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Adaptive synchronization of Chua's system with uncertain inputs*

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The synchronization of Chua's system, whose inputs include an unknown constant parameter, is studied in this paper. A constructive method is applied to designing an adaptive controller, in which only one variable information of the master system is needed. With the action of control signals, the parameter of the slave system will approach the corresponding unknown parameter in the master system. At the same time, the synchronization errors will also converge to zero asymptotically. Numerical simulations show that the proposed theoretical approach is very effective.

Keywords: chaos, synchronization, Chua's system, uncertain inputs
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1. Introduction

Since the chaotic synchronization was presented,[1] the tremendous interest has been aroused in studying the synchronization of chaotic systems in various fields such as biology, physics, chemistry, and engineering.[2−5] Over the last decade, many new types of chaotic synchronization have appeared: lag synchronization,[6] phase synchronization,[7] generalized synchronization,[8] adaptive synchronization,[9−13] and fractional-order synchronization.[14]

A number of methods in the literature are based on the assumption that variables of the master system are all available and the master system is entirely known, in which no uncertain inputs or perturbations are contained. But, as a rule, this case is very rare in reality. For example, in secure communication, a signal is transmitted from the master system to the slave system, where the transmitted signal is one of the state variables, which is kept unmeasurable. On the other hand, actual systems may contain some perturbations or inputs including unknown parameters. So, the synchronization of chaotic system with inputs containing uncertain parameters, whose available output is one, is worth being studied.

Recently, adaptive synchronization has been investigated in some papers. In Ref.[9], an approach to adaptive synchronization of two chaotic systems with unknown parameters was proposed by using adaptive backstepping, but the obtained control law required all state variables of the chaotic system to be known. An adaptive synchronization method using a single variable was addressed in Ref.[10], nevertheless the considered system is assumed to be entirely given there, which contained no uncertain inputs or perturbations. In addition, the addressed approach is not constructive and a Lyapunov function needs to be found for different systems. In Ref.[11], a novel method was provided for a class of chaotic system dependent linearly on unknown parameters, but an exceptive condition must be satisfied firstly and the designed controller also needs all variables of the chaotic system to be known. The same drawback also can be seen in Ref.[12]. A new approach to uncertain chaotic systems was proposed in Ref.[13]. However, in the studied case, the unknown parameters are included in nonlinear parts of the system, not in perturbations or inputs.

Chaotic synchronization is closely related to observer problem in control theory.[15] In general, chaotic synchronization can be thought of as the design problem of a feedback law for full observer using the known information of the plant, so as to ensure that the con-

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trolled receiver synchronizes with the plant. So the research on the nonlinear observer will give us much inspiration.[16−18]

In this paper, a constructive scheme for Chua's system is addressed. Such a strategy can not only guarantee that the synchronization errors between the master system and the slave system approach zero asymptotically, but also adjust the parameter of the slave system approximately to the unknown parameter of the master system.

The rest of this paper is organized as follows. In the next section, we briefly review the modified Chua circuit model and a master–slave formalism for synchronization. In Section 3, an adaptive method for Chua's system is described. Numerical simulations are given to verify the correctness of the theoretical analysis. Finally, conclusions are drawn in Section 4.

2. Model description

Chua's circuit is well known and has been extensively studied. The modified Chua system is different from the usual Chua system in that the piecewise-linear nonlinearity is replaced by an appropriate cubic nonlinearity which yields a similar behaviour. The modified Chua system can be represented as follows:

$$\begin{aligned} \dot{x} &= -\beta y, \\ \dot{y} &= z - y + x, \\ \dot{z} &= \alpha(y - z^3 - \gamma z), \end{aligned} \tag{1}$$

where α, β and γ are known parameters. With the perturbations added, which include one unknown parameter, the above Chua system is rewritten as

$$\begin{aligned} \dot{x}_1 &= -\beta y_1 + \theta\varphi_x(t), \\ \dot{y}_1 &= z_1 - y_1 + x_1 + \theta\varphi_y(t), \\ \dot{z}_1 &= \alpha(y_1 - z_1^3 - \gamma z_1) + \theta\varphi_z(t), \end{aligned} \tag{2}$$

where θ is an unknown parameter and $\varphi_x, \varphi_y, \varphi_z$ are bounded known perturbation functions. The behaviour of the perturbed chaotic system is different from that of the original chaotic system for the same initial states. For example, assuming

$$\begin{aligned} &\varphi_x = \sin(t), \\ &\varphi_y = \cos(2t), \\ &\varphi_z = \sin(t) + \cos(2t), \\ &\alpha = 10,\ \beta = 16,\ \gamma = -0.143,\ \text{and}\ \theta = 0.1, \end{aligned} \tag{3}$$

the respective dynamics behaviours are shown in Fig.1 and Fig.2 for the initial states $x(0) = x_1(0) = 0.04$, $y(0) = y_1(0) = 0.08$, $z(0) = z_1(0) = 0.06$. The system (2) is called the master system, then the slave system is taken in the following form:

$$\begin{aligned} \dot{x}_2 &= -\beta y_2 + \widehat{\theta}\varphi_x(t), \\ \dot{y}_2 &= z_2 - y_2 + x_2 + \widehat{\theta}\varphi_y(t), \\ \dot{z}_2 &= \alpha(y_2 - z_2^3 - \gamma z_2) + \widehat{\theta}\varphi_z(t). \end{aligned} \tag{4}$$

In the above master–slave formalism, $\varphi_x, \varphi_y, \varphi_z$ and α, β, γ are bounded known functions, and known parameters, respectively; both θ and $\widehat{\theta}$ are unknown constant parameters. Further, what should be noted is: 1) We suppose that only z_1 and z_2 are available or measurable, and 2) The unknown parameter is merged into the perturbations that are not related to the state variables. Our aim is to design an adaptive controller that can drive the slave system (4) to follow the master system (2).

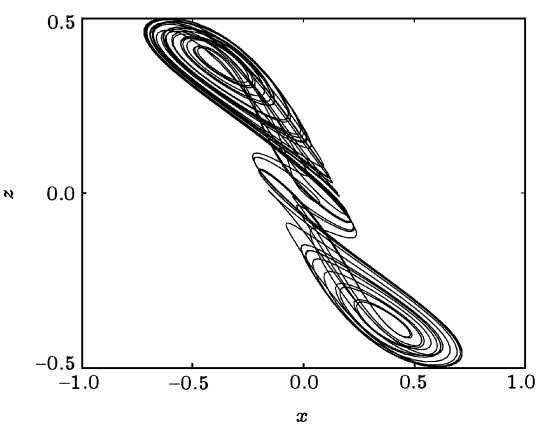


Fig.1. Phase portrait of the original chaotic system .

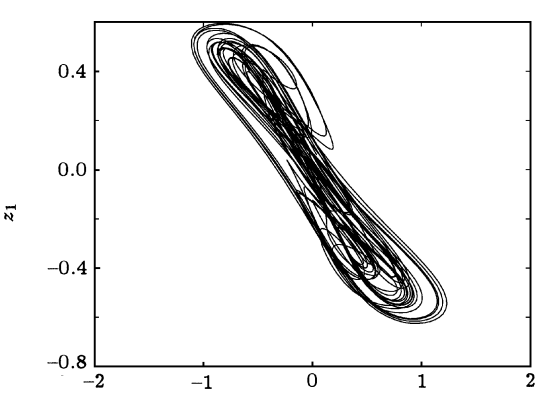


Fig.2. Phase portrait of the perturbed chaotic system .

3. Adaptive synchronization design

This section will be devoted to the design of the synchronization controller.

Above all, assume that the designed synchronization controller is taken as the form of

$$\begin{aligned} u_x =&(\psi_x(v_x, v_y, v_z) + k_x)(z_1 - z_2),\\ u_y =&(\psi_y(v_x, v_y, v_z) + k_y)(z_1 - z_2),\\ u_z =&(\psi_z(v_x, v_y, v_z)\\ &- \alpha(z_1^2 + z_1 z_2 + z_2^2) + k_z)(z_1 - z_2),\\ \dot{\widehat{\theta}} =&\lambda v_x(z_1 - z_2),\\ \dot{v}_x =&\phi_x(v_x, v_y, v_z, \varphi_x(t)),\\ \dot{v}_y =&\phi_y(v_x, v_y, v_z, \varphi_y(t)),\\ \dot{v}_z =&\phi_z(v_x, v_y, v_z, \varphi_z(t)), \end{aligned} \tag{5}$$

where v_x, v_y, v_z are assistant variables to be constructed, whose dynamics only depends on $\varphi_x(t)$, $\varphi_y(t)$, $\varphi_z(t)$, respectively, and λ, k_x, k_y, k_z are the designed parameters. After adding the above control laws to the slave system, we have

$$\begin{aligned} \dot{x}_2 =&-\beta y_2 + \widehat{\theta}\varphi_x(t) + (\psi_x + k_x)(z_1 - z_2),\\ \dot{y}_2 =&z_2 - y_2 + x_2 + \widehat{\theta}\varphi_y(t) + (\psi_y + k_y)(z_1 - z_2),\\ \dot{z}_2 =&\alpha(y_2 - z_2^3 - \gamma z_2) + \widehat{\theta}\varphi_z(t)\\ &+ (\psi_z - \alpha(z_1^2 + z_2^2 + z_1 z_2) + k_z)(z_1 - z_2),\\ \dot{\widehat{\theta}} =&\lambda v_x(z_1 - z_2). \end{aligned} \tag{6}$$

From the systems (2) and (6), the error dynamics between the master system and the slave system is obtained by

$$\begin{aligned} \dot{e}_x =&-\beta e_y + e_\theta\varphi_x(t) - (\psi_x + k_x)e_z,\\ \dot{e}_y =&e_z - e_y + e_x + e_\theta\varphi_y(t) - (\psi_y + k_y)e_z,\\ \dot{e}_z =&\alpha(e_y - \gamma e_z) + e_\theta\varphi_z(t) - (\psi_z + k_z)e_z. \end{aligned} \tag{7}$$

Because θ is an unknown constant, we have

$$\dot{e}_\theta = -\lambda v_x e_z. \tag{8}$$

Define that

$$\begin{aligned} \mu_x(t) &= e_x(t) - v_x(t)e_\theta(t),\\ \mu_y(t) &= e_y(t) - v_y(t)e_\theta(t),\\ \mu_z(t) &= e_z(t) - v_z(t)e_\theta(t), \end{aligned} \tag{9}$$

and substitute the expressions (8) and (9) into the expression (7), then we will have the following resulting system that can be regarded as an auxiliary system:

$$\begin{aligned} \dot{v}_x =&-\beta v_y + \varphi_x(t) - k_x v_z,\\ \dot{v}_y =&v_x - v_y + v_z + \varphi_y(t) - k_y v_z,\\ \dot{v}_z =&\alpha v_y - \alpha\gamma v_z + \varphi_z(t) - k_z v_z. \end{aligned} \tag{10}$$

It is obvious that if we choose

$$\begin{aligned} \psi_x(v_x(t), v_y(t), v_z(t)) =&\lambda v_x^2,\\ \psi_y(v_x(t), v_y(t), v_z(t)) =&\lambda v_x v_y,\\ \psi_z(v_x(t), v_y(t), v_z(t)) =&\lambda v_x v_z, \end{aligned} \tag{11}$$

then the system (7) will be transformed into a linear system. That is to say,

$$\begin{bmatrix} \dot{\mu}_x\\ \dot{\mu}_y\\ \dot{\mu}_z \end{bmatrix} = \begin{bmatrix} 0 & -\beta & -k_x\\ 1 & -1 & 1 - k_y\\ 0 & \alpha & -\alpha\gamma - k_z \end{bmatrix} \begin{bmatrix} \mu_x\\ \mu_y\\ \mu_z \end{bmatrix}. \tag{12}$$

Based on the linear system theory, we may choose

$$k_x = 0.8, \quad k_y = 2, \quad \text{and} \quad k_z = 2, \tag{13}$$

which ensure that the dynamics system (12) has an asymptotically stable equilibrium at (0,0,0).

From the expressions (8) and (9), the error dynamics of the parameter θ can be rewritten in the form of

$$\dot{e}_\theta = -\lambda v_x^2 e_\theta - \lambda v_x \mu_z. \tag{14}$$

Before considering the stability of the above equation, we should recall the following Lemma[19] first.

Lemma Consider a system

$$\begin{aligned} \dot{z} &= f(z, y),\\ \dot{y} &= Ay + p(z, y), \end{aligned} \tag{15}$$

and suppose that $p(z, 0) = 0$ for all z near 0 and

$$\frac{\partial p}{\partial y}(0, 0) = 0. \tag{16}$$

If $\dot{z} = f(z, 0)$ has an asymptotically stable equilibrium at $z = 0$ and the eigenvalue of A has a negative real part, then the above system has an asymptotically stable equilibrium at $(z, y) = (0, 0)$.

Since $v_x(t)$ is bounded, which is generated by the system (10), there exist positive constants T, m and n, such that

$$n \le \int_t^{t+T} v(\tau)^2 \mathrm{d}\tau \le m. \tag{17}$$

Aa a consequence , by applying the above lemma, we can conclude that $\mu_x, \mu_y, \mu_z e_\theta$ converge to zero. Finally, from the following relationships:

$$\begin{aligned} e_x(t) &= \mu_x(t) + v_x(t)e_\theta(t), \\ e_y(t) &= \mu_y(t) + v_y(t)e_\theta(t), \\ e_z(t) &= \mu_z(t) + v_z(t)e_\theta(t), \end{aligned} \tag{18}$$

it can be proven that e_x, e_y and e_z also converge to zero.

In the above statement, we have proposed an adaptive control law that guarantees that the synchronization errors converge to zero for an uncertain Chua system. In order to verify the efficacy of the addressed scheme, numerical simulations are carried out.

Let us suppose that the system whose phase portrait is shown in Fig.2 is called the master system, then it is also described by

$$\begin{aligned} \dot{x}_1 =& 16y_1 + 0.1\sin(t), \\ \dot{y}_1 =& z_1 - y_1 + x_1 + 0.1\cos(2t), \\ \dot{z}_1 =& 10(y_1 - z_1^3 + 0.143z_1) \\ & + 0.1(\sin(t) + \cos(2t)). \end{aligned} \tag{19}$$

On the other hand, the initial states of the slave system (4) are chosen as $x_2(0) = 1.54$, $y_2(0) = 1.58$ and $z_2(0) = 1.56$, and the initial value of the parameter $\widehat{\theta}$ is assumed to be 8.1.

Taking $\lambda = 1$, we apply the designed adaptive control signals to the slave system, then it will be changed into

$$\begin{aligned} \dot{x}_2 =& -16y_2 + (v_x^2 + 0.8)(z_1 - z_2) + \widehat{\theta}\sin(t), \\ \dot{y}_2 =& z_2 - y_2 + x_2 + (v_x v_y + 2)(z_1 - z_2) + \widehat{\theta}\cos(2t), \\ \dot{z}_2 =& 10(y_2 + 0.143z_2 - z_2^3) \\ & + (v_x v_z - 10(z_1^2 + z_1 z_2 + z_2^2) + 2)(z_1 - z_2) \\ & + \widehat{\theta}(\sin(t) + \cos(2t)), \\ \dot{\widehat{\theta}} =& v_x(z_1 - z_2), \\ \dot{v}_x =& -16v_y - 0.8v_z + \sin(t), \\ \dot{v}_y =& v_x - v_y + v_z - 2v_z + \cos(2t), \\ \dot{v}_z =& 10v_y + 1.43v_z - 2v_z + \sin(t) + \cos(2t), \end{aligned} \tag{20}$$

where the following assumption is made: $v_x(0) = 0.04, v_y(0) = 0.08$ and $v_z(0) = 0.06$.

A master–slave formalism is taken , which contains the master system (19) and the slave system (20). This synchronization system has been simulated for the above design. The corresponding synchronization errors are shown in Figs.3–5. The estimative error of the parameter θ illustrated in Fig.6 approaches zero evidently.

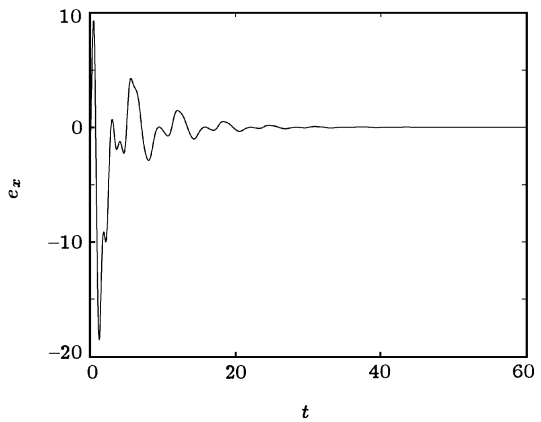


Fig.3. Synchronization error for x.

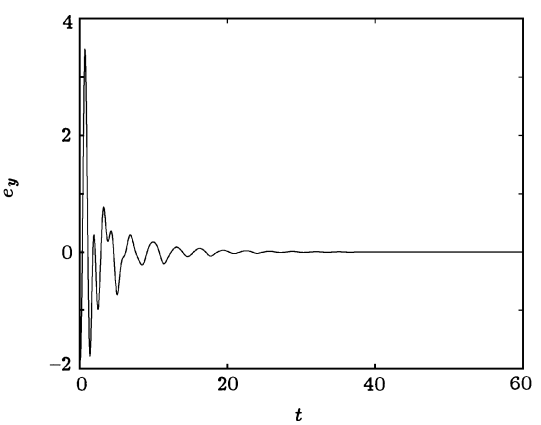


Fig.4. Synchronization error for y.

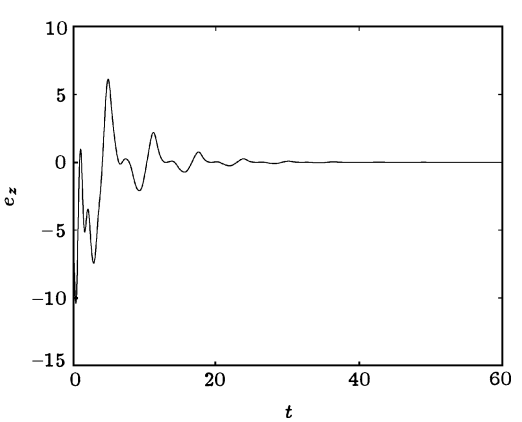


Fig.5. Synchronization error for z.

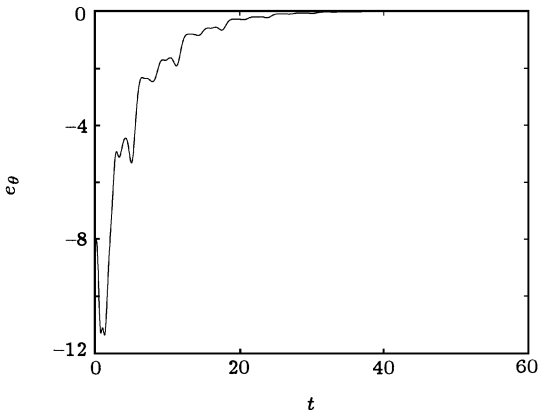


Fig.6. The dynamic error of θ.

4. Conclusions

In this work, we have studied the synchronization problem of Chua's system. The proposed approach provides a procedure for designing an adaptive synchronization controller for Chua's system with uncertain inputs or perturbations. This algorithm allows one to handle realistic situations such as unmeasured states and uncertainties.

The resulting synchronization controller is obtained by use of a constructive method , which needs one available variable of an uncertain Chua's system. Finally, the performance of the designed controller is verified by numerical simulations.

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