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Thomson scattering off a pair (electron–positron) plasma*

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Thomson scattering off a pair (electron–positron) plasma is theoretically investigated in the collisionless and collisional limits respectively. Our calculations show that the power spectrum of the Thomson scattering off a collisionless pair plasma is just proportional to the velocity distribution function of the particles in the plasma. Collective modes in the plasma do not have any effects on the Thomson scattering spectrum because of the correlation between the negatively- and positively-charged particles. In the collisional limit, the power spectrum of the Thomson scattering presents three spikes: two peaks correspond to two contra-propagating sound waves and one peak corresponds to an entropy wave.

Keywords: pair plasma, Thomson scattering, dynamic form factor
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1. Introduction

Ordinary plasmas consist of electrons and ions. The huge difference in mass between electrons and ions has a profound influence on the collective phenomena of the plasmas. For example, it is well known that a Debye sheath forms around a Langmuir probe in an ordinary plasma because the probe is charged by more mobile electrons due to their much lighter mass.[1] It is not difficult to image what will happen if singly charged positive ions have the same mass as the electrons. There should not be any sheath around the probe at all because the negatively- charged electron and positively-charged ion have the same mobility, so that the probe is equally charged by electrons and ions. Another example is ion-acoustic waves in an ordinary plasma. Obviously, this wave mode will disappear if the positively- charged ion has the same mass as the electron. These two simple examples just show the importance of the mass asymmetry between the negatively- charged particles and positively- charged particles of a plasma to the plasma properties. In fact, if several species of ions exist in a plasma, collective phenomena in the plasma become more complicated.[2–5]

On the other hand, there could exist a special kind of plasmas, i.e. the so-called pair plasmas, consisting of only negatively- and positively-charged particles with an equal mass and opposite charge number and number density. Very recently such kind of plasmas comes into reality: pair-ion plasma without electrons is successfully generated by using fullerene as an ion source.[6] Electron–positron plasma is another kind of pair plasmas, which could exist naturally under certain astrophysical conditions and can now be generated in laboratory. With the help of Penning trap to confine positrons, experiments on electron–positron beam-plasma have been performed.[7,8] The advent of ultra-intense lasers provides another method to produce dense electron–positron plasmas.[9] Linear and nonlinear collective modes in pair plasmas have been extensively investigated in theory.[10,11] Due to the mass symmetry of pair plasmas, it is found that the behaviours of the collective modes in pair plasmas are simpler than those in ordinary plasmas. In this paper, we present an important diagnostic theory for pair plasmas, i.e. Thomson scattering off a pair

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(electron–positron) plasma.

Thomson scattering is an important diagnostic technique in the study of plasma physics.[12] For an ordinary plasma, the total cross section of Thomson scattering is just determined by the electrons in the plasma due to their much lighter mass. Scatterings off the ions are always negligible. This situation is no longer valid for a pair plasma. For a pair plasma, positively- and negatively- charged have the same mass, and thereby they should contribute equally to the total cross section of Thomson scattering. This means that the usual Thomson theory needs to be modified. In this paper, we discuss the theory of Thomson scattering off a pair plasma in the collisionless and collisional limits separately.

The rest of the paper is organized as follows. We derive the general equations for the Thomson scattering off a pair plasma in Section 2. Then we discuss the dynamic form factor in the collisionless limit in Section 3, and in the collisional limit in Section 4. Finally, we summarize our discussion.

2. Basic equations

The basic process of Thomson scattering is that a charged particle oscillates under the action of the field of an incident electromagnetic wave and then emits a scattered electromagnetic wave. In the non-relativistic case, the motion of a charged particle under the action of an electromagnetic wave is described by the following equation:

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = \frac{q\boldsymbol{E}(\boldsymbol{r},t)}{m}, \tag{1}$$

where $\boldsymbol{E}$ is the electric field of the wave, m is the particle mass and q is the particle charge. Since the particle speed is much slower than the light speed, the Lorentz force is neglected. In this paper, we do not consider the Thomson scattering off magnetized pair plasmas. Hence no static magnetic field is assumed in the following calculations. As seen in Eq.(1), an oscillating electric dipole $\boldsymbol{d} = q\boldsymbol{r}$ is induced under the action of the incident wave. Scattered wave is then emitted due to this electric dipole via dipole radiation.

We assume that the electric field of the incident wave is $\boldsymbol{E} = \boldsymbol{E}_0\cos(\boldsymbol{k}_0\cdot\boldsymbol{r} - \omega_0 t)$, where $\boldsymbol{E}_0$ is the wave amplitude, and $\boldsymbol{k}_0$ and ω_0 are the wave vector and frequency of the incident wave respectively. At the position $\boldsymbol{R}$ far from the particle, the magnetic field of the scattered waves due to the induced electric dipole $\boldsymbol{d}$ is approximately given by[13]

$$\boldsymbol{B}(\boldsymbol{R},t) = \frac{1}{c^2R}\ddot{\boldsymbol{d}}(\tau)\times\boldsymbol{n},$$

where the dot indicates differentiation with respect to time, $\tau = t - |\boldsymbol{R}-\boldsymbol{r}|/c \approx t - R/c + \boldsymbol{n}\cdot\boldsymbol{r}/c$ is the retardation time, and $\boldsymbol{n}$ is the unit vector along $\boldsymbol{R}$. The total magnetic field of the scattered waves from a pair plasma is the sum of the scattered waves from individual particles, and given by

$$\begin{aligned}\boldsymbol{B}(\boldsymbol{R},t) =& \frac{q_+^2}{mc^2R}(\boldsymbol{E}_0\times\boldsymbol{n}) \\ &\times\sum_{j=1}^{N_+}\cos\{\boldsymbol{k}\cdot\boldsymbol{r}_j - \omega_0(t-R/c)\} \\ &+\frac{q_-^2}{mc^2R}(\boldsymbol{E}_0\times\boldsymbol{n}) \\ &\times\sum_{j=1}^{N_-}\cos\{\boldsymbol{k}\cdot\boldsymbol{r}_j - \omega_0(t-R/c)\},\end{aligned} \tag{2}$$

where $\boldsymbol{k} = \boldsymbol{k}_0 - (\omega_0/c)\boldsymbol{n}$ is the differential wave vector, $\boldsymbol{r}_j$ is the position of the jth particle, $q_\pm$ are the particle charges, and $N_\pm$ are the numbers of the positively- and negatively-charged particles in the plasma, respectively. In Eq.(2), we intentionally separate the contributions from the positively- and negatively-charged particles. For simplicity, we further assume that the pair plasma consists of electrons and positrons, in which $q_+ = -q_- = e$, $N_+ = N_- = N$, and $m = m_\mathrm{e}$, where m_e is the electron mass.

The power scattered into the solid angle $\mathrm{d}\Omega$ is given by $(c/4\pi)|\boldsymbol{B}|^2R^2\mathrm{d}\Omega$. What can be directly measured is the scattered power spectrum, which can be obtained by means of the Wiener–Khinchine theorem,[12]

$$\begin{aligned}&\frac{\mathrm{d}^2P_\mathrm{s}(\boldsymbol{R},\omega_\mathrm{s})}{\mathrm{d}\omega_\mathrm{s}\mathrm{d}\Omega} \\ &=\frac{c}{8\pi^2}\int_{-\infty}^{\infty}\mathrm{d}\tau\,\mathrm{e}^{-\mathrm{i}\omega_\mathrm{s}\tau}\overline{\boldsymbol{B}(R,t)\cdot B(R,t+\tau)}.\end{aligned} \tag{3}$$

Here the bar indicates either the time average, or equivalently, the ensemble average. Substituting Eq.(2) into Eq.(3), we can find that the power spectrum of the scattered light is closely related to the particle density fluctuations,[14]

$$\begin{aligned}\frac{\mathrm{d}^2P_\mathrm{s}(\boldsymbol{R},\omega_\mathrm{s})}{\mathrm{d}\omega_\mathrm{s}\mathrm{d}\Omega} =& \frac{V}{2\pi}I_0r_0^2\sin^2\beta\left[(\delta n_+^2)_{k,\omega} + (\delta n_-^2)_{k,\omega}\right. \\ &\left.+(\delta n_+\delta n_-)_{k,\omega} + (\delta n_-\delta n_+)_{k,\omega}\right],\end{aligned} \tag{4}$$

where $r_0 = e^2/m_\mathrm{e}c^2$ is the classical electron radius, $I_0 = (c/8\pi)E_0^2$ is the intensity of the incident wave, β is the angle between $\boldsymbol{E}_0$ and $\boldsymbol{n}$, $\omega = \omega_0 - \omega_\mathrm{s}$ is

the differential frequency, V is the scattering volume, and the symbols $(\cdots)_{k,\omega}$ denote the spectral densities of the particle density fluctuations and the correlations between them.[15] It should be pointed out that Eq.(5) is valid in the case where the electrons and positrons have the same number densities. As usual, we can define the dynamic form factor $S(\boldsymbol{k},\omega)$ of a pair (electron–positron) plasma with the equation

$$\frac{\mathrm{d}^2 P_\mathrm{s}(\boldsymbol{k},\omega_\mathrm{s})}{\mathrm{d}\omega_\mathrm{s}\mathrm{d}\Omega} = 2nVI_0 r_0^2 \sin^2\beta S(\boldsymbol{k},\omega), \tag{5}$$

where $n = N/V$ is the electron and also the positron number density. According to Eq.(5), the dynamic form factor $S(\boldsymbol{k},\omega)$ of a pair plasma is given by

$$S(\boldsymbol{k},\omega) = \frac{1}{4\pi n}\left[(\delta n_+^2)_{k,\omega} + (\delta n_-^2)_{k,\omega} + (\delta n_+\delta n_-)_{k,\omega} + (\delta n_-\delta n_+)_{k,\omega}\right]. \tag{6}$$

For a comparison, we write here the dynamic form factor of an ordinary plasma,

$$S_\mathrm{O}(\boldsymbol{k},\omega) = \frac{1}{2\pi n}(\delta n_-^2)_{k,\omega}. \tag{7}$$

We will not take into account any effects of annihilation of electrons and positrons in the following calculations. Since the electrons and positrons own the same mass, it is reasonable to assume that the pair plasma is in an equilibrium state with a temperature T. Therefore, the spectral densities $(\delta n_+^2)_{k,\omega}$, $(\delta n_-^2)_{k,\omega}$, and $(\delta n_+\delta n_-)_{k,\omega}$ can be calculated based on the fluctuation–dissipation theorem.[15] Following the standard procedure in Ref.[15], we have

$$(\delta n_-^2)_{k,\omega} = \frac{2T}{\omega}\frac{k^2}{4\pi e^2}\mathrm{Im}\frac{(1+\chi_+)\chi_-}{\epsilon_l}, \tag{8a}$$

$$(\delta n_+^2)_{k,\omega} = \frac{2T}{\omega}\frac{k^2}{4\pi e^2}\mathrm{Im}\frac{(1+\chi_-)\chi_+}{\epsilon_l}, \tag{8b}$$

$$(\delta n_+\delta n_-)_{k,\omega} = (\delta n_-\delta n_+)_{k\omega} = \frac{2T}{\omega}\frac{k^2}{4\pi e^2}\mathrm{Im}\frac{\chi_+\chi_-}{\epsilon_l}, \tag{8c}$$

where $\epsilon_l = 1 + \chi_+ + \chi_-$ is the longitudinal plasma permittivity, χ_+ and χ_- are the electron and positron susceptibilities respectively, and Im refers to the imaginary part of the quantity. Obviously, in the equilibrium state the electron susceptibility χ_- should be equal to the positron susceptibility χ_+, i.e.

$$\chi_+ = \chi_- = \chi. \tag{9}$$

Substituting Eq.(9) into Eq.(8), we obtain the dynamic form factor of a pair plasma as follows:

$$S(\boldsymbol{k},\omega) = \frac{1}{4n\pi}\frac{4T}{\omega}\frac{k^2}{4\pi e^2}\mathrm{Im}\chi. \tag{10}$$

Local density fluctuations slightly break the neutrality of the plasma. As a consequence, fluctuating electric fields are generated, and in turn have a strong influence on the density fluctuations. This is the reason why the plasma permittivity appears in Eq.(8). However, the permittivity disappears in the dynamic form factor of a pair plasma, as seen in Eq.(10). We will address ourselves to this subject in the next section.

3. Dynamic form factor in the collisionless limit

We first discuss the dynamic form factor of a collisionless pair plasma. In this case, the susceptibility χ can be easily obtained from the linearized Vlasov and Poisson equations, and given by

$$\chi = \frac{4\pi n e^2}{Tk^2} Z(\omega/\sqrt{2}kv_\mathrm{T}), \tag{11}$$

where $v_\mathrm{T} = (T/m_\mathrm{e})^{1/2}$ is the particle thermal velocity and $Z(x)$ is the plasma dispersion function defined as[16]

$$Z(x) = 1 - 2x\mathrm{e}^{-x^2}\int_0^\infty \mathrm{e}^{p^2}\mathrm{d}p + \mathrm{i}\pi^{1/2}x\mathrm{e}^{-x^2}. \tag{12}$$

Substituting Eq.(12) into Eqs.(10) and (11), we have

$$S(\boldsymbol{k},\omega) = \frac{1}{\sqrt{2\pi}kv_\mathrm{T}}\exp\left(-\frac{\omega^2}{2k^2v_\mathrm{T}^2}\right). \tag{13}$$

This result was also obtained by Sincell and Krolik with a different method.[17]

Equation (13) clearly shows that the dynamic form factor of a collisionless pair plasma does not depend on the collective motion of the plasma. It is just proportional to the velocity distribution function of the charged particles in the plasma. This result is remarkably different from that of an ordinary plasma. For an ordinary plasma, the dynamic form factor is strongly affected by the collective motion in the plasma.[12] In the so-called collective regime, i.e. $k\lambda_\mathrm{D} \lesssim 1$ where $\lambda_\mathrm{D} = (T/4\pi n_\mathrm{e}e^2)^{1/2}$ is the electron Debye length, the dynamic form factor of an ordinary plasma is greatly enhanced at the frequencies corresponding to the weakly damped eigenmodes in the plasma like electron plasma waves and ion-acoustic waves. Only in the so-called incollective regime, i.e. $k\lambda_\mathrm{D} \gg 1$, can the dynamic form factor be proportional to the electron velocity distribution. In an incollective regime, all collective wave modes are heavily damped. As shown in Eq.(13), however, the dynamic

form factor of a collisionless pair plasma is always in a Gaussian form no matter what the parameter $k\lambda_{\rm D}$ is. This result does not mean that there exist no weakly damped waves in a pair plasma. In fact, it is easy to demonstrate that there exist weakly damped waves in a pair plasma in the case of $k\lambda_{\rm D} \ll 1$ by solving the dispersion equation $\epsilon_l = 1 + 2\chi = 0$. The frequency and damping rate of the waves are respectively given by

$$\omega_{\rm p}^2 \approx 2\omega_{\rm pe}^2[1 + (3/2)(k\lambda_{\rm D})^2]$$

and

$$\frac{\gamma}{\omega_{\rm p}} = -\sqrt{\frac{\pi}{8}}\frac{\omega_{\rm p}^3}{\omega_{\rm pe}^3(k\lambda_{\rm D})^3}\exp\left[-\frac{\omega_{\rm p}^2}{2\omega_{\rm pe}^2(k\lambda_{\rm D})^2}\right],$$

where $\omega_{\rm pe} = 4\pi ne^2/m_{\rm e}$ is the electron plasma frequency. We can see that $|\gamma/\omega_{\rm p}| \ll 1$ in the case of $k\lambda_{\rm D} \ll 1$. These weakly damped eigenmodes, however, have not any effects on the dynamic form factor of the Thomson scattering. This fact should be attributed to the mass symmetry of the pair (electron–positron) plasma.

The reason that the dynamic form factor (6) is not affected by any collective motions can be seen more clearly from the kinetic equation of fluctuations. As seen in Eq.(6), the dynamic form factor can be rewritten as

$$S(\boldsymbol{k},\omega) = \frac{1}{4\pi N}[(\delta n_+ + \delta n_-)^2]_{k,\omega}. \tag{14}$$

On the other hand, the exact distribution functions

$$\mathcal{F}_+(\boldsymbol{r},\boldsymbol{v},t) = \sum_{i=1}^{N}\delta[\boldsymbol{r} - \boldsymbol{r}_{+j}(t)]\delta[\boldsymbol{v} - \boldsymbol{v}_{+j}(t)],$$

$$\mathcal{F}_-(\boldsymbol{r},\boldsymbol{v},t) = \sum_{i=1}^{N}\delta[\boldsymbol{r} - \boldsymbol{r}_{-j}(t)]\delta[\boldsymbol{v} - \boldsymbol{v}_{-j}(t)],$$

satisfy the Klimontovich equations[18]

$$\frac{\partial\mathcal{F}_+}{\partial t} + \boldsymbol{v}\cdot\frac{\partial\mathcal{F}_+}{\partial\boldsymbol{r}} - \frac{e}{m}\frac{\partial\varphi}{\partial\boldsymbol{r}}\cdot\frac{\partial\mathcal{F}_+}{\partial\boldsymbol{v}} = 0,$$

$$\frac{\partial\mathcal{F}_-}{\partial t} + \boldsymbol{v}\cdot\frac{\partial\mathcal{F}_-}{\partial\boldsymbol{r}} - \frac{e}{m}\frac{\partial\varphi}{\partial\boldsymbol{r}}\cdot\frac{\partial\mathcal{F}_-}{\partial\boldsymbol{v}} = 0,$$

where φ is the electric potential in the plasma. These exact distribution functions can be written as

$$\mathcal{F}_{+,-} = f(\boldsymbol{v}) + \delta f_{+,-}, \tag{15}$$

where $f(\boldsymbol{v})$ is the ensemble averaged velocity distribution function, and $\delta f_{+,-}$ are the fluctuations of the exact distribution functions. The density fluctuations are functionals of the fluctuations $\delta f_{+,-}$,

$$\delta n_{+,-} = \int \delta f_{+,-}\,{\rm d}^3\boldsymbol{v}. \tag{16}$$

Neglecting the influence of collisions on the evolutions of the fluctuations, it is easy to find that the fluctuations $\delta f_{+,-}$ satisfy the following equations:[18]

$$\frac{\partial\delta f_+}{\partial t} + \boldsymbol{v}\cdot\frac{\partial\delta f_+}{\partial\boldsymbol{r}} - \frac{e}{m}\frac{\partial\delta\varphi}{\partial\boldsymbol{r}}\cdot\frac{\partial f(\boldsymbol{v})}{\partial\boldsymbol{v}} = 0, \tag{17a}$$

$$\frac{\partial\delta f_-}{\partial t} + \boldsymbol{v}\cdot\frac{\partial\delta f_-}{\partial\boldsymbol{r}} + \frac{e}{m}\frac{\partial\delta\varphi}{\partial\boldsymbol{r}}\cdot\frac{\partial f(\boldsymbol{v})}{\partial\boldsymbol{v}} = 0, \tag{17b}$$

where $\delta\varphi$ is the fluctuating potential of the plasma that depends on the particle fluctuations through the Poisson equation,

$$\nabla^2\delta\varphi = -4\pi e\int(\delta f_+ - \delta f_-)\,d^3\boldsymbol{v}.$$

Summing the equations (17a) and (17b), we obtain

$$\frac{\partial}{\partial t}(\delta f_+ + \delta f_-) + \boldsymbol{v}\cdot\frac{\partial}{\partial\boldsymbol{r}}(\delta f_+ + \delta f_-) = 0. \tag{18}$$

This equation clearly shows that the evolution of the fluctuating quantity $(\delta f_+ + \delta f_-)$ does not depend on any self-consistent electric fields in the plasma although the evolutions of δf_+ and δf_- themselves involve the electric fields. Therefore, the density fluctuation $(\delta n_+ + \delta n_-)$ can not be affected by any collective modes, but it is essentially caused by the self-consistent fields in the plasma.

Physically, the fact that Thomson scattering off a pair plasma is independent of the collective motion in the plasma can be understood in the following way. The motion of a particle in the plasma consists of two parts: one is the thermal motion, the other is collective motion. Accordingly, the scattering spectrum due to either the positively- or the negatively- charged particles can also be separated into two parts: incollective and collective parts. For example, we can rewrite Eqs.(8a) and (8b) respectively as

$$(\delta n_{-,+}^2)_{k,\omega} = \frac{2T}{\omega}\frac{k^2}{4\pi e^2}\left({\rm Im}\chi - {\rm Im}\frac{\chi^2}{\epsilon_l}\right), \tag{19a}$$

$$(\delta n_-\delta n_+)_{k,\omega} = \frac{2T}{\omega}\frac{k^2}{4\pi e^2}{\rm Im}\frac{\chi^2}{\epsilon}. \tag{19b}$$

Obviously, the first term on the right-hand side of Eq.(19a) originates from the thermal motion of the particles and the second term stems from the collective motion. Since the electrons and positrons have the opposite charge number, there is a phase difference π between their collective motions at any time. Therefore, the correlation between the electrons and positrons has an opposite sign as seen in Eq.(19b), and

then cancels the influence of collective motions of the particles on the Thomson scattering spectrum.

4. Dynamic form factor in the collisional limit

The conclusion we obtained in the last section is valid only in the case where the pair plasma is collisionless because the susceptibility χ is obtained from the Vlasov equation. Collisions in a plasma can affect the fluctuations. In general, it is quite difficult to calculate the effect of collisions on fluctuations in a plasma. However, in the collisional limit, the problem becomes easier to solve, because frequent collisions lead a plasma to an equilibrium state so quickly that the plasma could be accurately described with a set of hydrodynamic equations. What we have to do is to calculate the transport coefficients of the pair plasma. After we obtain the hydrodynamic equations, we can calculate the density fluctuations with the aid of the theory of hydrodynamic fluctuations.

The hydrodynamic equations of a pair plasma in the collisional limit can be obtained by solving the transport equations, which are given by

$$\begin{aligned}&\frac{\partial f_-}{\partial t}+\boldsymbol{v}\cdot\nabla f_- -\frac{e}{m_{\rm e}}\boldsymbol{E}\cdot\frac{\partial f_-}{\partial \boldsymbol{v}}\\ =&C(f_-,f_-)+C(f_-,f_+),\end{aligned}\tag{20a}$$

$$\begin{aligned}&\frac{\partial f_+}{\partial t}+\boldsymbol{v}\cdot\nabla f_+ +\frac{e}{m_{\rm e}}\boldsymbol{E}\cdot\frac{\partial f_+}{\partial \boldsymbol{v}}\\ =&C(f_+,f_+)+C(f_+,f_-),\end{aligned}\tag{20b}$$

where $f_\pm$ are the ensemble averaged distribution functions of the positively- and negatively- charged particles, $\boldsymbol{E}$ is the electric field in the plasma, and $C(f,f)$ is the collisional term. Transport coefficients can be obtained from the set of equations in the collisional limit.

We assume that the plasma is in local-equilibrium. Hence the distribution functions of the charged particles can be written as

$$f_-=f_0+f'_-,\quad f_+=f_0+f'_+,\tag{21}$$

where f_0 is the local-equilibrium distribution function, and $f'_\pm$ are the small deviations. Here we assume that the negatively- charged particles have the same local-equilibrium distribution function as the positively- charged particles. The local-equilibrium distribution function f_0 depends on time and space position through the hydrodynamic variables n, T and $\boldsymbol{u}$,

$$f_0=\frac{n}{(2\pi)^{3/2}v_{\rm T}^3}\exp\left[-\frac{m_{\rm e}(\boldsymbol{v}-\boldsymbol{u})^2}{2T}\right],\tag{22}$$

where n is the charge number density, T is the temperature, and $\boldsymbol{u}$ is the charge flow velocity. Substituting the expression (21) into Eq.(20), and linearizing the two resulting equations, we have

$$\frac{\partial f_0}{\partial t}+\boldsymbol{v}\cdot\nabla f_0-\frac{e}{m_{\rm e}}\boldsymbol{E}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}=C(f_0,f'_-)+C(f'_-,f_0)+C(f_0,f'_+)+C(f'_-,f_0),\tag{23a}$$

$$\frac{\partial f_0}{\partial t}+\boldsymbol{v}\cdot\nabla f_0+\frac{e}{m_{\rm e}}\boldsymbol{E}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}=C(f_0,f'_+)+C(f'_+,f_0)+C(f_0,f'_-)+C(f'_+,f_0).\tag{23b}$$

Summing the two equations of Eq.(23), we have

$$\frac{\partial f_0}{\partial t}+\boldsymbol{v}\cdot\nabla f_0=C(f_0,f'_-+f'_+)+C(f'_-+f'_+,f_0).$$

Let

$$F=f'_-+f'_+,$$

we have

$$\frac{\partial f_0}{\partial t}+\boldsymbol{v}\cdot\nabla f_0=C(f_0,F)+C(F,f_0).\tag{24}$$

Substituting the local-equilibrium distribution function (22) into the left hand side of transport equation (24), we obtain the transport equation by following the standard procedure in Ref.[18],

$$f_0\left\{\left(\frac{mv^2}{2T}-\frac{5}{2}\right)\boldsymbol{v}\cdot\nabla\ln T+\frac{m}{T}\left[v_\alpha v_\beta-\frac{v^2}{3}\delta_{\alpha\beta}\right]V_{\alpha\beta}\right\}=C(f_0,F)+C(F,f_0),\tag{25}$$

where

$$V_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_a}\right). \quad (26)$$

The transport equation (25) can be solved by using the well-known Chapman–Enskog method.[18] Expanding the small deviation F in a series of the Sonine polynomials, we obtain the thermal conductivity κ and viscosity η of the pair plasma,[19]

$$\kappa = 3.95 n v_{\mathrm{T}}^2 \tau, \quad (27)$$

and

$$\eta = 0.965 n T \tau, \quad (28)$$

where τ is the collision time defined as

$$\tau = \frac{3\sqrt{m_{\mathrm{e}}}T^{3/2}}{4\sqrt{\pi}e^4 n \ln \Lambda}, \quad (29)$$

and $\ln \Lambda$ is the Coulomb logarithm.

The hydrodynamic equations for the pair plasma are then given by

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{u}) = 0,$$

$$m_{\mathrm{e}} n \left(\frac{\partial u_\alpha}{\partial t} + u_\beta \frac{\partial u_\alpha}{\partial x_\beta}\right) = -\frac{\partial p}{\partial x_\alpha} - \frac{\partial \pi_{\alpha\beta}}{\partial x_\beta},$$

$$\frac{3}{2} n \left(\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T\right) = -p \nabla \cdot \boldsymbol{u} + \nabla \cdot (\kappa \nabla T) - \pi_{\alpha\beta} \frac{\partial u_\alpha}{\partial x_\beta},$$

where $p = nT$ is the thermal pressure, and $\pi_{\alpha\beta}$ is the viscous stress tensor defined as

$$\pi_{\alpha\beta} = -\eta \left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta} \nabla \cdot \boldsymbol{u}\right).$$

Based on the theory of hydrodynamic fluctuations,[20] we obtain the dynamic form factor,

$$S(k,\omega) = \frac{c_{\mathrm{p}} - c_{\mathrm{v}}}{c_{\mathrm{p}}} \frac{2\kappa k^2/n c_{\mathrm{p}}}{(\kappa k^2/n c_{\mathrm{p}})^2 + \omega^2} + \frac{c_{\mathrm{v}}}{c_{\mathrm{p}}} \left[\frac{\gamma k^2}{(\gamma k^2)^2 + (\omega + c_{\mathrm{s}} k)^2} + \frac{\gamma k^2}{(\gamma k^2)^2 + (\omega - c_{\mathrm{s}} k)^2}\right], \quad (30)$$

where $c_{\mathrm{v}} = 3/2$ is the specific heat at constant volume, $c_{\mathrm{p}} = 5/2$ is the specific heat at constant pressure, $c_{\mathrm{s}} = (5T/3m_{\mathrm{e}})^{1/2}$ is the adiabatic sound speed of the pair plasma, and

$$\gamma = \frac{1}{2}\left[\frac{4}{3}\frac{\eta}{m_{\mathrm{e}} n_0} + \frac{\kappa}{n_0}\left(\frac{1}{c_{\mathrm{v}}} - \frac{1}{c_{\mathrm{p}}}\right)\right] = 1.17 v_{\mathrm{T}}^2 \tau. \quad (31)$$

As seen in Eq.(30), the scattering spectrum is no longer proportional to the velocity distribution function of the charged particles in the collisional limit. It is determined by the transport process in the plasma. The spectrum presents three spikes. Two peaks locating at $\omega = \pm k c_{\mathrm{s}}$ correspond to the two contrapropagating acoustic waves. The other locating at $\omega = 0$ corresponds to the entropy waves. The entropy waves are non-propagation waves because no pressure fluctuations arise in the processes. It should be emphasized that Eq.(30) is valid in the case of $\omega\tau \ll 1$ and $k\tau v_{\mathrm{T}} \ll 1$.

A few more remarks should be made about the sound waves. The sound speed c_{s} is close to the thermal speed of the charges in the plasma. If the collisions in the plasma are rare, the sound waves are heavily damped due to the Landau damping. Therefore, these waves can only exist in the collisionless limit in which it is the frequent collisions that break off the wave-particle interactions, and make the Landau damping negligible. In this case, the damping of the sound waves is determined by some dissipation processes in the plasmas, such as the thermal conduction and viscosity, as seen in Eq.(31).

5. Summary

In summary, Thomson scattering off a pair (electron–positron) plasma has been theoretically investigated in two limits separately. The power spectrum of the Thomson scattering off a pair plasma is just proportional to the velocity distribution function of the particles in the collisionless limit, and presents three peaks corresponding to two sound waves and an entropy wave in collisional limit. In the collisionless limit, collective modes in the plasma do not have any effects on the Thomson scattering spectrum because the correlation between the negatively- and positively-charged particles can cancel the effect of the collective motions on the scattering spectrum when the plasma is in mass symmetry. The calculation of the dynamic form factor is much more difficult in the intermediate-collision regime. In principle, this problem can be solved with the fluctuation–dissipation theorem and

the linearized Fokker–Planck equations, as we have done in Ref.[21].

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