{19} - 1.5\times10^{19}$ m^{-3}, 1–6, 0.4–1 keV, and 0.4–0.5 V. By fitting the experimental

data to modeled HXR spectra, τ_r can be inferred from the runaway electron energy ε_r, i.e. from the inverse of the slope of the HXR spectrum.

Firstly we measure the bremsstrahlung radiation spectra by pulse height analysis (PHA). Using the expression (3), we can experimentally deduce the value of τ_r, as a global measurement, assuming that the inverse slope of the HXR spectrum is the mean energy of runaway electrons in the plasma. The variation of ε_r ranges from 60 keV to 1.2 MeV, which corresponds to the runaway electron confinement time varying from 2 ms to 16 ms.

2.1.2. D_r determined from the hard x-ray spectra

To relate runaway electron confinement time τ_r to runaway electron diffusion coefficient D_r, we consider a stationary runaway electron density. From $(\partial n_r/\partial t = \nabla(D_r n_r') + s_r$ where s_r is the reduction rate of runaway electrons and n_r' is the density gradient, we obtain the runaway electron flux:

$$\Gamma_r(r) = -D_r n_r' = \frac{1}{r}\int_0^r s_r s \mathrm{d}s\,.$$

Therefore, the runaway density can be inferred as follows: $n_r = \int_r^a \Gamma_r(r)/D_r \mathrm{d}r$.

The runaway electron confinement time is expressed as follows

$$\tau_r = \frac{\int n_r \mathrm{d}V}{\Gamma_r(a)A} = \frac{\int_0^a n_r r \mathrm{d}r}{\Gamma_r(a)a}\,, \qquad (4)$$

where A is the area of the electron flux impinging on the limiter. For the HL-1M, if D_r is constant, considering s_r calculated by Connor and Hastie,[4] τ_r can be approximated by

$$\tau_r = \frac{a^2}{4D_r}\left[1 - \left(\frac{r_{SR}}{a}\right)^2\right]\,. \qquad (5)$$

where r_{SR} is the radius for the profile s_r produced by the runaway electron maximum rate. HL-1M plasma conditions for this profile were calculated using the tokamak parameters: $B_t = 1.9$ T, $V_L = 2$ V, and $I_p = 140$ kA, and experimental profiles of electron density and temperature, as well as the safety factor q and Z_{eff} obtained from the measurements of electron cyclotron emission (ECE) and HCN laser interferometer. s_r has a maximum of around $a/2$. Nevertheless, for these conditions the calculated runaway electron density profile is peaked in the plasma centre.

2.2. Runaway electron diffusion coefficient obtained using magnetic perturbation techniques

In the following sections we present experimental results from the HL-1M using three perturbation techniques to measure the runaway electrons diffusion coefficient, and thus the level of magnetic fluctuations.

2.2.1. Runaway electron diffusion coefficient deduced from sawtooth perturbation

If we consider that runaway electrons diffuse radially after each internal impingement on the limiter, the simplest model that we can adopt is similar to the heat pulse diffusion model described by Soler and Callen[5], where what we should consider is the runway electron density instead of the temperature. The time Δt_r for a pulse to reach its peek through a medium of size 'a' with a diffusion coefficient D_r can be written using the dimensional method as

$$\Delta t_r = a^2/PD_r\,, \qquad (6)$$

where P is a constant that takes into account the geometry and pulse features of the problem involved. For a single pulse in cylindrical co-ordinates it is easy to show that $P = 4$. For sawtooth pulses, if the pulse is created by the flattening of a peaked profile inside the $q = 1$ surface, the perturbation has a negative component in the axial zone and a positive one in the zone close to $q = 1$. Figure 1 shows the signals from a central chord SXR detector and from an HXR detector during the applying of sawtooth pulses. Therefore, the pulse has a bipolar feature. In this case, considering the modifications by Zheng,[6] P depends on the inverse radius position of the initial bipolar pulse. For the HL-1M tokamak value of inverse radius r_s obtained from SXR is about 5–6 cm for $B_t = 2$ T. P has an approximate value of 14. Adopting a peaked profile for the runway electron density and the same inverse radius for runway electrons and SXR, we can estimate the runway electron diffusion coefficient D_r from the experimental values of time for the runaway electron density to reach its peak as $D_r = a^2/(14\Delta t_r)$. Experimental results for Δt_r exhibit a clear increase from 0.1 ms to 0.3 ms in a range of B_t from 1.4 T to 2.5 T. It must be noted that under the same plasma conditions, both τ_r runaway electron confinement time and Δt_r have the same upward tendency. Experimentally, τ_r is a factor of 3 to 5 greater than Δt_r, which can also be deduced from the expression (2). D_r is the average effect of electron locus along the runaway electron direction. Supposing that runaway electron density pulse radially propagates from $q = 1$ surface to the limiter quickly at a uniform velocity, we can approximately obtain $r = (a + r_s)/2$; thus runaway electron diffusion coefficient is (2–5) m^2/s.

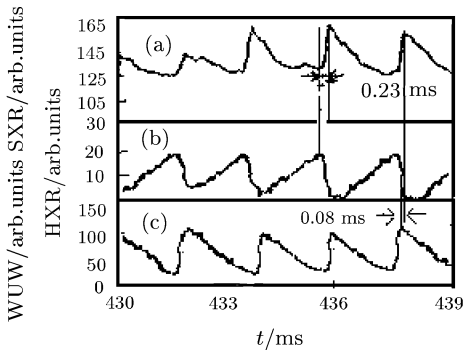


Fig.1. Signals from a central chord SXR detector (b) and from an HXR detector (a) during the applying of sawtooth and microwave radiation ($\lambda = 3$ cm) intensity sawtoothing signals.

2.2.2. Runaway electron diffusion coefficient deduced from microwave radiation

Consider that runaway electrons diffuse radially after each internal impingement on the limiter, which can cause the additional runaway electron wave to diffuse in plasma. The microwave radiation ($\lambda = 3$ cm) intensity sawtoothing signals and sawtooth oscillation of the hard x-ray signals are recorded simultaneously (see Fig.1). The simplest mode that we adopt is similar to the diffusion model described above. The time $\Delta t_{\rm r}$ for a pulse to reach its peak with a diffusion coefficient $D_{\rm r}$ can be written using dimensional method as

$$\Delta t_{\rm r} = (\Delta X)^2 / P D_{\rm r} \,. \tag{7}$$

The plasma density $n_{\rm w}$ at which microwave stops propagating at a frequency of below $f = 10$ GHz is referred to as cut-off density. For the microwave radiation ($\lambda = 3$ cm) cut-off density $n_{\rm w} \approx 1.2 \times 10^{18}$ m^{-3} appears at a radius $r_{\rm w}$ of electron density profile, as shown in Fig.2. We can estimate the runaway electron diffusion coefficient $D_{\rm r}$ from the experimentally obtained time $\Delta t_{\rm r}$.

If $\Delta X = a - r_{\rm w}$, than

$$D = (\Delta X)^2 / C \Delta t_{\rm r} \,, \tag{8}$$

where $\Delta t_{\rm r}$ is for a pulse to reach its peak in the propagation from $r_{\rm w}$ through $(a - r_{\rm w})$ to the limiter. The pulse has a bipolar feature, and C depends on the inverse radial position of the initial bipolar pulse and the microwave frequency. For the HL-1M tokamak C has an approximate value of 30 and field distance $r_{\rm w} = 160$ mm. Assuming the profile for the runaway electron density to peak and the inverse radius for runaway electrons and HXR to be the same as their previous values, we can estimate the runaway electron diffusion $D_{\rm r}$ from the experimental values of $\Delta t_{\rm r}$. Supposing a runaway electron wave pulse radially propagating from $q = 1$ surface to the limiter quickly at a uniform velocity, we can approximately have $r = (a + r_{\rm w})/2$, thus runaway electron diffusion coefficient is $(4 - 6)$ m^2/s. If the experimental condition is allowable, we can measure the microwave radiations at wavelengths $\lambda = 1$ cm and $\lambda = 2$ cm at the same time to obtain runaway electron diffusion coefficients at other different locations.

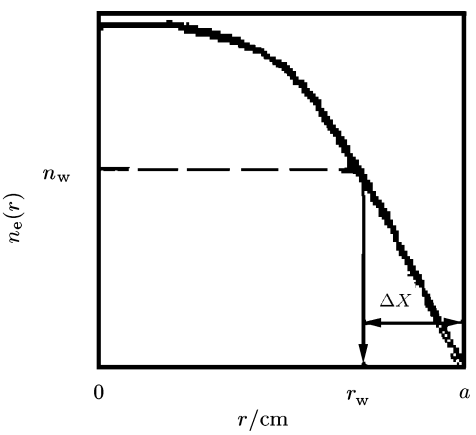


Fig.2. Plasma electron density $n_{\rm e}(r)$ as a function of plasma radius r.

2.2.3. Plasma shift experiment

An increase in current flowing in the vertical field coils causes both the plasma position[7] and the runaway electron orbits inward shifts, with the plasma current remaining constant, but they can be changed in an induced electric field. This induced electric field accelerates not only the runaway electrons, but also bulk electrons. This acceleration can be a source term in the runaway electron diffusion equation. Electrons with the energy of the order of 60 keV can be easily accelerated from bulk electrons due to free fall. By letting the runaway electron flux to the detector meet a nearly steady state condition before shifting the plasma and examining its transient response, the time evolution of the original radial profile of the runaway electrons is unnecessarily known. For the shift experiments, however, the motion of plasma changes the induced electric field, causing changes in loop voltage being of the order of one volt. In a shorter time scale the effects of induction, collisions, and drifts can be ignored even though these effects can be important on a longer time scale. The shifting of small position of plasma can leave a longer distance for the runaway electrons to diffuse before reaching the limiter at the

outer radius of the plasma. Since the runaway orbits are shifted outwards with respect to flux surfaces, due to the induced electric field the runaway electrons should not encounter the limiter on the inner side of the plasma. The plasma displacement, hard x-ray signal and the best fit to diffusion equation model are shown in Fig. 3.

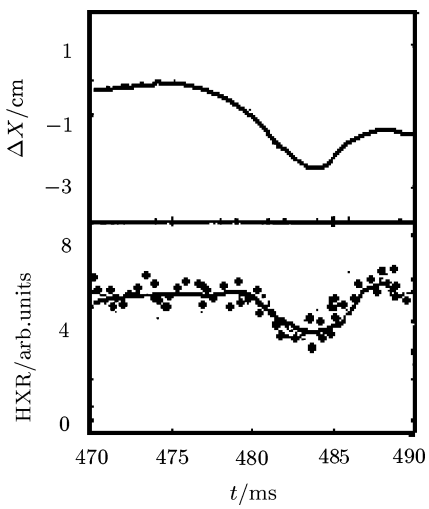


Fig.3. Plasma displacement (upper) and hard x-ray signal (lower) during plasma shift. The solid line is for the best fit to a diffusion equation model.

The hard x-ray detector viewing the high field side of the poloidal limiter does not indicate an increase in hard x-ray flux as the plasma moves inwards. We are concerned only with the shorter time scale of the transient signal of the hard x-ray, and thus the effects of collisions, drift and induced electric field can be ignored even though these effects can be important on a larger time scale. The following simple diffusion equation:

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(D_{\mathrm{r}}\frac{\partial N}{\partial x}\right), \qquad (9)$$

can describe the behaviour of runaway electrons,[7] where a slab model in x-direction is used since the change in minor radius is small compared with the initial minor radius. D_{r} is assumed to be constant independents of both time and space. A numerical fit [7] to the hard x-ray flux during the shift measurements indicates that the diffusion coefficient of runaway electrons is 9 m^2/s. Unfortunately, the runaway electron diffusion coefficient D_{r} is unlikely to be a constant across the minor radius as indicated in analytical models. Measurement of the background electron thermal conductivity χ_{e} in the HL-1M shows that it is much larger on the edge than in the centre. If the actual runaway electron diffusion coefficient D_{r} has a similar radial dependence, then the plasma shift measurements provide a measure of D_{r} near the edge rather than in the core since the shorter time evolution of a plasma slightly increasing in size would be expected to be dominated by the local value of D_{r}. According to the measured distance of the plasma shift, this D_{r} measurement should be characteristic of the plasma conditions on the edge for a distance of 20 mm.

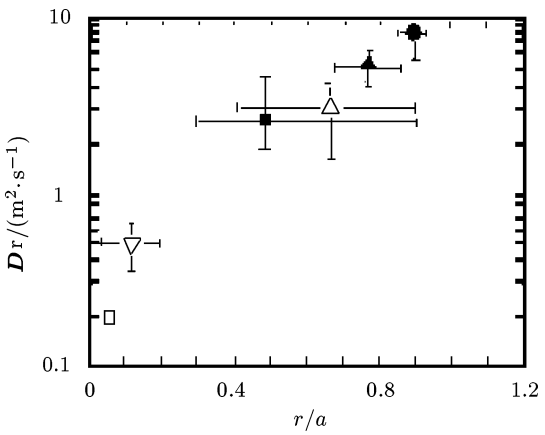


Fig.4. Local runaway electron diffusion coefficient $\boldsymbol{D}_{\mathrm{r}}$ as a function of plasma radius r. 鈴 shift experiment, 測 from microwave radiation Δ from sawtooth perturbation 觀 HXR bremsstrahlung spectra, ∇ experimental core diffusion coefficient □ from diffusion model.

Now, we still cannot measure the runaway electron diffusion in core, D_{ro}, but can calculate the diffusion coefficient of electrons in the core to be $\sim 0.2\ \mathrm{m}^2/\mathrm{s}$, using the definite linear diffusion equation model presented by Myra and Catto.[8] The difference in value of diffusion coefficient ΔD_{r} between D_{r} obtained using Hard x-ray (HXR) bremsstrahlung spectra and D_{r} obtained from experiment in the impingement of sawtooth of SXR and HXR, is $\sim 0.5\ \mathrm{m}^2/\mathrm{s}$. The experimental result is in good agreement with the simulation result.

The typical results for local runaway electron diffusion coefficient D_{r} in the HL-1M tokamak are shown in Fig. 4. Consequently, results for D_{r} obtained from hard x-ray bremsstrahlung radiation and three perturbation techniques point out that those values come from a resulting local effect along runaway electron trajectories.

3. Magnetic turbulence

Assuming that internal stochastic magnetic field results in the runaway electron diffusion, which can be related to the runaway electron diffusion coefficient by $D_{\mathrm{r}} < uD_{\mathrm{m}}$, where u is the runaway electron velocity

and $D_{\rm m}$ is the magnetic diffusion coefficient. We can further express $D_{\rm m}$[8] as

$$D_{\rm m} = \pi q R(\langle(\tilde{b}_{\rm r}/B_{\rm t})^2\rangle D_{\rm r} = D_{\rm m}\nu_{\rm II}\gamma = qR(\tilde{b}/B_{\rm t})^2\nu_{\rm II}\gamma \approx \chi_{\rm e}\nu_{\rm r}/\nu_{\rm th}\,. \quad (10)$$

As an example where $u = c$, $q = 3.5$, $D_{\rm r} = 1\ {\rm m}^2/{\rm s}$ and $R = 1.02$ m, we observe $\langle(\tilde{b}_{\rm r}/B_{\rm t})^2\rangle$ being more than 3×10^{-8}. Under the experimental condition similar to that of this experiment, $(\tilde{b}_{\rm r}/B_{\rm t})^2$ is found to be around 4×10^{-8} using magnetic probes behind the limiter in the HL-1M. Such an agreement between magnetic probe measurements and the runaway perturbation technique on the edge validates the effectiveness of runaway perturbation technique for measuring magnetic fluctuations; it also indicates that the runaway electron diffusion is due to internal magnetic field fluctuations. Figure 5 shows the magnetic fluctuation as a function of plasma radius in the HL-1M tokamak.

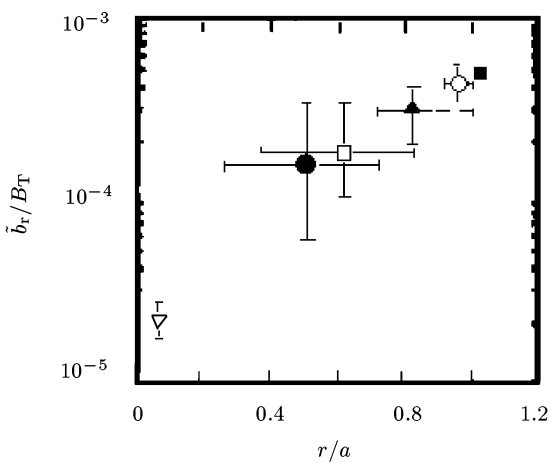


Fig.5. The magnetic fluctuation as a function of plasma radius. 觀 Magnetic prob on the edge ○ Shift experiment 測 Microwave radiation □ From impingement of sawtooth oscillations 鈴 From hard x-ray bremsstrahlung spectra ∇ Diffusion equation model.

We can confirm that magnetic fluctuation decreases from the edge to the inner minor radius, as observed in the HL-1M tokamak.

The relation between magnetic diffusion and runaway electrons has been discussed in several articles. Several drift orbit effects reduce particle diffusion in the presence of magnetic turbulence. The two dominant mechanisms, which reduce the partial diffusion of runaway electrons, are (i) a profile averaging over the radial mode resulting from the guiding centre radial drift departing from a surface and (ii) a poloidal drift-induced detuning of the wave-particle resonance away from the rational surfaces where the magnetic fluctuation is strongest. The fall-off of runaway diffusion with energy is primarily governed at high energies by the mechanism (i).

4. Conclusions

The runaway electron diffusion coefficient is obtained using two methods: a stationary method in which a confinement time is deduced from HXR bremsstrahlung radiation, and the magnetic perturbation technique. Both methods give local values for $D_{\rm r}$ in the range of $(0.4 - 10)\ {\rm m}^2{\cdot}{\rm s}^{-1}$ with a dependence on toroidal magnetic field as $B_{\rm t}^{-1.5}$. Comparing this scaling with the relationship (10), we can say that changes in $B_{\rm t}$ can affect the structure of the magnetic turbulence since the averaging drift effect is unimportant in this type of discharge.

The deduced magnetic fluctuation level is about $(2 - 5) \times 10^{-4}$, corresponding to the average of magnetic fluctuation levels measured inside and on the plasma edge using other diagnostics in the HL-1M tokamak. The $(\tilde{b}_{\rm r}/B_{\rm t})$ value is dependent on toroidal magnetic field; it proportionally decreases with the increase of toroidal magnetic field. We can confirm that magnetic fluctuations increase from the centre to the edge, as observed in the HL-1M tokamak.

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