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Direct acceleration of electrons using two crossed Laguerre–Gaussian laser beams in vacuum*

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The basic physical characteristics of electrons accelerated by two linearly polarized and circularly symmetric crossed Laguerre–Gaussian (LG) laser beams with equal frequency and amplitude in vacuum are studied in detail. The condition, under which electrons can be accelerated effectively, and the energy gain are discussed.

Keywords: electron acceleration, crossed Laguerre–Gaussian (LG) beams, slippage distance, energy gain

PACC: 4260, 4262

1. Introduction

The advent of ultrashort high-power laser technology capable of building up terawatt and petawatt laser systems has given rise to a renewed interest in the possibility of directly accelerating electrons in vacuum by ultrahigh-power lasers.[1−4] Scully and Zubairy proposed the idea that particles were directly accelerated by using the longitudinal electric field of a focused laser beam.[5] Then Esarey and Sprangle made a detailed study of the laser acceleration of electrons in vacuum.[6,7] The methods of the electron acceleration by a single and two crossed beams, such as by a single asymmetric Hermite–Gaussian (HG) beam, Bessel beam and two crossed Gaussian beams, two crossed plane waves etc., were proposed and analysed.[5−10]

Like HG beams, Laguerre–Gaussian beams are of considerable importance both in laser theory and practical applications.[11] The purpose of the present paper is to study the direct acceleration of electrons by two linearly polarized and circularly symmetric crossed LG beams with equal frequency and amplitude. In Section 2 the theoretical formalism of electric-field components of two linearly polarized and circularly symmetric crossed LG beams is presented. The basic physical characteristics of using the resulting longitudinal electric field on the z-axis of two crossed LG beams are discussed in Section 3. Finally, the main results obtained in the paper are summarized in Section 4.

2. Theoretical formulism

Consider two linearly polarized and circularly symmetric LG beams with equal frequency and amplitude in vacuum. The first LG beam propagates along the z_1-axis and the second one along the z_2-axis. The z_1- and z_2-axes are rotated through angles θ and $-\theta$ with respect to the z-axis, respectively, as shown in Fig.1. The transverse electric-field components in the cylindrical coordinate system read.[11]

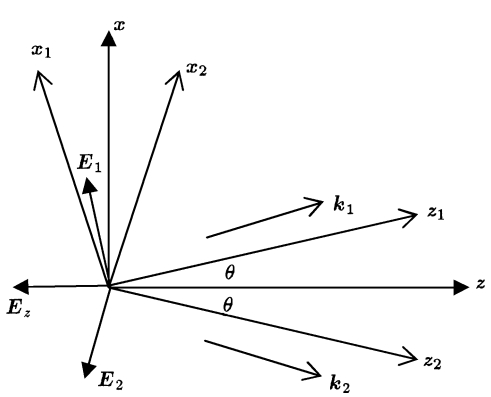


Fig.1. Rectangular coordinate system and electric fields of two crossed LG laser beams.

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$$E_{xj}(r_j,\varphi_j,z_j,t) = E_0\frac{w_0}{w_j}L_p\left(\frac{2r_j^2}{w_j^2}\right) \times \exp\left(-\frac{r_j^2}{w_j^2}\right)\exp(-i\psi_j), \quad (1a)$$

$$E_{yj}(r_j,\varphi_j,z_j,t) = 0, \quad (1b)$$

where j=1, 2 (unless otherwise stated), $E_0 > 0$ is the amplitude (a constant), $r_j=(x_j^2+y_j^2)^{1/2}$, $w_j(z_j) = w_0(1+z_j^2/Z_{\rm R}^2)^{1/2}$ is the width of the corresponding Gaussian beam, w_0 is the waist width, $Z_{\rm R}=\pi w_0^2/\lambda$ is the Rayleigh length, the wavelength λ is related to the wave number k and frequency ω by $k=2\pi/\lambda=\omega/c$ with c being the speed of light in vacuum, $L_p(\cdot)$ denotes the Laguerre polynomial with index p, and the phase term ψ_j is

$$\psi_j = kz_j - \omega t + \frac{z_j r_j^2}{Z_{\rm R}w_j(z_j)^2} - (2p+1)\arctan\left(\frac{z_j}{Z_{\rm R}}\right) + \phi_{0j}, \quad (2)$$

where ϕ_{0j} is the initial phase (a constant), t is the time.

Recalling the relation between the transverse and longitudinal electric-field components $\boldsymbol{E}_\perp$ and E_z, i.e, $E_z = -(\mathrm{i}/k)\nabla\cdot\boldsymbol{E}_\perp$,[12] and taking into consideration $\partial L_p(u)/\partial u = -L_{P-1}^1(u)$, in the paraxial approximation the longitudinal electric-field component E_z of the jth LG beam is obtained and in the rectangular coordinate system is expressed as

$$E_{zj} = \frac{2\mathrm{i}E_0w_0x_j}{kw_j^3}\exp\left(-\frac{r_j^2}{w_j^2}\right)\left[2L_{P-1}^1\left(\frac{2r_j^2}{w_j^2}\right) + \left(1+\mathrm{i}\frac{z_j}{Z_{\rm R}}\right)L_{\rm p}\left(\frac{2r_j^2}{w_j^2}\right)\right]\exp(-\psi_j). \quad (3)$$

From Eqs.(1) and (3) the total transverse and longitudinal electric-field components are given by

$$E_x(x,y,z) = [E_{x1}(x_1,y_1,z_1)+E_{x2}(x_2,y_2,z_2)]\cos\theta + [E_{z1}(x_1,y_1,z_1) - E_{z2}(x_2,y_2,z_2)]\sin\theta, \quad (4a)$$

$$E_y(x,y,z) = 0, \quad (4b)$$

$$E_z(x,y,z) = -[E_{x1}(x_1,y_1,z_1)-E_{x2}(x_2,y_2,z_2)]\sin\theta + [E_{z1}(x_1,y_1,z_1) + E_{z2}(x_2,y_2,z_2)]\cos\theta, \quad (4c)$$

where $x_1 = x\cos\theta - z\sin\theta$, $y_1 = y$, $z_1 = x\sin\theta + z\cos\theta$, $x_2 = x\cos\theta + z\sin\theta$, $y_2 = y$, $z_2 = -x\sin\theta + z\cos\theta$.

Substituting $y = y_1 = y_2$=0, x=0 into Eqs.(4), the resulting transversal and longitudinal electric-field components on the z axis are expressed as

$$E_x(0,0,z) = \frac{2E_0w_0}{kw^3zZ_{\rm R}}\exp(-\alpha^2)\left\{2z\cos\theta\left(Z_{\rm R}^2+z^2\right)L_{\rm p}(2\alpha^2) + \mathrm{i}Z_{\rm R}\left[-2w^2\alpha^2\left(L_{\rm p}(2\alpha^2)+2L_{p-1}^1(2\alpha^2)\right)\right]\right\}\exp(-\mathrm{i}\psi_{\rm M})\cos\frac{\phi_{01}-\phi_{02}}{2}, \quad (5a)$$

$$E_z(0,0,z) = \frac{2E_0w_0\sin\theta}{kw^3}\exp(-\alpha^2)\left\{\mathrm{i}2Z_{\rm R}\,L_{\rm p}(2\alpha^2) - 2z\cos\theta\left[L_{\rm p}(2\alpha^2)+2L_{p-1}^1(2\alpha^2)\right]\right\}\exp(-\mathrm{i}\psi_{\rm M})\sin\frac{\phi_{01}-\phi_{02}}{2}, \quad (5b)$$

where $w = w_0(1+z^2\cos^2\theta/Z_{\rm R}^2)^{1/2}$, $\alpha = z\sin\theta/w$,

$$\psi_{\rm M} = kz\cos\theta - \omega t + \frac{z^3\cos\theta\sin^2\theta}{Z_{\rm R}w^2} - (2p+1)\arctan\left(\frac{z\cos\theta}{Z_{\rm R}}\right) + \frac{(\phi_{01}+\phi_{02})}{2}. \quad (6)$$

Equations (5) indicate that the resulting transverse and longitudinal-field components on the z-axis depend on the amplitude E_0, initial phase ϕ_{0j}, mode index p, angle θ and propagation distance z in general.

3. Laser-accelerated electrons in vacuum

The basic idea of laser-accelerated electrons in vacuum is to use the axial longitudinal electric-field component $E_z(0,0,z)$ on the z-axis for accelerating electrons, which requires $E_z(0,0,z)$ to be non-zero and maximized. From Eqs.(5) it can be seen that, if $\phi_{01}-\phi_{02}=\pi$, the resulting transverse field component $E_x(0,0,z)$ on the z-axis vanishes and the resulting longitudinal-field component on the z-axis reaches a maximum. It means that the initial phase difference

$(\phi_{01}-\phi_{02})$ plays a key role in accelerating electrons, and the two circularly symmetric crossed LG beams with a π-rad phase difference are favourable for directly accelerating electrons.

Assume that $\phi_{01}-\phi_{02}=\pi$, the real part of Eq.(5b) is written as

$$E_z(0,0,z) = -\frac{4E_0 w_0 \alpha}{kw^2}\exp(-\alpha^2) \times \left\{ \frac{Z_{\rm R}}{z} L_{\rm p}\left(2\alpha^2\right)\cos\psi_t + \cos\theta[L_{\rm p}\left(2\alpha^2\right) + 2L_{p-1}^1\left(2\alpha^2\right)]\sin\psi_t \right\}, \quad (7)$$

where

$$\psi_t = kz\cos\theta - \omega t + \frac{z^3\cos\theta\sin^2\theta}{Z_{\rm R}w^2} - (2p+1)\arctan\left(\frac{z\cos\theta}{Z_{\rm R}}\right) + \phi_{01}. \quad (8)$$

The corresponding transversal and longitudinal magnetic-field components can be found from the relation $\boldsymbol{B} = -(\mathrm{i}/\omega)\nabla\times\boldsymbol{E}$ and given by

$$B_x(0,0,z) = B_y(0,0,z) = B_z(0,0,z) = 0. \quad (9)$$

If $\theta << 1$, one has $\alpha\to 0$, $L_p(0)=1$, and $L_p^1(0)=p+1$. Then Eq.(7) becomes

$$E_z(0,0,\hat{z}) = -\frac{2E_0\theta}{(1+\hat{z}^2)^{3/2}}\left[1+(2p+1)^2\hat{z}^2\right]^{1/2} \times \exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right)\cos\psi, \quad (10)$$

where

$$\psi = -\frac{1}{\theta_0^2}\int\frac{\mathrm{d}\hat{z}}{\gamma_z^2} - \frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - (2p+1)\arctan\hat{z} - \arctan\left[(2p+1)\hat{z}\right] + \phi_{01}, \quad (11)$$

$t = \int \mathrm{d}z/v_z$, $\gamma_z = (1-v_z^2/c^2)^{-1/2}$, v_z and t being the electron velocity along the z axis, and time respectively. $\hat{z} = z/Z_{\rm R}$ is the propagation distance normalized to the Rayleigh length , $\theta_0 = w_0/Z_{\rm R}$ is the diffraction angle.

From Eq.(11) the phase velocity of the accelerating field can be expressed as[13]

$$v_{\rm ph} = \omega/|\nabla\psi| = c\left\{1-\left[\frac{\left(1-\hat{z}^2\right)\theta^2+(2p+1)\left(1+\hat{z}^2\right)\theta_0^2+\tau^2\theta_0^2}{2\left(1+\hat{z}^2\right)^2}\right]\right\}^{-1}, \quad (12)$$

where $\tau = \left(1+\hat{z}^2\right)(2p+1)^{1/2}\left[1+\hat{z}^2(2p+1)^2\right]^{-1/2}$. Near the waist $|z|\le Z_{\rm R}$, if

$$\frac{\left(1-\hat{z}^2\right)\theta^2+(2p+1)\left(1+\hat{z}^2\right)\theta_0^2+\tau^2}{2\left(1+\hat{z}^2\right)^2} << 1,$$

we have

$$\frac{v_{\rm ph}}{c} \approx 1+\frac{\theta^2+2(2p+1)\theta_0^2}{2} = 1+\frac{1}{2\gamma_{\rm c}^2} > 1, \quad (13)$$

where $\gamma_c=[\theta^2+2(2p+1)\theta_0^2]^{-1/2}$ is the critical energy. Equation (13) implies that the phase velocity $v_{\rm ph}$ is greater than the speed of light c in vacuum and the axial field slips ahead of the electrons. The slippage distance $z_{\rm s}$, which is defined as the distance over which an electron phase slips by π, is

$$z_{\rm s} \approx \pi\left(\omega\left|v_{\rm ph}^{-1}-v_z^{-1}\right|\right)^{-1} = \frac{\gamma_{\rm c}^2\lambda}{1+\gamma_{\rm c}^2/\gamma_z^2} = \frac{\lambda}{\theta^2+2(2p+1)\theta_0^2}. \quad (14)$$

The normalized accelerating longitudinal field $-E_z(0,0,z)/E_0$ of two circularly symmetric crossed LG beams versus the normalized distance z/λ given by Eq.(10) is shown in Fig.2(a) at w_0/λ=5, p=0, 1, 2 and Fig.2(b) at p=1, w_0/λ=5, 8, 15, and θ=0.1rad. As can be seen, both the waist width-to-wavelength ratio w_0/λ and mode index p can affect the accelerating field, and the slippage distance $z_{\rm s}$ increases as p decreases and w_0 increases, which is consistent with the result from Eq.(14).

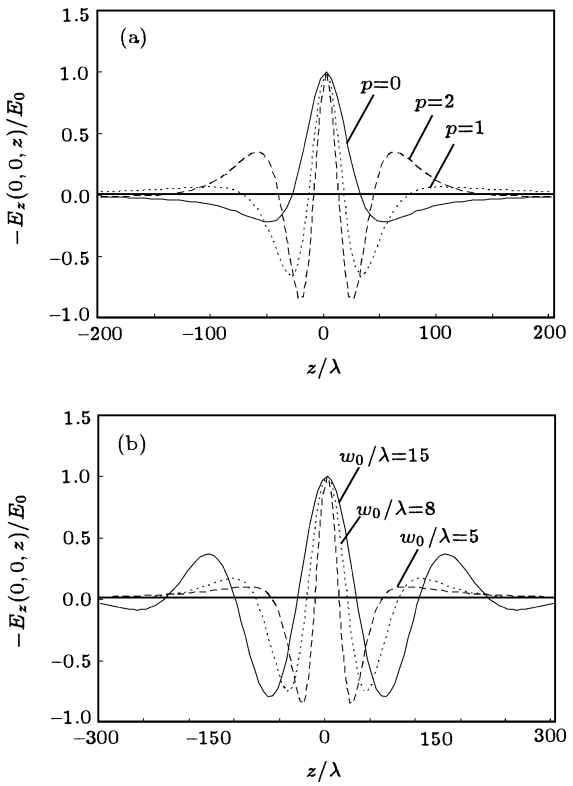


Fig.2. Normalized accelerating axial field $-E_z(0,0,z)/E_0$ versus normalized axial distance z/λ for two crossed LG beams at (a) θ=0.1rad, w_0/λ=5, p=0, 1, 2, and (b) θ=0.1rad, p=1, w_0/λ=5, 8, 15.

In the low-energy limit $\gamma_z << \gamma_c$, slippage is dominated by the low electron velocity, i.e. the first term in Eq.(11), whereas in the high-energy limit $\gamma_z >> \gamma_c$, slippage is dominated by diffraction, i.e. the rest terms except the first one in Eq.(11). In the case of our interest the accelerating axial electric field can be written as the gradient of a potential, i.e. $E_z = -Z_R^{-1}\partial U/\partial\hat{z}$, where

$$U(\hat{z}) = \frac{2E_0\theta w_0}{\theta_0}\int \frac{1}{(1+\hat{z}^2)^{3/2}}\left[1+(2p+1)^2\hat{z}^2\right]^{1/2} \times \exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right) \times \cos\left[-\frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - (2p+1)\arctan\hat{z} - \arctan\left((2p+1)\hat{z}\right) + \phi_{01}\right]\mathrm{d}\hat{z}. \quad (15)$$

Equation (15) indicates that the potential is dependent on the waist width w_0, mode index p, amplitude E_0, angle θ and axial distance z. The influences of w_0/λ and p on the potential are shown in Fig.3 by using Eq.(15), where the normalized accelerating potential $U(z)/U_0$ versus the normalized distance z/λ is plotted at (a) p=0, 1, 2, w_0/λ=5, and (b) w_0/λ=5, 8, 15, p=1, and θ=0.1rad. The energy gain ΔW from the injection point z_I to the extraction point z_F is determined by $\Delta W = e[U(z_F) - U(z_I)]$ with e being electron charge. In the special case of $\hat{z}_F = -\hat{z}_I = \hat{z}_0$, the energy gain is given by

$$\Delta W = \frac{2eE_0\theta w_0}{\theta_0}\int_{-\hat{z}_0}^{\hat{z}_0} \frac{1}{(1+\hat{z}^2)^{3/2}}\left[1+\hat{z}^2(2p+1)^2\right]^{1/2} \times \exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right) \times \cos\left[-\frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - (2p+1)\arctan\hat{z} - \arctan\left(\hat{z}(2p+1)\right) + \phi_{01}\right]\mathrm{d}\hat{z}. \quad (16)$$

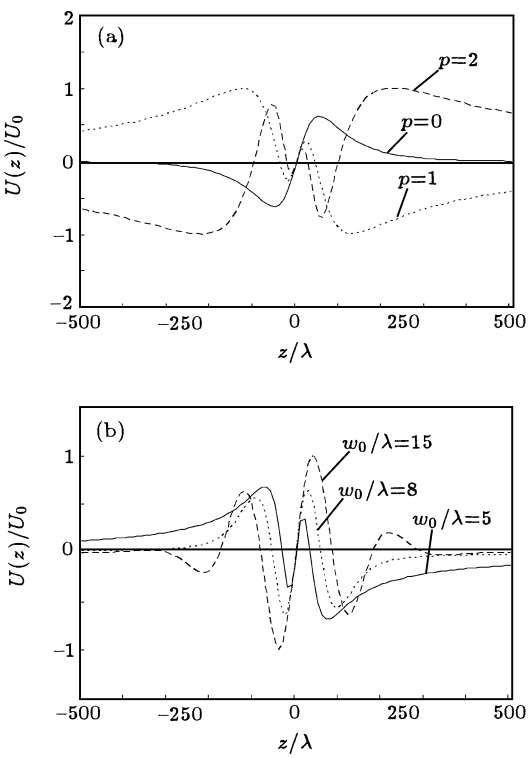


Fig.3. Normalized $U(z)/U_0$ versus normalized axial distance z/λ for two crossed LG beams at (a) θ=0.1rad, w_0/λ=5, p=0, 1, 2, and (b) θ=0.1rad, p=1, w_0/λ=5, 8, 15.

For p=0 Eqs.(10), (11), (15) and (16) reduce to

$$E_z(0,0,\hat{z}) = -\frac{2E_0\theta}{1+\hat{z}^2}\exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right)\cos\psi_0, \tag{17}$$

$$\psi_0 = -\frac{1}{\gamma_z^2\theta_0^2}\int \mathrm{d}\hat{z} - \frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - 2\arctan\hat{z} + \phi_{01}, \tag{18}$$

$$U(\hat{z}) = \frac{2E_0 Z_{\mathrm{R}}\theta_0^2}{\theta}\exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right)\sin\left[\frac{\hat{z}\theta^2}{(1+\hat{z}^2)\,\theta_0^2} - \phi_{01}\right], \tag{19}$$

$$\Delta W = \frac{4eE_0 Z_{\mathrm{R}}\theta_0^2}{\theta}\cos\phi_{01}\exp\left[-\frac{(\hat{z}_0\theta/\theta_0)^2}{1+\hat{z}_0^2}\right]\sin\left[\frac{\hat{z}_0\theta^2}{(1+\hat{z}_0^2)\theta_0^2}\right], \tag{20}$$

respectively. Eqs.(17)–(20) are consistent with those presented in Refs.[5,6].

For p=1, Eqs.(10), (11), (15) and (16) become

$$E_z(0,0,\hat{z}) = -\frac{2E_0\theta}{(1+\hat{z}^2)^{3/2}}\left(1+9\hat{z}^2\right)^{1/2}\exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right)\cos\psi_1, \tag{21}$$

$$\psi_1 = -\frac{1}{\gamma_z^2\theta_0^2}\int \mathrm{d}\hat{z} - \frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - 3\arctan\hat{z} - \arctan 3\hat{z} + \phi_{01}, \tag{22}$$

$$\begin{aligned} U(\hat{z}) =& \frac{2E_0 w_0\theta_0}{(1+\hat{z}^2)\theta^3}\exp\left(-\frac{(\hat{z}\theta/\theta_0)^2}{1+\hat{z}^2}\right)\left\{4\hat{z}\theta^2\cos\left(\frac{\hat{z}(\theta/\theta_0)^2}{1+\hat{z}^2} - \phi_{01}\right)\right. \\ &\left. - \left[\left(3\hat{z}^2-1\right)\theta^2 + 4\left(\hat{z}^2+1\right)\theta_0^2\right]\sin\left[\frac{\hat{z}\,(\theta/\theta_0)^2}{1+\hat{z}^2} - \phi_{01}\right]\right\}, \end{aligned} \tag{23}$$

$$\begin{aligned} \Delta W =& \frac{4eE_0 w_0\theta_0}{(1+\hat{z}_0^2)\,\theta^3}\exp\left(-\frac{(\hat{z}_0\theta/\theta_0)^2}{1+\hat{z}_0^2}\right)\cos\phi_{01}\left\{4\hat{z}_0\theta^2\cos\left(\frac{\hat{z}_0(\theta/\theta_0)^2}{1+\hat{z}_0^2}\right)\right. \\ &\left. - \left[\left(3\hat{z}_0^2-1\right)\theta^2 + 4\left(\hat{z}_0^2+1\right)\theta_0^2\right]\sin\left[\frac{\hat{z}_0\,(\theta/\theta_0)^2}{1+\hat{z}_0^2}\right]\right\}, \end{aligned} \tag{24}$$

respectively. From Eqs.(20) and (24) we see that $\Delta W \to 0$ as $\hat{z}_0 \to \infty$, which is in agreement with that obtained from the Lawson–Woodward theorem.[14,15]

A finite energy gain ΔW takes place over a finite interaction range $2z_0$. From Eq.(24), it follows that the energy gain ΔW is maximized when $\hat{z}_0$ satisfies

$$\tan\left[\frac{\hat{z}_0\,(\theta/\theta_0)^2}{1+\hat{z}_0^2}\right] = \frac{3\hat{z}_0^4 - 12\hat{z}_0^2 + 1}{2\hat{z}_0\,(3-5\hat{z}_0^2)}. \tag{25}$$

For the two crossed LG beams with index p=1, and λ=1μm, θ=0.1rad, w_0/λ=5, 15, we have the optical interaction distances z_{s}=29 μm, 78.7 μm, and diffraction angles θ_0=63mrad, 21mrad, respectively. Within the interaction distance $z_{\mathrm{s}} = 2z_0$, ΔW (Mev)=16.8$P^{1/2}$ (TW)$^{1/2}$ (P-laser power) for w_0/λ=5 and ΔW (Mev)=14.6$P^{1/2}$ (TW)$^{1/2}$ for w_0/λ=15. Therefore, for P=20 TW we obtain the maximum energy gains ΔW =75 Mev, 65 Mev and average accelerating gradients $< E_z >= \Delta W/ez_{\mathrm{s}}$=2.577 TV/m, 0.827 TV/m for w_0/λ=5 and 15, respectively. Similarly, for p=0, under the same condition we have z_{s}=55μm, 92μm, ΔW=133 Mev, 75.4 Mev, and $< E_z >$=2.41 TV/m, 0.82 TV/m for w_0/λ=5 and 15, respectively. The results show that for a fixed laser power P, ΔW increase as w_0/λ and p decrease. However, z_{s} increases with the increase of w_0/λ and decrease of p.

4. Conclusions

In this paper the direct acceleration of electrons by using two linearly polarized and circularly symmetric crossed LG beams with equal frequency and amplitude is proposed and studied. It is shown that the initial phase difference plays a key role. For two linearly polarized crossed LG beams with a phase difference of π-rad the resulting axial transverse electric field, axial transverse and longitudinal magnetic fields disappear, and the resulting longitudinal electric field achieves a maximum, which can be used to effectively accelerate electrons in vacuum.

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