Vol 15 No 5, May 2006
1009-1963/2006/15(05)/0958-05
Chinese Physics
© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

The design and artificial realization of a controller of pulse coupling feedback*

Lü Ling(吕 翎)[a)†], Guo Zhi-An(郭治安)[b)], Luan Ling(栾 玲)[a)],
Zou Cheng-Ye(邹成业)[a)], and Zhao Hong-Yan(赵鸿雁)[a)]

[a)] *College of Physics and Electronic Technology, Liaoning Normal University, Dalian 116029, China*

[b)] *Department of Mathematics and Physics, Dalian Jiaotong University, Dalian 116028, China*

(Received 12 October 2005; revised manuscript received 6 December 2005)

In this paper a controller of pulse coupling feedback (PCF) is designed to control chaotic systems. Control principles and the technique to select the feedback coefficients are introduced. This controller is theoretically studied with a three dimensional (3D) chaotic system. The artificial simulation results show that the chaotic system can be stabilized to different periodic orbits by using the PCF method, and the number of the periodic orbits are $2^n \times 3^m$p (n and m are integers). Therefore, this control method is effective and practical.

Keywords: chaos control, pulse coupling feedback, 3D chaotic system, periodic orbit
PACC: 0545

1. Introduction

Since the parameter perturbation method[1] (OGY method) was initially proposed by Ott, Grebogi and Yorke for controlling unstable periodic orbits of chaotic attractor to expected targets, the study of chaos control has been developed rapidly. By now, according to different control targets, the initial OGY method has been developed in two types of the parameter control method and the variable control method.[2−15] The control method of oscillatory absorber proposed by Kapitaniak, Kocarev and Chua is just a variable control method, of which the control mechanism is to couple the chaotic system uncontrolled with a oscillatory absorber to modify the chaotic system by it, so as to obtain steady periodic orbits from the chaotic attractor of the system uncontrolled.[16] Their work uses the difference between a variable of the original chaotic system and a variable of the additional oscillatory absorber as a feedback term to couple the system with the absorber and to disturb the system to get new dynamic behaviour. But the technique needs an additional subsystem, so it requires redesigning the system and adjusting the parameters of the system, which is hard to fulfill in practice. A controller of pulse coupling feedback (PCF) is designed to control chaotic systems in this paper. Here the differences between some variables of the controlled system are used directly as feedback term and coupling is realized within the system. The coupling is of pulse form to diminish the cost of control. Artificial simulation is made with a three dimensional (3D) chaotic system, and the results show that the coupling feedback coefficients g and the impulse interval d are feedback parameters. Along with the change of g and d, the system undergoes a transition from chaos into stable states, and the number of the periodic orbits are $2^n \times 3^m$p (n and m are integers). Meanwhile, the power spectrum and the phase diagram also exhibits the change of the dynamic behaviour of the system, which shows that the controller has better capability, and it can introduce a new method for constructing chaotic controller.

2. Designing of the controller

Suppose the dynamic equations of an N-dimensional discrete chaotic system can be described as

$$x_{n+1}^{(i)} = f_i(\{x_n^{(i)}\}, \{\mu_k\}),$$
$$(i = 1, 2 \ldots N, \quad k = 1, 2 \ldots l), \qquad (1)$$

*Project supported by the National Natural Science Foundation of China (Grant No 20373021) and Natural Science Foundation of Liaoning Province, China (Grant No 2050790).
†Corresponding author. E-mail: luling1960@yahoo.com.cn

where $x_n^{(i)} \in R^N$ are system variables, and $\mu_k \in R$ are system parameters. When the PCF method is used to control the system (1), the control scheme can be summarized as follows.

When system variables $x_n^{(i)}$ and $x_n^{(j)}$ are taken as the objects of pulse coupling control and coupling is made by using the difference of the two variables, the controller is designed as

$$u_{ij} = g_{ij}(x_n^{(i)} - x_n^{(j)}). \tag{2}$$

The coupling is taken in the form of pulse in the method in order to diminish cost of the control, namely, the control is added only once for every d times of iteration, i.e.

$$g_{ij} = \begin{cases} g & n = md, \\ 0 & n \neq md, \end{cases} \quad (m = 1, 2, 3 \cdots), \tag{3}$$

where g_{ij} are coupling feedback coefficients, and d is impulse interval.

Using the method proposed in the article to control the system (1) and $n = md$, we can turn the controlled equations to the following form:

$$\begin{cases} x_{n+1}^{(1)} = f_1(\{x_n^{(i)}\}, \{\mu_k\}), \\ \cdots \\ x_{n+1}^{(i)} = f_i(\{x_n^{(i)}\}, \{\mu_k\}) + g_{ij}(x_n^{(i)} - x_n^{(j)}), \\ \cdots \\ x_{n+1}^{(j)} = f_j(\{x_n^{(i)}\}, \{\mu_k\}) + g_{ji}(x_n^{(j)} - x_n^{(i)}), \\ \cdots \end{cases} \tag{4}$$

However, when $n \neq md$, the equations remain in the same form as Eq.(1).

When control is added, the eigenvalue of Eq.(1) will be changed according to the coupling feedback coefficients g_{ij} and the impulse interval d which play a role of adjusting parameters. The maximum Lyapunov exponent of the system can be changed from positive into negative by appropriately adjusting the value of g_{ij} and d, and consequently the orbits of the system can be changed from chaos into periodic orbits. This is the control principle of the PCF controller. Since an N-dimensional continuous chaotic system can be changed into a discrete one, the PCF controller can be applied to both continuous chaotic systems and discrete ones.

3. Artificial simulation

Recently, Liu and Chen introduced a 3D chaotic system which was realized by the circuit in Fig.1(a). The dynamic equations of the system are written as follows:[17]

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = ax - d_1 yz, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -by + d_2 xz, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = -cz + d_3 xy, \end{cases} \tag{5}$$

where $d_i = 1 (i = 1, 2, 3)$, $a = 5$, $c = 3.3$ and b is an adjustable parameter. The three variables x, y, z in

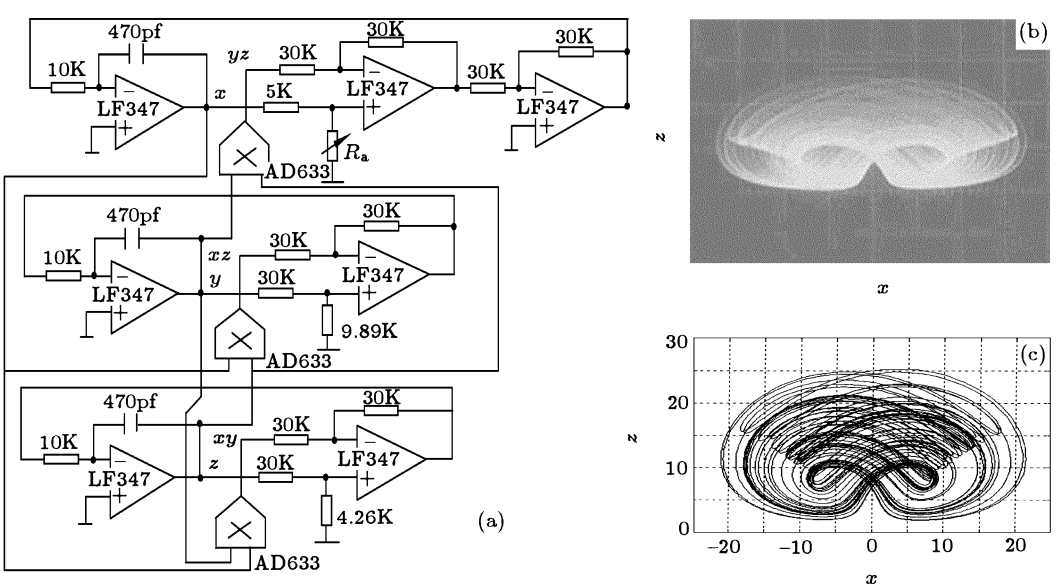


Fig.1. (a) Circuitry realization of the new chaotic system. (b), (c) The chaotic attractor in the (x, z) phase space.

Equations (5) are produced from the top, middle and bottom parts of the circuit respectively. The chaotic phase diagram, Fig.1(b), which is obtained from experimental observation on R_a=5kΩ, conforms with that of artificial simulation, Fig.1(c).

The steady state solution (x_0, y_0, z_0) of the 3D system is given by $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{\mathrm{d}z}{\mathrm{d}t} = 0$,

$$(x_0, y_0, z_0) = (\pm\sqrt{bc}, \pm\sqrt{ac}, \pm\sqrt{ab}). \tag{6}$$

With perturbations of x', y', z' , the linearized equations of the perturbed steady state become

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a & -z_0 & -y_0 \\ z_0 & -b & x_0 \\ y_0 & x_0 & -c \end{bmatrix}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{7}$$

and the eigenvalue equation is

$$\begin{vmatrix} a-\omega & -z_0 & -y_0 \\ z_0 & -b-\omega & x_0 \\ y_0 & x_0 & -c-\omega \end{vmatrix} = 0. \tag{8}$$

That is

$$\omega^3 + c_2\omega^2 + c_1\omega + c_0 = 0, \tag{9}$$

where

$$\begin{aligned} c_2 &= b + c - a, \\ c_1 &= -ab - ac + bc - x_0^2 + y_0^2 + z_0^2, \\ c_0 &= -abc + 2x_0y_0z_0 + ax_0^2 + by_0^2 + cz_0^2. \end{aligned}$$

According to Eq.(9) and the definition of Lyapunov exponent,[18] which is the basic quantity to describe the chaotic system, the maximum Lyapunov exponent of the 3D system can be calculated. The evolution of the maximum Lyapunov exponent of the system against the parameter b is shown in Fig.2 in which Lyapunov exponent is positive, this is the essential character of a nonlinear system to be in chaos. Therefore, when $d_i = 1 (i = 1, 2, 3)$, $a = 5$, $c = 3.3$, the system is in chaos. Figure 3 is the phase diagram of the 3D chaotic system. As is shown, all the orbits are in finite regions, and the orbits have the ergodic property and special self-similar structures. As is seen, the orbits form bands. If the scale of the phase space is magnified, more small bands of orbits within the original bands can be found. All of these are typical characters of a chaotic attractor.

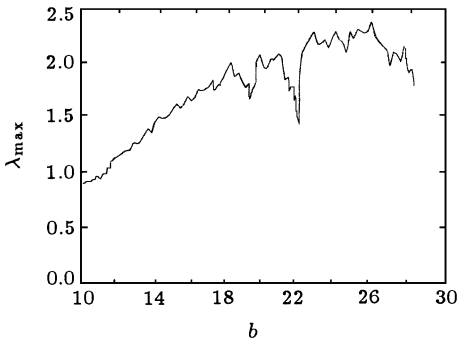


Fig.2. Maximum Lyapunov exponent of 3D chaotic system.

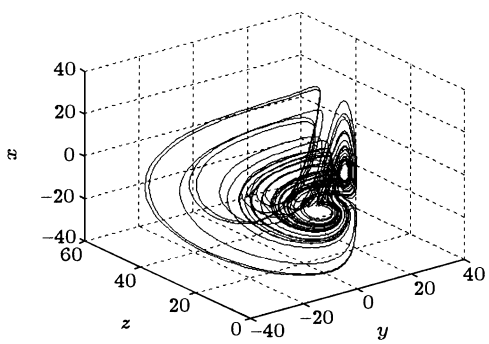


Fig.3. The chaotic attractor in the (x, y, z) phase space.

When PCF control is added to the 3D chaotic system, the continuous chaotic system can be rewritten in discrete form

$$\begin{cases} x_{n+1} = x_n + (ax_n - y_nz_n)\Delta t, \\ y_{n+1} = y_n + (-by_n + x_nz_n)\Delta t, \\ z_{n+1} = z_n + [-cz_n + x_ny_n]\Delta t. \end{cases} \tag{10}$$

Taking x_n and y_n as coupling variables, and adding the coupling feedback terms $g_{xy}(x_n - y_n)$ and $g_{yx}(y_n - x_n)$ to the first and the second equations in Eq.(10), and with the forms of g_{xy} and g_{yx} being the same as those in Eq.(3), iteration of 1.0×10^4 times with b =15, $g_{xy} = g_{yx} = g$, we obtain the artificial simulation result as shown in Fig.4. The orbits of the system are obviously changed into n closed curves. Comparing the maximum Lyapunov exponents of the 3D system before and after control, we see that the maximum Lyapunov exponent $\lambda_{\max} > 0$ when b =15 and $\lambda_{\max} < 0$ when g=0.98, d=3. That is, the maximum Lyapunov exponent of the system is changed from positive to negative after the control. This is the typical character of periodic orbits. Therefore the dynamic property of the system is changed from a chaotic state into a periodic one. And this means that the controlled system changes from a chaotic attractor in periodic orbits. The attractor has an infinite

number of self-similar structures embedded in it and limited in finite phase space. The simulation result of 1p orbit is shown in Fig.4(a). Keeping the impulse interval d the same but changing the feedback parameter g, so long as the maximum Lyapunov exponent of the system remains negative (though its magnitude changed), we have the results of 2p, 4p,... $2^n \times 3^m$p (n and m are integrals) orbits, as shown in Fig.4(b), 4(c) and 4(d).

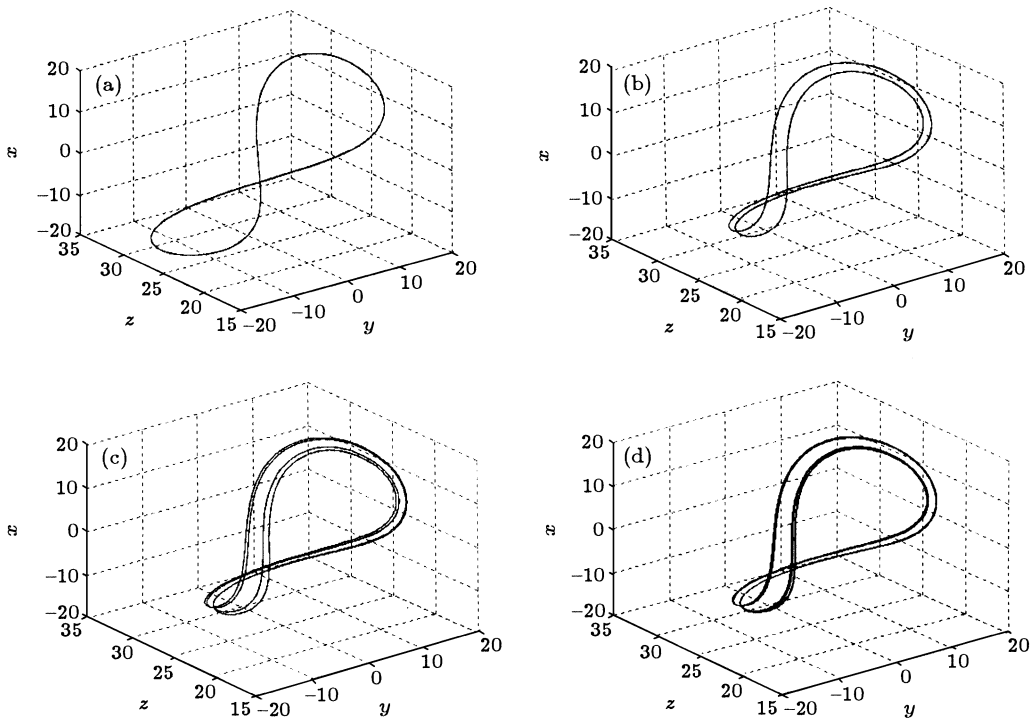


Fig.4. The periodic orbits in the controlled system. (a) 1p periodic orbit (d=3, g= 0.98), (b) 2p periodic orbit (d=3, g= 0.78), (c) 4p periodic orbit (d=3, g= 0.71), (d) 6p periodic orbit (d=3, g= 0.66).

Power spectrum analysis is an effective method in determining the dynamical behaviour of the system. Power spectrum can be evaluated by using the Fourier transform of a variable in the phase space. In the power spectrum periodic motions correspond to spikes, while a chaotic motion corresponds to continuous spectrum with spikes and noise background.

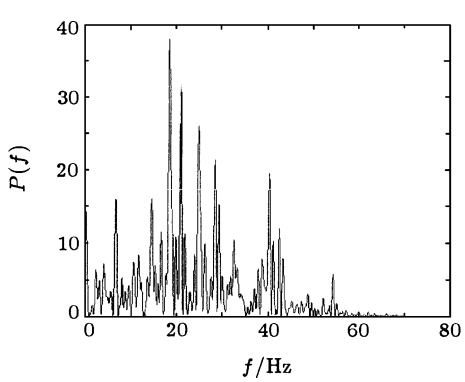


Fig.5. The power spectrum in the chaos system.

The power spectrums of the system before and after control are calculated by fast Fourier transform, as shown in Fig.5 and Fig.6. Figure 5 is the power spectrum of the system uncontrolled. As is seen, it is a continuous spectrum with spikes and noise background. Figure 6 is the power spectrum of the 2p orbit, having only spikes without noise background, which is a typical character of periodic motion.

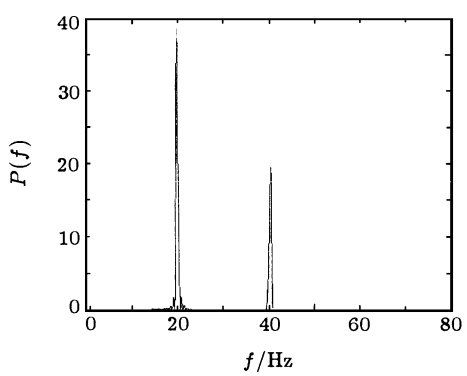


Fig.6. The power spectrums of 2p orbit.

It is also found in the artificial simulation that when the impulse interval d takes other values the periodic orbits can also be obtained easily though they change a little in shape. This shows that the control is effective and sensitive to the 3D system. Meanwhile, the number of periodic orbits obtained can be very rich by adjusting the parameters g and d. This means that the system can be stabilized in many periodic orbits, such as 1p, 2p, 3p, 4p, ... $2^n \times 3^m$p (n and m are integers), which shows that the system can be controlled in a large range. Furthermore, as is seen, the control parameters, the coupling feedback parameter g and the impulse interval d, are both external ones instead of parameters of the controlled system. Therefore, this method overcomes the shortcomings of the control method of oscillatory absorber proposed by Kapitaniak, Kocarev and Chua, which requires obtaining the detailed information within the system before controlling to redesign the system and adjust the parameters of the system. The method provides a useful reference to the transition from theoretical study to application, and it is hopeful to apply it to a practical system of information and communication.

4. Conclusion

The 3D chaotic system is well controlled by using the PCF controller presented in this paper. Artificial simulation results show that the maximum Lyapunov exponent, the phase diagrams and the power spectrums all can be changed when the control is added to the system. When the system parameters are kept unchanged and only the external controlling parameters are adjusted, the maximum Lyapunov exponent of the system can change from positive to negative, correspondingly the power spectrum changes from a continuous spectrum with spikes and noise background into a spectrum with only spikes, and the orbits of the system in the phase diagrams change from the chaotic attractor into $2^n \times 3^m$ p (n and m are integers) periodic orbits.

References

[1] Ott E, Grebogi C and Yorke J A 1990 *Phys. Rev. Lett.* **64** 1196

[2] Wang Q Y, Lu Q S and Wang H X 2005 *Chin. Phys.* **14** 2189

[3] Yue L J and Shen K 2005 *Chin. Phys.* **14** 1760

[4] Liu Y Z and Fei S M 2005 *Acta Phys. Sin.* **54** 3486 (in Chinese)

[5] Trimper S and Zabrocki K 2004 *Phys. Lett.* A **321** 205

[6] Jiang P Q, Wang B H, Xia Q H and Bu S L 2004 *Acta Phys. Sin.* **53** 3280 (in Chinese)

[7] Fu W B and Tang J S 2004 *Acta Phys. Sin.* **53** 2889 (in Chinese)

[8] Alvarez-Ramirez J, Espinosa-Paredes G and Puebla H 2003 *Phys. Lett.* A **316** 196

[9] Tian Y C, Tadé M O and Levy D 2002 *Phys. Lett.* A **296** 87

[10] Gao X P, Fan Z P, Zhang Q L and Wang Y Q 2002 *Acta Phys. Sin.* **51** 2216 (in Chinese)

[11] Chen L Q 2001 *Phys. Lett.* A **281** 327

[12] Yang L F, Dolnik M, Zhabotinsky A M and Epstein I R 2000 *Phys. Rev.* E **62** 6414

[13] Konishi K, Kokame H and Hirata K 2000 *Phys. Rev.* E **62** 384

[14] Yang X S 2000 *Acta Phys. Sin.* **49** 1919 (in Chinese)

[15] Mirus K A and Sprott J C 1999 *Phys. Lett.* A **254** 275

[16] Kapitaniak T, Kocarev L and Chua L O 1993 *Int. J. Bifur. Chaos* **3** 459

[17] Liu W B and Chen G R 2004 *Int. J. Bifur. Chaos* **14** 1395

[18] Lü L 2000 *Nonlinear Dynamics and Chaos* (Dalian: Dalian Publishing House) p130 (in Chinese)