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Hyperchaos evolved from the Liu chaotic system

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This paper introduces a new hyperchaotic system by adding an additional state into the third-order Liu chaotic system. Some of its basic dynamical properties, such as the hyperchaotic attractor, Lyapunov exponent, fractal dimension and the hyperchaotic attractor evolving into chaotic, periodic, quasi-periodic dynamical behaviours by varying parameter d are studied briefly. Various attractors are illustrated not only by computer simulation but also by conducting an electronic circuit experiment.

Keywords: hyperchaos, circuit implementation, Liu chaotic system
PACC: 0545

1. Introduction

Hyperchaos, with more than one positive exponent, has been studied with increasing interest in the last few years due to its great potential in technological applications, such as in the fields of secure communication,[1] lasers,[2] and so on. So, the generation, control, synchronization and application of hyperchaos have recently become a central topic in research.[3−10] During the past thirty years, a large number of various hyperchaotic systems have been proposed by several researchers.[5−8] However, the two most well known hyperchaotic systems are the hyperchaotic Chua circuit[5] and the Rossler system.[7]

In this paper, the interesting problem of generating hyperchaos by adding an additional state into the third-order Liu chaotic system is studied which relies on one multiplier and one quadratic term.[11] Some of its basic dynamical properties, such as the hyperchaotic attractor, Lyapunov exponent and fractal dimension evolving into periodic, quasi-periodic dynamical behaviours by varying the system's parameter d have been discussed in detailed numerical and theoretical analyses. Furthermore, various attractors are illustrated not only by computer simulation but also by the experiment on an electronic circuit.

2. New hyperchaotic system

First, consider the following Liu chaotic system:[11]

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= bx - kxz, \\ \dot{z} &= -cz + hx^2, \end{aligned} \tag{1}$$

where a, b, k, c, h are constants and x, y, z are variables of the system, $\dot{x}$, $\dot{y}$, $\dot{z}$ are derivatives of x, y, z respectively. Taking a =10, b =40, c =2.5, k =1, h =4, the Lyapunov exponents of this chaotic system are obtained to be

$$\lambda_{L_1} = 1.64328, \quad \lambda_{L_2} = 0, \quad \lambda_{L_3} = -14.42.$$

Apparently, there is only one positive Lyapunov exponent, that is to say, Liu chaotic system is a three-dimensional autonomous chaotic system.

On the basis of Liu chaotic system, a new hyperchaotic system which is judged by two positive Lyapunov exponents can be generated by adding an additional state w. The four-dimensional autonomous differential equations that describe the system are shown as follows

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= bx - kxz + w, \\ \dot{z} &= -cz + hx^2, \\ \dot{w} &= -dx, \end{aligned} \tag{2}$$

where d is a parameter to be tuned, its value can be varied within a certain range. First, the dynamical system (2) can be characterized with its Lyapunov

exponents that are computed numerically by Wolf method.[12]

1. For periodic orbits, there are one zero and three negative Lyapunov exponents.

2. For quasi-periodic orbits, there are two zero and two negative Lyapunov exponents.

3. For chaotic orbits, there are one positive, one zero and two negative Lyapunov exponents.

4. For hyperchaotic orbits, there are two positive values in the four Lyapunov exponents.

The Lyapunov exponents of the system (2) have different values when the parameter d is gradually varied, and one can observe different dynamical behaviours of the system (2) at different parameter d. The bifurcation diagram of x with increasing d is given in Fig.1. The spectrum of Lyapunov exponents on the $d-\lambda$ plane, i.e., a plot of the four Lyapunov exponents versus d, is obtained as shown in Fig.2. It can be observed that the bifurcation diagram well coincides with the spectrum of Lyapunov exponents. As d increases, the system (2) undergoes the following routes dynamically:

(1) $0 < d \leq 42$, $\lambda_{1,2} > 0$, the new system (2) is hyperchaotic (Fig.3(a));

(2) $42 < d \leq 52.4$, $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_{3,4} < 0$, the new system (2) is chaotic (Fig. 3(b));

(3) $52.4 < d \leq 64.2$, $\lambda_1 = 0$, $\lambda_{2,3,4} < 0$, the new system (2) is periodic (Fig.3(c));

(4) $64.2 < d \leq 67.2$, $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_{3,4} < 0$, the new system (2) is chaotic (Fig. 3(d));

(5) $67.2 < d \leq 75$, $\lambda_1 = 0$, $\lambda_{2,3,4} < 0$, the new system (2) is periodic (Fig.3(e));

(6) $75 < d \leq 88.6$, $\lambda_{1,2} = 0$, $\lambda_{3,4} < 0$, the new system (2) is quasi-periodic (Fig.3(f));

(7) $88.6 < d \leq 90.4$, $\lambda_1 = 0$, $\lambda_{2,3,4} < 0$, the new system (2) is periodic (Fig.3(g));

(8) $90.4 < d \leq 100$, $\lambda_{1,2} = 0$, $\lambda_{3,4} < 0$, the new system (2) is quasi-periodic (Fig.3(h)).

Let a =10, b =40, c =2.5, k =1, h =4, d =10.6,the typical hyperchaotic dynamics can be obtained from system (2). Its Lyapunov exponents can be obtained

$$\lambda_1 = 1.1491, \quad \lambda_2 = 0.12688,$$

$$\lambda_3 = 0, \quad \lambda_4 = -13.767.$$

The Lyapunov dimension of this system (2) is

$$D_{\mathrm{L}} = j + \frac{1}{|\lambda_{j+1}|}\sum_{i=1}^{j}\lambda_i = 3 + \frac{\lambda_1 + \lambda_2}{|\lambda_4|}$$
$$= 3 + \frac{1.1491 + 0.12688}{|-13.767|} = 3.0927$$

Apparently, the Lyapunov dimension of this system (2) is fractal. Its portrait is shown in Fig.4.

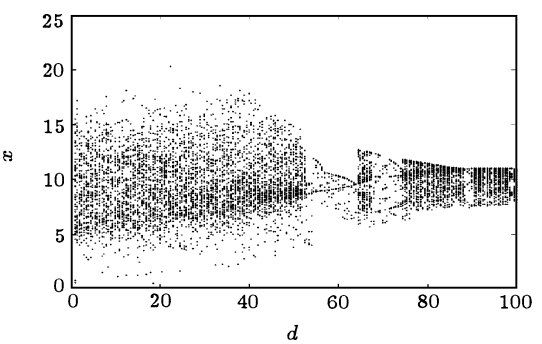


Fig.1. Bifurcation diagram for increasing d with $a = 10$, $b = 40$, $c = 2.5$, $k = 1$, h=4.

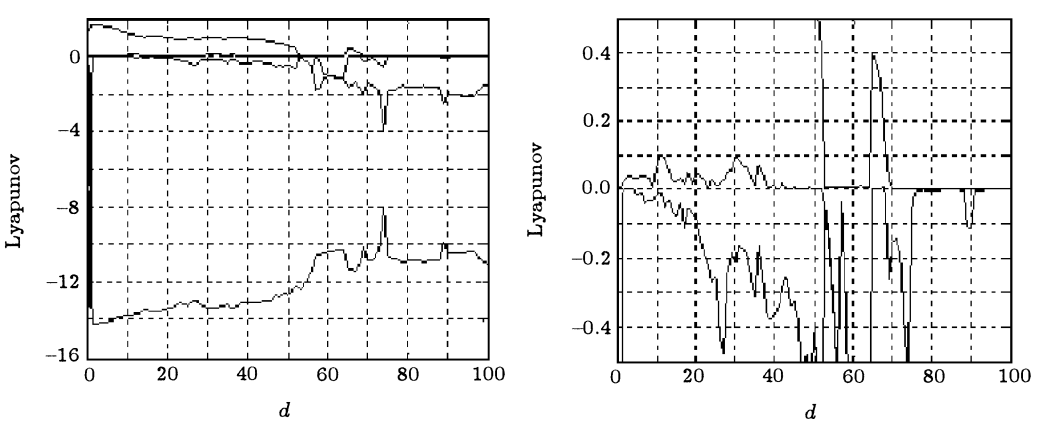


Fig.2. Lyapunov exponents versus d.

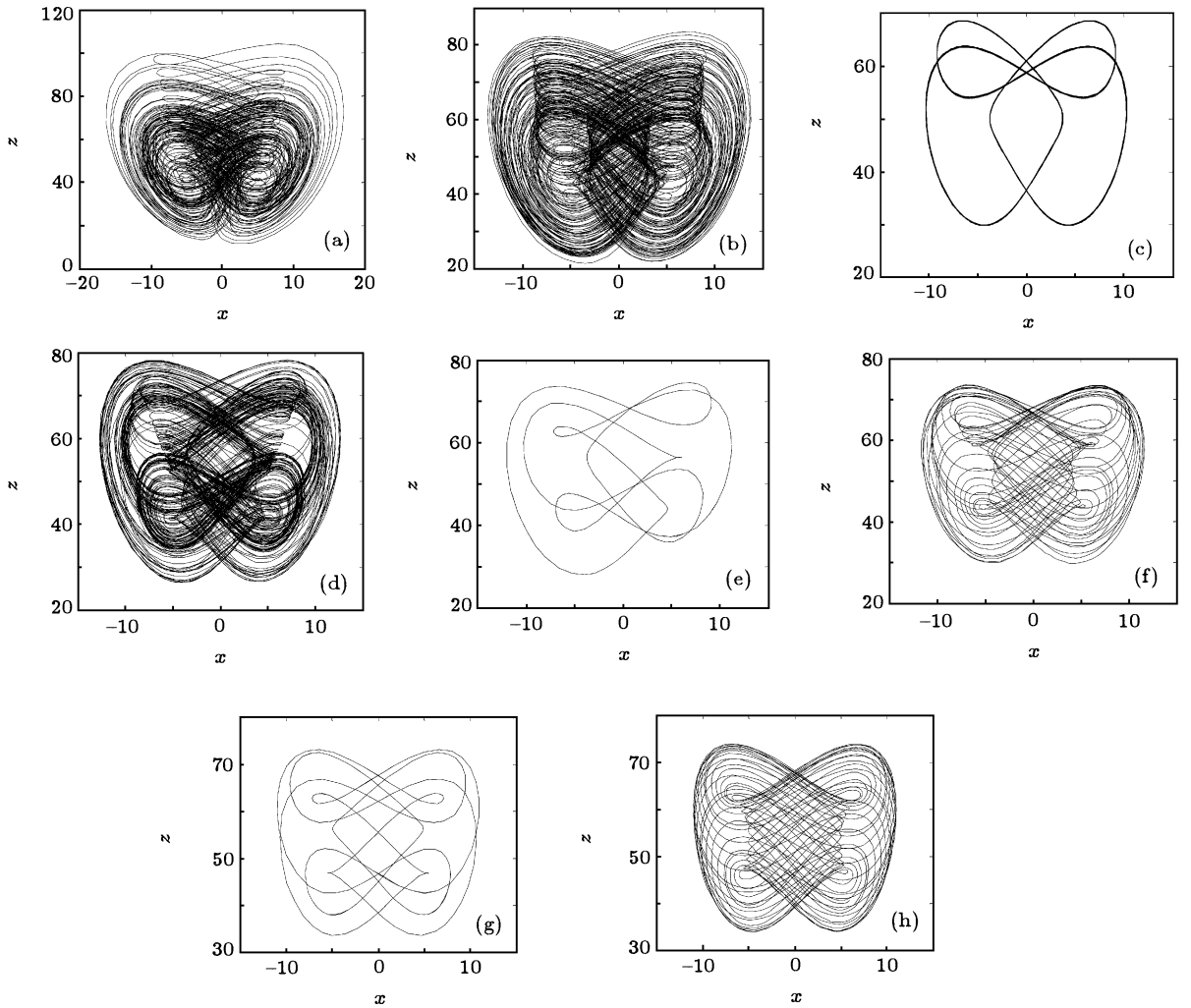


Fig.3. Phase portraits of system (2) in $x-z$ plane for increasing d with $a=10$, $b=40$, $c=2.5$, $k=1$, h=4. (a) hyperchaotic attractor, d=20; (b) chaotic attractor, d=51; (c) periodic attractor, d=60; (d) chaotic attractor, d=65; (e) periodic attractor, d=70.4; (f) quasi-periodic attractor, d=77; (g) periodic attractor, d=89; (h) quasi-periodic attractor, d=92.

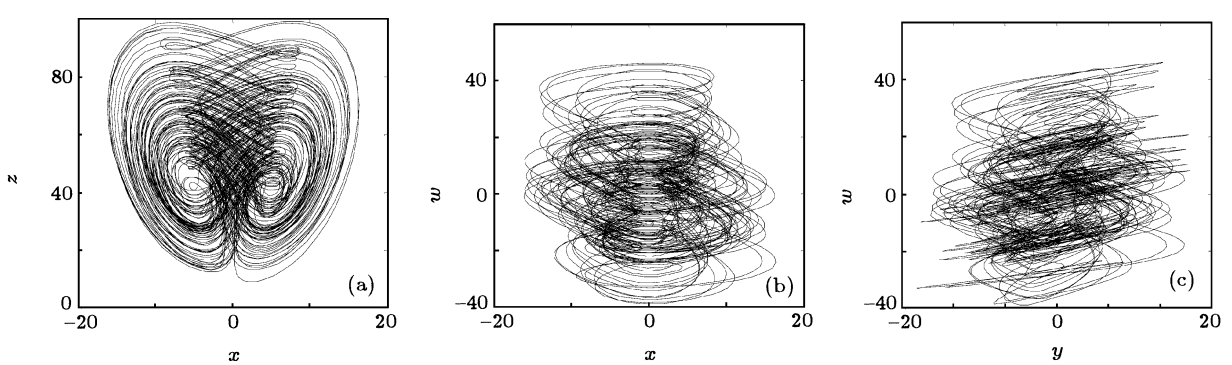


Fig.4. Phase portrait of hyperchaotic system, with $a=10$, $b=40$, $c=2.5$, $k=1$, h=4, d=10.6 (a) $z-x$, (b) $w-x$, (c) $w-y$.

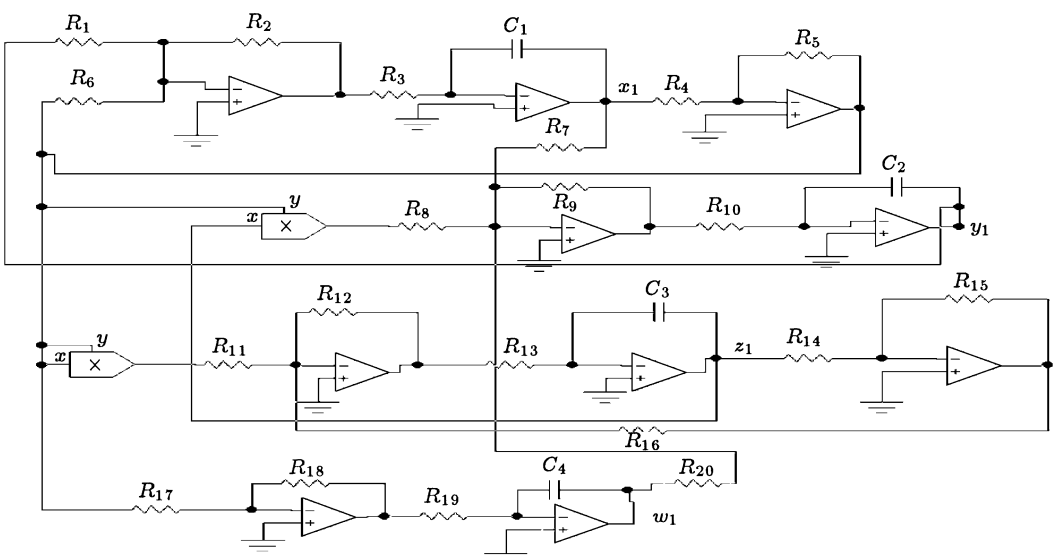
3. Circuit experimental confirmation of the hyperchaotic system

In this Section, an electronic circuit is designed to realize the new hyperchaotic system. Because of the restriction in the acceptable voltage of amplifier and multiplier, we must decrease the amplitude of every variable of system (2). So, let $x = 10x_1$, $y = 10y_1$, $z = 10z_1$, $w = 10w_1$. The system (2) can be changed into the following form:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1), \\ \dot{y}_1 &= bx_1 - 10kx_1z_1 + w_1, \\ \dot{z}_1 &= -cz_1 + 10hx_1^2, \\ \dot{w}_1 &= -dx_1. \end{aligned} \tag{3}$$

The designed circuitry for realizing system (3) is presented in Fig.5. It consists of four channels to conduct the integration of the four state variables, x_1, y_1, z_1 and w_1, respectively. Operational amplifiers (LM741), analog multipliers (AD633), linear resistors and capacitors are employed to perform the required addition, subtraction and multiplication operations.

The observations measured in the experimental setup are shown in Figs.6(a)–6(h) by varying R_{17}. By comparing Figs.3(a)–3(h), a good qualitative agreement between the numerical simulation and the experimental measurement is observed.



Op-Amps: LM741. Multiplier: AD633.

Fig.5. Circuitry realization of the hyperchaotic system (3). Resistance/kΩ: R_1, R_2, R_4, R_7, R_{12}, R_{14}, R_{15}, R_{18}=10; R_3, R_{10}, R_{13}, R_{19}=100; R_5, R_6=20; R_8=8; R_9, R_{16}=40; R_{11}=1; R_{20}=400. Capacitance/μF: C_1, C_2, C_3, C_4=1.

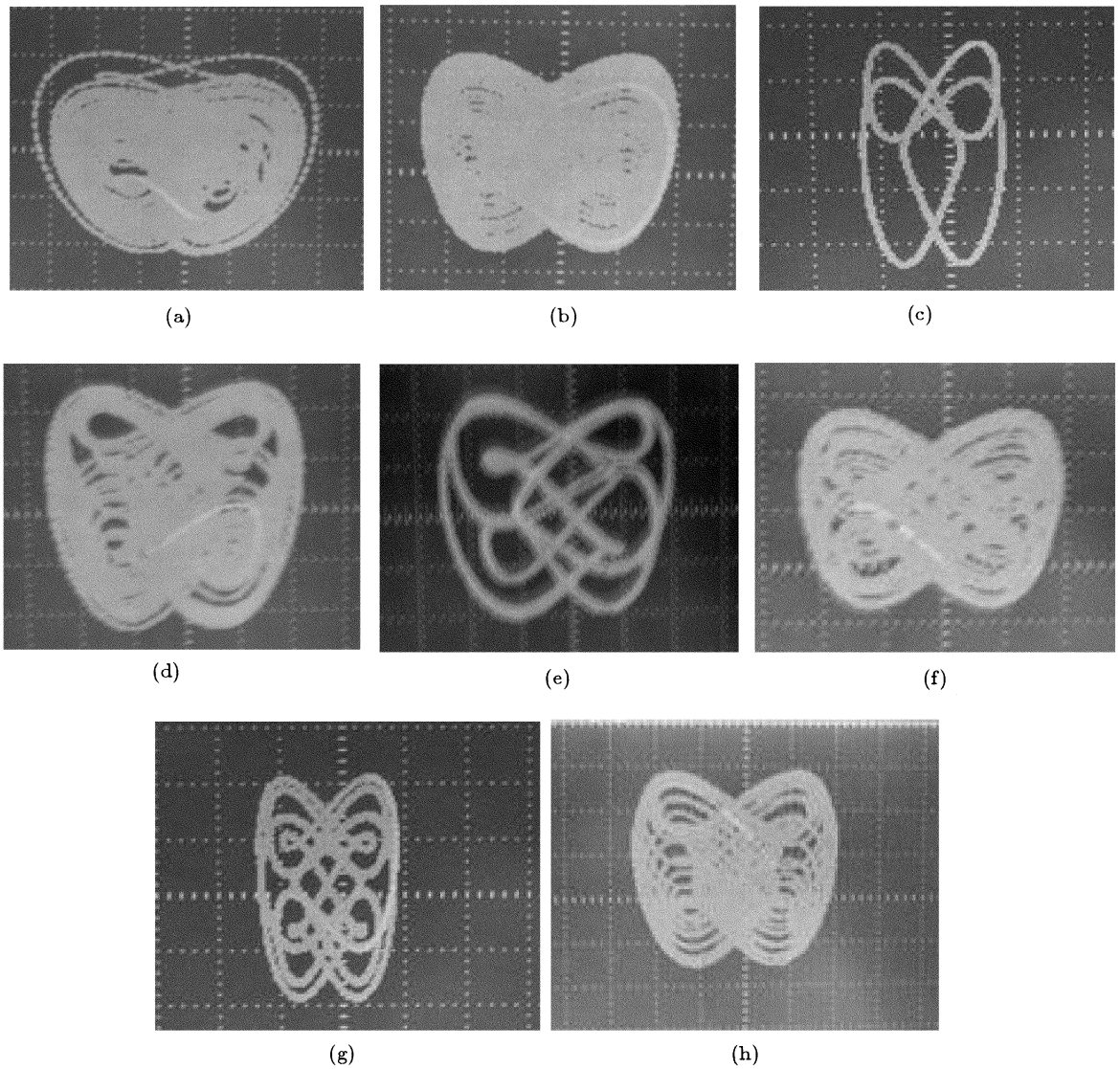


Fig.6. Phase portrait of system (3) in $x-z$ plane versus decreasing R_{17}, measured by using the designed experimental set-up: (a) hyperchaotic attractor, R_{17}=10kΩ (0.5V/div, 2V/div); (b) chaotic attractor, R_{17}=3.922kΩ (0.5V/div, 1V/div); (c) periodic attractor, R_{17}=3.334kΩ (1V/div, 1V/div); (d) chaotic attractor, R_{17}=3.077kΩ (0.5V/div, 1V/div); (e) periodic attractor, R_{17}=2.84kΩ (0.5V/div, 1V/div); (f) quasi-periodic attractor, R_{17}=2.6kΩ (0.5V/div, 1V/div); (g) periodic attractor, R_{17}=2.247kΩ (1V/div, 1V/div); (h) quasi-periodic attractor, R_{17}=2.174kΩ (0.5V/div, 1V/div).

4. Conclusion

In this paper, we have presented a modified Liu hyperchaotic system. It is a fourth-order autonomous nonlinear system. The hyperchaotic orbits, chaotic orbits, periodic orbits and quasi-periodic orbits have been observed by varying the parameter d not only by computer simulation, but also by electronic circuit experiment. It is expected that more detailed and thorough analysis will be provided in near future.

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