Vol 15 No 5, May 2006
1009-1963/2006/15(05)/0947-06

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Stochastic resonance in a bias linear system with multiplicative and additive noise

Guo Feng(郭 锋)[a)†], Zhou Yu-Rong(周玉荣)[b)c)],
Jiang Shi-Qi(蒋世奇)[a)d)], and Gu Tian-Xiang(古天祥)[a)]

[a)] *School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China*
[b)] *School of Life Science and Technology, University of Electronic Science and Technology of China, Chengdu 610054, China*
[c)] *School of Information and Electric Engineering, Panzhihua University, Panzhihua 617000, China*
[d)] *China Gas Turbine Establishment, Jiangyou 621703, China*

(Received 26 October 2005; revised manuscript received 3 December 2005)

In this paper, the stochastic resonance in a bias linear system subjected multiplicative and additive dichotomous noise is investigated. Using the linear-response theory and the properties of the dichotomous noise, this paper finds the exact expressions for the first two moments and the signal-to-noise ratio (SNR). It is shown that the SNR is a non-monotonic function of the correlation time of the multiplicative and additive noise, and it varies non-monotonously with the intensity and asymmetry of the multiplicative noise as well as the external field frequency. Moreover, the SNR depends on the system bias, the intensity of the cross noise between the multiplicative and additive noise, and the strength and asymmetry of the additive noise.

Keywords: stochastic resonance, bias linear system, signal-to-noise ratio
PACC: 0540, 0250

1. Introduction

Stochastic forces can play a crucial role in influencing the deterministic kinetics. A representative example is the stochastic resonance (SR) phenomenon observed in systems driven by random and external periodic forces.[1−21] The original work on SR by Benzi *et al* was in the context of modelling the switch of the Earth's climate between ice ages and periods of relative warmth.[1,2] Typically SR is a nonlinear effect that accounts for the optimum response of a dynamical system to an external force at positive noise intensity. The term SR in a broad sense means the non-monotonic behaviour of the output signal as a function of some characteristics of the noise (noise intensity or noise correlation time) or of a periodic force (amplitude or frequency).

Researchers have investigated SR by applying various kinds of methods. Fauve and Heslot,[3] and McNamara *et al*[4] observed the SR by experiment. Wiesenfeld *et al*[4,5] studied the SR phenomenon in the two-state mode by the adiabatic elimination theory. Dykman *et al*[6] and Hu *et al*[7,8] introduced the linear-response theory and perturbation theory to investigate the SR. Zhou *et al*[9] employed the residence-time distribution to explain the SR as a resonance synchronization phenomenon.

The SR phenomenon has been studied in a variety of nonlinear systems with additive and multiplicative noise.[10−14] It was concluded that non-linearity, periodic and random force are the essential ingredients for the onset of SR. On the other hand, behaviour similar to SR has also been found in linear systems subjected to multiplicative coloured or dichotomous noise.[15−17] It was suggested that noise multiplicativity and time correlation are the necessary conditions for the SR to occur in linear systems.

As widely used in actual systems such as modern communication systems[18,19], bias signal is adopted to improve the quality of communication. However, only few papers involve the effect of the bias signal in a linear system. In this paper, we investigate the SR phenomenon in a bias linear system with multiplicative and additive dichotomous noise. According to the expression of the signal-to-noise ratio (SNR), we find that the SNR decreases as the increase of the

†Corresponding author. E-mail: guofen9932@163.com

bias of the system and the increase of the intensity of the additive noise, while the SNR increases as the increase of the asymmetry of the additive noise and the increase of the intensity of the cross noise between the multiplicative and additive noise.

2. The bias linear system and its output SNR

Consider a bias linear system described by the following stochastic differential equation:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -[a+\xi(t)]\,x(t)+\xi(t)\,A\cos(\Omega t)+r+\eta(t), \tag{1}$$

where r is the bias of the system. $\xi(t)$ and $\eta(t)$ are dichotomous noises with zero mean and their correlation functions are described as follows:

$$\begin{aligned}
&\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0,\\
&\langle\xi(t)\,\xi(s)\rangle = \sigma_1 \exp\left[-\lambda_1|t-s|\right],\\
&\langle\eta(t)\,\eta(s)\rangle = \sigma_2 \exp\left[-\lambda_2|t-s|\right],\\
&\langle\xi(t)\,\eta(s)\rangle = \langle\eta(t)\,\xi(s)\rangle = \sigma_3 \exp\left[-\lambda_3|t-s|\right].
\end{aligned} \tag{2}$$

Where σ_3 is the intensity of the cross noise between the multiplicative noise $\xi(t)$ and additive noise $\eta(t)$. $\xi(t)$ and $\eta(t)$ take the values J_1, $-K_1$ and J_2, $-K_2$ respectively. $J_i>0$, $K_i>0$, $i=1,2$. We define p_1 as the transition rate of $\xi(t)$ from J_1 to $-K_1$ and p_2 the reverse rate, q_1 as the transition rate of $\eta(t)$ from J_2 to $-K_2$ and q_2 the reverse one. Δ_1 and Δ_2 denote the asymmetries of the dichotomous noise $\xi(t)$ and $\eta(t)$ respectively. These parameters satisfy the following equations:

$$\begin{aligned}
&p_2J_1 - p_1K_1 = 0,\\
&q_2J_2 - q_1K_2 = 0,\\
&\Delta_1 = J_1 - K_1, \quad \Delta_2 = J_2 - K_2,\\
&\lambda_1 = p_1 + p_2, \quad \lambda_2 = q_1 + q_2,\\
&\sigma_1 = J_1K_1, \qquad \sigma_2 = J_2K_2.
\end{aligned} \tag{3}$$

There are different ways to solve the linear stochastic equation (1). In order to find the first two moments, we average Eq.(1) and multiply it by $2x$, then obtain the following two equations:

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = -a\langle x\rangle - \langle\xi x\rangle + r, \tag{4}$$

$$\begin{aligned}
\frac{\mathrm{d}\langle x^2\rangle}{\mathrm{d}t} =& -2a\langle x^2\rangle - 2\langle\xi x^2\rangle + 2\langle\eta x\rangle + 2r\langle x\rangle\\
&+ 2A\cos(\Omega t)\langle\xi x\rangle.
\end{aligned} \tag{5}$$

Applying the well-known Shapiro–Loginov formula,[20] we obtain the following equation:

$$\frac{\mathrm{d}\langle\xi x\rangle}{\mathrm{d}t} = \left\langle\xi\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle - \lambda_1\langle\xi x\rangle. \tag{6}$$

Multiplying Eq.(1) by $\xi(t)$, averaging, and then combining it with Eq.(6), we have:

$$\frac{\mathrm{d}\langle\xi x\rangle}{\mathrm{d}t} = -\langle\xi^2 x\rangle - (a+\lambda_1)\langle\xi x\rangle + \sigma_1 A\cos(\Omega t) + \sigma_3. \tag{7}$$

Equation (7) contains the moment $\langle\xi^2 x\rangle$, one can apply the properties of dichotomous noise $\xi(t)$ to decouple this term,

$$\langle\xi^2 x\rangle = \sigma_1\langle x\rangle + \Delta_1\langle\xi x\rangle. \tag{8}$$

Substituting Eq.(8) into Eq.(7), one has:

$$\begin{aligned}
\frac{\mathrm{d}\langle\xi x\rangle}{\mathrm{d}t} =& -\sigma_1\langle x\rangle - (a+\lambda_1+\Delta_1)\langle\xi x\rangle\\
&+ \sigma_1 A\cos(\Omega t) + \sigma_3.
\end{aligned} \tag{9}$$

Equations (4) and (9) are equations for two unknown functions $\langle x\rangle$ and $\langle\xi x\rangle$. Solving these equations, we can obtain the asymptotic value at $t\to\infty$ of the first moment $\langle x\rangle$,

$$\langle x\rangle = f_1\cos(\Omega t) + f_2\sin(\Omega t) + f_3, \tag{10}$$

where

$$\begin{aligned}
f_1 =& A\sigma_1\left(\Omega^2 - c_1c_2\right)\left[\left(\Omega^2+c_1^2\right)\left(\Omega^2+c_2^2\right)\right]^{-1},\\
f_2 =& -A\sigma_1\Omega\left(2a+\lambda_1+\Delta_1\right)\\
&\times\left[\left(\Omega^2+c_1^2\right)\left(\Omega^2+c_2^2\right)\right]^{-1},\\
f_3 =& \left[r\left(a+\lambda_1+\Delta_1\right)-\sigma_3\right]\left(c_1c_2\right)^{-1},\\
c_{1,2} =& (1/2)\left(2a+\lambda_1+\Delta_1\right)\\
&\pm(1/2)\sqrt{\left(\lambda_1+\Delta_1\right)^2+4\sigma_1}.
\end{aligned} \tag{11}$$

In a similar manner, we can obtain the following equations:

$$\begin{aligned}
\frac{\mathrm{d}\langle\xi x^2\rangle}{\mathrm{d}t} =& -2\sigma_1\langle x^2\rangle - (2a+\lambda_1+2\Delta_1)\langle\xi x^2\rangle\\
&+ 2\langle\xi\eta x\rangle + 2\left[r+\Delta_1 A\cos(\Omega t)\right]\langle\xi x\rangle\\
&+ 2\sigma_1 A\cos(\Omega t)\langle x\rangle,
\end{aligned} \tag{12}$$

$$\begin{aligned}
\frac{\mathrm{d}\langle\eta x\rangle}{\mathrm{d}t} =& -(a+\lambda_2)\langle\eta x\rangle - \langle\xi\eta x\rangle\\
&+ \sigma_3 A\cos(\Omega t) + \sigma_2,
\end{aligned} \tag{13}$$

$$\begin{aligned}
\frac{\mathrm{d}\langle\xi\eta x\rangle}{\mathrm{d}t} =& -\sigma_1\langle\eta x\rangle - (a+\lambda_1+\lambda_2+\Delta_1)\langle\xi\eta x\rangle\\
&+ r\sigma_3 + \Delta_1\sigma_3 A\cos(\Omega t) + \Delta_2\sigma_3.
\end{aligned} \tag{14}$$

According to Eqs.(5) and (12)–(14), one obtains four equations for the four variables, $\langle x^2\rangle$ $\langle \xi x^2\rangle$, $\langle \eta x\rangle$ and $\langle \xi \eta x\rangle$. Omit the complicated calculation process, here we give only the stationary value $t \to \infty$ of the second moment:

$$\left\langle x^2 \right\rangle_{\rm st} = \left[\left(2a + 2\Delta_1 + \lambda_1\right) f_8 - 2 f_9 \right] \left(c_5 c_6\right)^{-1}, \quad (15)$$

where

$$\begin{aligned}
f_8 =& 2 f_6 + 2 r f_3 + A f_4, \\
f_9 =& 2 f_7 + 2 r f_5 + A \Delta_1 f_4 + A \sigma_1 f_1, \\
f_4 =& A \sigma_1 \left[\Omega^2 \left(a + \lambda_1 + \Delta_1\right) + a c_1 c_2 \right] \\
& \times \left[\left(\Omega^2 + c_1^2\right) \left(\Omega^2 + c_2^2\right) \right]^{-1}, \\
f_5 =& - a \left[r \left(a + \lambda_1 + \Delta_1\right) - \sigma_3 \right] \left(c_1 c_2\right)^{-1} + r, \\
f_6 =& \left[\sigma_2 \left(a + \lambda_1 + \lambda_2 + \Delta_1\right) - \sigma_3 \left(\Delta_2 + r\right) \right] \left(c_3 c_4\right)^{-1}, \\
f_7 =& - \left(a + \lambda_2\right) \left[\sigma_2 \left(a + \lambda_1 + \lambda_2 + \Delta_1\right) \right. \\
& \left. - \sigma_3 \left(\Delta_2 + r\right) \right] \left(c_3 c_4\right)^{-1} + \sigma_2, \\
c_{3,4} =& \left(1/2\right) \left(2a + \lambda_1 + 2\lambda_2 + \Delta_1\right) \\
& \pm \left(1/2\right) \sqrt{\left(\lambda_1 + \Delta_1\right)^2 + 4\sigma_1}, \\
c_{5,6} =& \left(1/2\right) \left(4a + \lambda_1 + 2\Delta_1\right) \\
& \pm \left(1/2\right) \sqrt{\left(\lambda_1 + 2\Delta_1\right)^2 + 16\sigma_1}, \qquad (16)
\end{aligned}$$

and $c_i, f_j (i = 1, 2, j = 1, 2, 3)$ have been defined earlier.

Our final aim is to calculate the stationary correlation function $\langle x(t+\tau)x(t)\rangle$. From the solution of the linear equation Eq.(1), one can obtain the following formula:

$$\begin{aligned}
x\left(t + \tau\right) =& x\left(t\right) \exp\left(-a\tau\right) g\left(\tau\right) \\
& + r \int_0^\tau g\left(u\right) \exp\left(-au\right) {\rm d}u \\
& + A \int_0^\tau h_2\left(u\right) \exp\left(-au\right) \\
& \times \cos\left[\Omega\left(t + \tau - u\right)\right] {\rm d}u \\
& + \int_0^\tau h_1\left(u\right) \exp\left(-au\right) {\rm d}u, \qquad (17)
\end{aligned}$$

where

$$\begin{aligned}
g\left(u\right) &= \left\langle \exp\left[- \int_0^u \xi\left(s\right) {\rm d}s \right] \right\rangle, \\
h_1\left(u\right) &= \left\langle \eta\left(t - u\right) \exp\left[- \int_0^u \xi\left(s\right) {\rm d}s \right] \right\rangle, \\
h_2\left(u\right) &= \left\langle \xi\left(t - u\right) \exp\left[- \int_0^u \xi\left(s\right) {\rm d}s \right] \right\rangle. \qquad (18)
\end{aligned}$$

By calculating the integrals presented in Eq.(18), one obtain:[21,22]

$$\begin{aligned}
g\left(u\right) &= \frac{\kappa_1}{\kappa_1 - \kappa_2} \exp\left(-\kappa_2 u\right) - \frac{\kappa_2}{\kappa_1 - \kappa_2} \exp\left(-\kappa_1 u\right), \\
h_1\left(u\right) &= \frac{\sigma_3}{\lambda_3} \exp\left(-\lambda_3 u - 1\right) g\left(u\right), \\
h_2\left(u\right) &= \frac{\sigma_1}{\lambda_1} \exp\left(-\lambda_1 u - 1\right) g\left(u\right), \qquad (19)
\end{aligned}$$

where $\kappa_{1,2} = c_{1,2} - a$, and $c_i (i = 1, 2)$ have been defined earlier.

Multiplying Eq.(17) by $x(t)$ and averaging, we obtain the asymptotic expression of the correlation function,

$$\begin{aligned}
\left\langle x\left(t + \tau\right) x\left(t\right) \right\rangle =& \left\langle x^2 \right\rangle_{\rm st} \exp\left(-a\tau\right) g\left(\tau\right) + r \left\langle x \right\rangle \int_0^\tau \exp\left(-au\right) g\left(u\right) {\rm d}u \\
& + A \left\langle x \right\rangle \int_0^\tau h_2\left(u\right) \exp\left(-au\right) \cos\left[\Omega\left(t + \tau - u\right)\right] {\rm d}u + \left\langle x \right\rangle \int_0^\tau h_1\left(u\right) \exp\left(-au\right) {\rm d}u. \qquad (20)
\end{aligned}$$

Averaging Eq.(20) over the external period $2\pi/\Omega$, one can obtain the stationary value of the correlation function:

$$\begin{aligned}
\left\langle x\left(t + \tau\right) x\left(t\right) \right\rangle_{\rm st} =& \left\langle x^2 \right\rangle_{\rm st} \exp\left(-a\tau\right) g\left(\tau\right) + m_1 \int_0^\tau \exp\left(-au\right) g\left(u\right) {\rm d}u + m_2 \int_0^\tau h_2\left(u\right) \exp\left(-au\right) \cos\left(\Omega u\right) {\rm d}u \\
& + m_3 \int_0^\tau h_2\left(u\right) \exp\left(-au\right) \sin\left(\Omega u\right) {\rm d}u + f_3 \int_0^\tau h_1\left(u\right) \exp\left(-au\right) {\rm d}u, \qquad (21)
\end{aligned}$$

where

$$m_1 = r f_3, \quad m_2 = \left(A/2\right) \left[f_1 \cos\left(\Omega\tau\right) - f_2 \sin\left(\Omega\tau\right) \right], \quad m_3 = \left(A/2\right) \left[f_1 \sin\left(\Omega\tau\right) + f_2 \cos\left(\Omega\tau\right) \right], \qquad (22)$$

and $f_i, h_j (i = 1, 2, 3, j = 1, 2), g\left(u\right)$ were defined earlier.

Applying the Fourier transform on both sides of Eq.(21), we obtain its one-sided averaged power spectrum defined for positive frequency Ω:

$$S\left(\omega\right) = S_{\rm s}\left(\Omega\right) \delta\left(\omega - \Omega\right) + S_{\rm n}\left(\omega\right), \qquad (23)$$

where

$$S_{\rm s}(\Omega) = A\pi\sigma_3(c_2-c_1)[\exp(1)f_{10}]^{-1} \times \{f_1[\lambda_3^3+(2c_1+2c_2-a)\lambda_3^2+(\Omega^2+(c_1+c_2)(c_1+c_2-a)+c_1c_2)\lambda_3+a\Omega^2 + c_1c_2(c_1+c_2-a)]+f_2\Omega[\lambda_3^2+2(c_1+c_2-a)\lambda_3+\Omega^2+(c_1+c_2)(c_1+c_2-a)-c_1c_2]\}, \quad (24)$$

$$S_{\rm n}(\omega) = S_{\rm n1}(\omega)+S_{\rm n2}(\omega), \quad (25)$$

$$S_{\rm n1}(\omega) = (c_1-c_2)^{-1}\{\langle x^2\rangle_{\rm st}[2c_2(c_1-a)(c_2^2+\omega^2)^{-1}+2c_1(a-c_2)(c_1^2+\omega^2)^{-1}] - rf_3[2(c_1-a)(c_2^2+\omega^2)^{-1}+2(a-c_2)(c_1^2+\omega^2)^{-1}]\} - 2f_3\sigma_1[\exp(1)\lambda_1(c_1-c_2)]^{-1}\{(c_1-a)[(\lambda_1+c_2)^2+\omega^2]^{-1}+(a-c_2)[(\lambda_1+c_1)^2+\omega^2]^{-1}\},$$

$$S_{\rm n2}(\omega) = -A\sigma_3(\lambda_3+c_1)(a-c_2)[(c_2+\lambda_3)^2+\Omega^2] \times \{f_2\Omega+f_1(\lambda_3+c_1)\}\{\exp(1)f_{10}[(\lambda_3+c_1)^2+\omega^2]\}^{-1} + A\sigma_3(\lambda_3+c_2)(a-c_1)\{\exp(1)f_{10}[(\lambda_3+c_2)^2+\omega^2]\}^{-1} \times \{f_2\Omega[(c_1+\lambda_3)^2+\Omega^2]+f_1(\lambda_3+c_2)[(c_1+\lambda_3^2)^2+\Omega^2]\}, \quad (26)$$

$$f_{10} = \lambda_3(c_1-c_2)[(\lambda_3+c_2)^2+\Omega^2][(\lambda_3+c_1)^2+\Omega^2], \quad (27)$$

and $c_i, f_j (i=1,2, j=1,2,3)$ have been defined earlier. Where $S_{\rm s}(\Omega)$ is the power density connected with the output signal, $S_{\rm n}(\omega)$ is the power spectrum associated with the noise background. SNR is defined as the ratio between the power densities of the signal and the noise background at the frequency $\omega=\Omega$:

$$\text{SNR} = \frac{S_{\rm s}(\Omega)}{S_{\rm n}(\omega=\Omega)}, \quad (28)$$

where $S_{\rm s}(\Omega)$ and $S_{\rm n}(\omega)$ have been defined earlier.

3. Discussions and conclusion

Up to now, we have obtained the expressions of the first two moments and the output SNR. Now let us discuss the influence of the noise and the external field on SNR and draw some conclusions.

By virtue of Eq.(28), the effects of the external frequency, the noise intensity, the asymmetry and correlation time as well as the bias of the system on the SNR are discussed through Figs.1–5.

In Fig.1, we plot the curve of the SNR versus the correlation time $\tau_2=\lambda_2^{-1}$ of the additive noise with varied additive noise asymmetry Δ_2. As seen in Fig.1, the SNR is a non-monotonic function of τ_2, a maximum exists on the curve of the SNR, i.e., stochastic resonance in a broad sense occurs, which is absent in Ref.[16]. Moreover, the SNR increases with the increase of Δ_2, the maximum-value position of the SNR curve shifts to the right when Δ_2 increases.

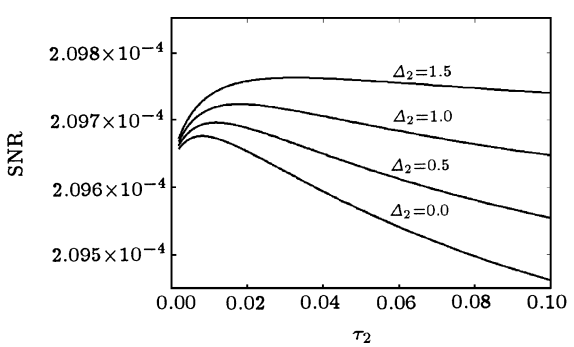


Fig.1. SNR as a function of the correlation time $\tau_2=\lambda_2^{-1}$ of the additive noise for λ_1=100, $\lambda_3=10$, $\Delta_1=1$, $a=10$, $\sigma_1=0.5$, σ=0.0012, $\sigma_3=0.01$, $r=10$, $A=1$, $\Omega=1$ with varied Δ_2.

The curve of the SNR versus the multiplicative noise intensity σ_1 with varied system bias r is plotted in Fig.2. The curve exhibits a maximum and we observe clearly the conventional stochastic resonance in this case, which is not presented in Ref.[16]. Moreover, the SNR decreases with the increase of the system bias r, the position of its peak moves to the left with the increase the bias r.

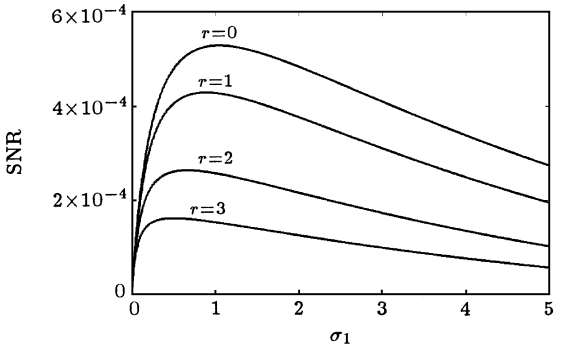


Fig.2. SNR as a function of the multiplicative noise intensity σ_1 for λ_1=5, λ_2=5, λ_3=5, $\Delta_1 = 1$, $\Delta_2 = 1$, $a = 2$, σ_2=0.1, σ_3=0.01, A=1, $\Omega = 1$ with varied r.

In Fig.3, we analyse the influence of the bias r on the SNR when the SNR is plotted versus the external frequency Ω. As seen in Fig.3, the SNR increases initially when Ω increases, and then reaches a maximum for some intermediate Ω. Hence, the bona fide stochastic resonance takes place. At the same time, the SNR decreases with the increase of r, which is a similar effect shown in Fig.2.

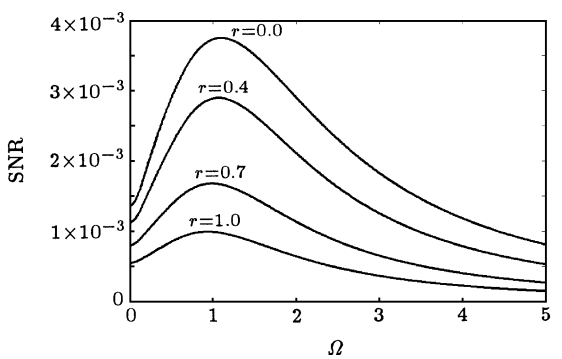


Fig.3. SNR as a function of the external frequency Ω for $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$, Δ_1=1, $\Delta_2 = 1$, $a = 0.6$, σ_1=1, σ_2=1, σ_3=0.1, $A = 1$ with varied r.

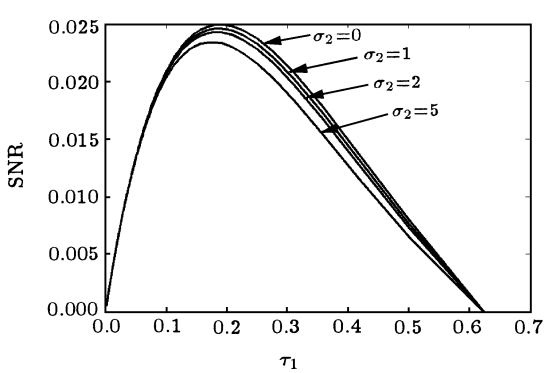


Fig.4. SNR as a function of the correlation time $\tau_1 = \lambda_1^{-1}$ of the multiplicative noise for $\lambda_2 = 1$, $\lambda_3 = 1$, $\Delta_1 = 0.2$, $\Delta_2 = 1$, $a = 1$, $\sigma_1 = 2$, $\sigma_3 = 1$, $r = 0$, $A = 1$, $\Omega = 2$ with varied σ_2.

In Fig.4, the SNR is shown as a function of the correlation time $\tau_1 = \lambda_1^{-1}$ of the multiplicative noise for different values of the additive noise strength σ_2. Here we again observe SR phenomenon in a broad sense. As shown in Fig.4, the SNR decreases slowly as σ_2 increases.

The curve of the SNR versus the asymmetry Δ_1 of the multiplicative noise for different values of the cross noise strength σ_3 is given in Fig.5. As shown in the figure, the SNR also behaves non-monotonously as the asymmetry Δ_1 varies. The SNR increases as the noise strength σ_3 increases, whereas the SNR decreases with the increase of the noise strength σ_2, as shown in Fig.4. In addition, by comparing Fig.5 with Fig.1, we can see that the effect of the asymmetry Δ_1 of the multiplicative noise on the SNR is different from that of the asymmetry Δ_2 of the additive noise, since the latter influences on the SNR monotonically.

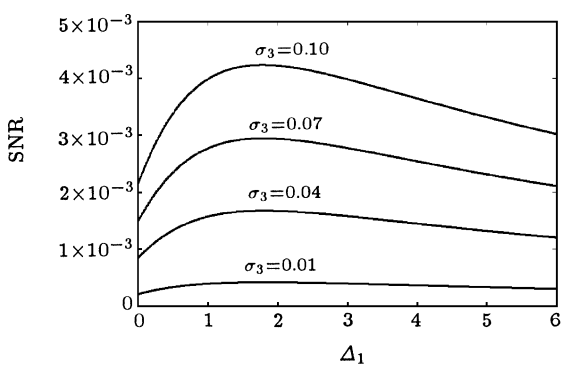


Fig.5. SNR as a function of the asymmetry Δ_1 of the multiplicative noise for $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 1$, $\Delta_2 = 1$, $a = 1$, $\sigma_1 = \sigma_2 = 1$, $r = 0$, $A = 1$, $\Omega = 0.2$ with varied σ_3 .

In conclusion, we have studied the SR phenomenon in a bias linear system subject to multiplicative and additive dichotomous noise. The output SNR shows non-monotonic behaviour when it is plotted versus the correlation time of additive noise, the strength, asymmetry and correlation time of the multiplicative noise as well as the external frequency. The conventional SR, the bona fide SR and the SR in a broad sense can be observed in the bias linear system. Furthermore, the SNR decreases as the increase of the system bias, the effect of the asymmetry of the multiplicative noise on the SNR is different from that of the additive noise, the influence of the intensity of the cross noise on the SNR is opposite to that of the additive noise.

References

[1] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys.* A **14** L453

[2] Benzi R, Parisi G, Sutera A and Vulpiani A 1982 *Tellus* **34** 10

[3] Fauve S and Heslot F 1983 *Phys. Lett.* A **97** 5

[4] McNamara B, Wiesenfeld K and Roy R 1988 *Phys. Rev. Lett.* **60** 2626

[5] McNamara B and Wiesenfeld K 1989 *Phys. Rev.* A **39** 4854

[6] Dykman MI, Mannella R and McClintock PVE 1990 *Phys. Rev. Lett.* **65** 2606

[7] Hu G, Nicolis G and Nicolis C 1990 *Phys. Rev.* A **42** 2030

[8] Hu G 1994 *Stochastic Force and Nonlinear System* (Shanghai: Shanghai Scientific and Technology Education Publishing House) (in Chinese)

[9] Zhou T, Moss F and Jung P 1990 *Phys. Rev.* A **42** 3161

[10] Xu W, Jin Y F, Li W and Ma S J 2005 *Chin. Phys.* **14** 1077

[11] Wang J F, Liu F, Wang J Y, Cheng G and Wang W 1997 *Acta Phys. Sin.* **46** 2305 (in Chinese)

[12] Zhang G J and Xu J X 2005 *Acta Phys. Sin.* **54** 557 (in Chinese)

[13] Kang Y M, Xu J X and Xie Y 2003 *Acta Phys. Sin.* **52** 2712 (in Chinese)

[14] Zhu H J, Li R and Wen X D 2003 *Acta Phys. Sin.* **52** 2404 (in Chinese)

[15] Barzykin A V and Scki K 1998 *Phys. Rev.* E **57** 6555

[16] Berdichevsky V and Gitterman M 1999 *Phys. Rev.* E **60** 1494

[17] Zhang L Y, Cao L and Wu D J 2003 *Acta Phys. Sin.* **52** 1174 (in Chinese)

[18] Fu H Y 2000 *Modern Telecom Transmission* (Beijing: Post and Telecom Publishing House) (in Chinese)

[19] Wang J, Cao L and Wu D J 2004 *Chin. Phys. Lett.* **21** 246

[20] Shapiro V E and Loginov V M 1978 *Physica* A **91** 563

[21] Kubo R 1986 *Nonequilibrium Statistical Mechanics* part 2 (Berlin: Springer)

[22] Fulinski A 1995 *Phys. Rev.* E **52** 4523