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Scheme for concentration of the unknown GHZ entangled states*

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We propose two schemes for concentrating unknown nonmaximally tripartite GHZ entangled states via cavity quantum electrodynamics (QED) techniques. The finial pure states obtained from the two schemes are shared by two or three parties. Our schemes only require large-detuned interaction between two driven atoms and the quantized cavity mode, which is insensitive to both the cavity decay and thermal field, thus the schemes are well within current experimental technology.

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1. Introduction

Quantum entanglement is one of the most important resources for fundamental studies in quantum mechanics. The nonlocal property of entanglement plays an important role in quantum information processing, such as quantum teleportation,[1−3] quantum dense coding,[4] quantum cloning,[5] quantum cryptography[6] and quantum secret sharing,[7] etc. In recent years, there has been a rapid development in quantum information. However, if we want to achieve a faithful quantum information processing, maximally entangled states are the preferred resource.[1] Because local operation and classical communication cannot generate entanglement between distant systems, the distribution of maximally entangled states is needed in quantum information processing.

But, due to the decoherence during storage or transmission of qubits through channels, the amount of entanglement will rapidly decrease, thus the entangled states shared by different users are usually nonmaximally entangled pure states, or in the more general case mixed entangled states. To overcome this problem, many quantum distillation or concentration protocols have been suggested. Bennett *et al*[8] presented the first entanglement concentration scheme, where the entanglement of the total system is transferred to the smaller number of maximally entangled pairs. Recently, Bose *et al*[9] proposed another physical scheme to concentrate nonmaximally entangled photonic polarization states using entanglement swapping. Experimental entanglement distillation schemes for the nonmaximally entangled photonic polarization states have also been proposed in Refs.[10, 11]. Cavity quantum electrodynamics(QED) technology provides an active tool to do quantum information processing with atoms.[12−16] Cao *et al*[17,18] discussed the scheme for the concentration of nonmaximally atomic entangled states.

Here, we will propose two schemes for concentrating the unknown Greenberger–Horne–Zeilinger (GHZ) entangled states in cavity QED. This paper is organized as follows. Section 2 discusses the concentration for two pairs of unknown nonmaximally entangled GHZ states , and two users share the final maximally

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entangled state. Three users share the final maximally entangled state is discussed in Section 3, and Section 4 presents the conclusion and some discussions.

2. Concentration for two users sharing the final maximally entangled states

In this section, we present the scheme for concentration of maximally entangled state from the two identical unknown nonmaximally tripartite GHZ states by local operation. Suppose the two identical nonmaximally tripartite GHZ states are

$$|\psi\rangle_{123} = a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123}, \tag{1a}$$

$$|\psi\rangle_{456} = a|ggg\rangle_{456} + \mathrm{i}b|eee\rangle_{456}, \tag{1b}$$

where $|a|^2+|b|^2=1$, atoms 1, 2, 4 and 5 belong to Alice and atoms 3 and 6 are sent to Bob. The combined state of the system is

$$\begin{aligned}|\psi\rangle =&(a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123})\\ &\otimes (a|ggg\rangle_{456} + \mathrm{i}b|eee\rangle_{456}),\end{aligned} \tag{2}$$

which can be rewritten as

$$\begin{aligned}|\psi\rangle = \frac{1}{\sqrt{2}}\Big[&(a^2|gggg\rangle_{2356} \pm \mathrm{i}b^2|eeee\rangle_{2356}) \otimes |\phi^{\pm}\rangle_{14}\\ &+ (\mathrm{i}ab|ggee\rangle_{2356} \pm ab|eegg\rangle_{2356}) \otimes |\psi^{\pm}\rangle_{14}\Big],\end{aligned} \tag{3}$$

where $|\phi^{\pm}\rangle_{14} = 1/\sqrt{2}(|gg\rangle_{14} \pm \mathrm{i}\,|ee\rangle_{14})$ and $|\psi^{\pm}\rangle_{14} = 1/\sqrt{2}(|ge\rangle_{14} \pm \mathrm{i}\,|eg\rangle_{14})$ are four Bell states of atom pairs (1, 4). From the above equation we can see that, if Alice operates Bell state measurement (BSM) on atom pair (1, 4), the state of the rest particles will collapse to one of the following states:

$$|\psi\rangle_1 = \frac{\mathrm{i}ab}{\sqrt{2}}(|ggee\rangle_{2356} + \mathrm{i}|eegg\rangle_{2356}), \tag{4a}$$

$$|\psi\rangle_2 = \frac{\mathrm{i}ab}{\sqrt{2}}(|ggee\rangle_{2356} - \mathrm{i}|eegg\rangle_{2356}), \tag{4b}$$

$$|\psi\rangle_3 = \frac{1}{\sqrt{2}}(a^2|gggg\rangle_{2356} + \mathrm{i}b^2|eeee\rangle_{2356}), \tag{4c}$$

$$|\psi\rangle_4 = \frac{1}{\sqrt{2}}(a^2|gggg\rangle_{2356} - \mathrm{i}b^2|eeee\rangle_{2356}). \tag{4d}$$

But the BSM is complex for experimental realization,[19] so here we investigate separate atomic measurements to achieve the goal of joint BSM by using optical cavity.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity and meanwhile the two atoms are driven by a classical field. In the case of $\omega_0 = \omega$ and $\Omega >> \delta >> g$ (Ω is the Rabi frequency of the classical field, δ is the detuning between atomic transition frequency ω_0 and cavity frequency ω, g is the coupling constant between cavity and particle), there is no energy exchange between the atomic system and the cavity, thus the scheme is insensitive to both the cavity decay and the thermal field. Then the interaction between the single-mode cavity and the atoms can be described, in the rotating-wave approximation, as[20]

$$\begin{aligned}H_{\mathrm{eff}} =&\frac{\lambda}{2}\Big[\sum_{j=1,4}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|)\\ &+ \sum_{j,k=1,4,j\neq k}(S_j^+ S_k^+ + S_j^+ S_k^- + \mathrm{H.c.})\Big],\end{aligned} \tag{5}$$

where $\lambda = \dfrac{g^2}{2\delta}$, $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, and $|e_j\rangle$, $|g_j\rangle$ are the excited and ground states of the jth atom, respectively. If the two atoms are simultaneously sent into the cavity, they interact with the cavity. By using the evolution operator of the system $U(t) = \mathrm{e}^{-\mathrm{i}H_0 t}\mathrm{e}^{-\mathrm{i}H_{\mathrm{eff}}t}$ together with $H_0 = \Omega\sum\limits_{j=1,4}\left(S_j^+ + S_j^-\right)$, H_{eff} being the effective Hamiltonian, it is easy to verify the evolutions by adjusting the interaction time $\lambda t = \pi/4$ and modulating the driving field $\Omega t = \pi$:

$$|g\rangle_1|g\rangle_4 \to \frac{1}{\sqrt{2}}(|g\rangle_1|g\rangle_4 - \mathrm{i}\,|e\rangle_1|e\rangle_4), \tag{6a}$$

$$|g\rangle_1|e\rangle_4 \to \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_4 - \mathrm{i}\,|e\rangle_1|g\rangle_4), \tag{6b}$$

$$|e\rangle_1|g\rangle_4 \to \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_4 - \mathrm{i}\,|g\rangle_1|e\rangle_4), \tag{6c}$$

$$|e\rangle_1|e\rangle_4 \to \frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_4 - \mathrm{i}\,|g\rangle_1|g\rangle_4). \tag{6d}$$

We can assume, without loss of generality, the quantum state of atom pair (1, 4) is

$$|\phi^+\rangle_{14} = 1/\sqrt{2}(|gg\rangle_{14} + \mathrm{i}\,|ee\rangle_{14}). \tag{7}$$

As the atom pair is sent to the cavity, they will simultaneously interact with the cavity mode. By adjusting the interaction time $\lambda t = \pi/4$ and modulating the driving field $\Omega t = \pi$, the states of the atom pair will evolve, according to Eq.(6), as

$$\begin{aligned}|\phi^+\rangle_{14} =&\frac{1}{\sqrt{2}}(|gg\rangle_{14} + \mathrm{i}\,|ee\rangle_{14})\\ \to&\frac{1}{2}\Big[(|gg\rangle_{14} - \mathrm{i}\,|ee\rangle_{14}) + \mathrm{i}(|ee\rangle_{14} - \mathrm{i}\,|gg\rangle_{14})\Big]\\ =&|gg\rangle_{14}.\end{aligned} \tag{8}$$

Similarly, the other three Bell-states of the atom pair would evolve into

$$|\phi^-\rangle_{14} = \frac{1}{\sqrt{2}}(|gg\rangle_{14} - \mathrm{i}\,|ee\rangle_{14}) \to -\mathrm{i}\,|ee\rangle_{14},$$

$$|\psi^+\rangle_{14} = \frac{1}{\sqrt{2}}(|ge\rangle_{14} + \mathrm{i}\,|eg\rangle_{14}) \to |eg\rangle_{14}\,,$$

$$|\psi^-\rangle_{14} = \frac{1}{\sqrt{2}}(|ge\rangle_{14} - \mathrm{i}\,|eg\rangle_{14}) \to -\mathrm{i}\,|ge\rangle_{14}\,. \quad (9)$$

In this way, we can achieve the goal of BSM with unity probability of success by two separated single-atom measurements.

Without loss of generality, we assume the collapsed state is in the form of Eq.(4a). After measurement, Alice performs a Hadamard gate operation on particle 2, which leads the state to

$$|\psi\rangle = \frac{\mathrm{i}ab}{2}\Big[|g\rangle_2(|gee\rangle_{356} + \mathrm{i}|egg\rangle_{356}) + |e\rangle_2(|gee\rangle_{356} - \mathrm{i}|egg\rangle_{356})\Big]. \quad (10)$$

Then Bob sends atom 3 through a classical field and adjusts the phase and amplitude so that it undergoes the transitions $|g\rangle_3 \to |e\rangle_3$, $|e\rangle_3 \to |g\rangle_3$. After a von Neumann measurement on atom 2, if the measured result is $|g\rangle_2$, the state of atoms 3, 5 and 6 is $ab/2(|eee\rangle_{356} + \mathrm{i}|ggg\rangle_{356})$, and if the result is $|e\rangle_2$, the state of the three atoms is $ab/2(|eee\rangle_{356} - \mathrm{i}|ggg\rangle_{356})$. In both the above two cases the final states are maximally entangled states. Therefore, the unknown nonmaximally entangled GHZ states are concentrated into one maximally entangled state. The final maximally entangled states are shared by two users. When the collapsed state is in the form of Eq.(4b), we also gain the maximally entangled state under the same procedures. But if we select Eq.(4c) or Eq.(4d), we cannot concentrate the unknown nonmaximally entangled GHZ states. So the unknown nonmaximally entangled GHZ states are concentrated into one maximally entangled GHZ state with a successful probability $P = |ab|^2$.

3. Concentration with three users sharing the final maximally entangled states

In this section we present the physical scheme for concentration of maximally entangled state from three identical unknown nonmaximally tripartite GHZ states. In this case three users share the residual maximally entangled states. Three identical tripartite GHZ class states are

$$|\psi\rangle_{123} = a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123},$$

$$|\psi\rangle_{456} = a|ggg\rangle_{456} + \mathrm{i}b|eee\rangle_{456},$$

$$|\psi\rangle_{789} = a|ggg\rangle_{789} + \mathrm{i}b|eee\rangle_{789}, \quad (11)$$

where atoms 1, 4 and 7 belong to Alice, atoms 2, 5 and 8 belong to Bob, and atoms 3, 6 and 9 belong to Charlie. The combined state of the system is

$$|\psi\rangle = (a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123}) \otimes (a|ggg\rangle_{456} + \mathrm{i}b|eee\rangle_{456}) \otimes (|a|ggg\rangle_{789} + \mathrm{i}b|eee\rangle_{789}). \quad (12)$$

Now Alice performs a Controlled-Not (C-NOT) gate operation[15] on atoms 4 and 7 with atom 4 used as target qubit and atom 7 used as control qubit. Then we obtain

$$|\psi\rangle = (a\,|ggg\rangle_{123} + \mathrm{i}b\,|eee\rangle_{123}) \otimes \Big[(a^2\,|ggg\rangle_{456} + \mathrm{i}ab\,|eee\rangle_{456})\,|ggg\rangle_{789} + (\mathrm{i}ab|egg\rangle_{456} - b^2|gee\rangle_{456})|eee\rangle_{789}\Big], \quad (13)$$

which can be expressed as

$$|\psi\rangle = (a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123}) \otimes \Big[(a^2|gggg\rangle_{5689} \pm \mathrm{i}ab|ggee\rangle_{5689})|\phi^\pm\rangle_{47} - (b^2|eeee\rangle_{5689} \mp \mathrm{i}ab|eegg\rangle_{5689})|\psi^\pm\rangle_{47}\Big], \quad (14)$$

where $|\phi^\pm\rangle_{47}$ and $|\psi^\pm\rangle_{47}$ are Bell states. Then Alice performs a BSM on atoms 4 and 7. If the measured result is $|\phi\rangle_{47}$, the whole state will collapse into

$$|\psi\rangle_{1235689} = 1\Big/\sqrt{2}(a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123}) \otimes (a^2|gggg\rangle_{5689} + ab|ggee\rangle_{5689}). \quad (15)$$

Then Bob performs C-NOT gate operation on atoms 5 and 8 with atom 5 used as target qubit and atom 8 used as control qubit. Thus we obtain

$$|\psi\rangle_{1235689} = (a|ggg\rangle_{123} + \mathrm{i}b|eee\rangle_{123}) \otimes (a^2|gggg\rangle_{5689} + ab|egee\rangle_{5689}), \quad (16)$$

which can be expanded as

$$|\psi\rangle_{1235689} = 1/2\Big[(a^2b|ggege\rangle_{13569} \pm \mathrm{i}ab|eeggg\rangle_{13569})|\psi^\pm\rangle_{28} + (a^3|ggggg\rangle_{13569} \pm ab^2|eeege\rangle_{13569})|\phi^\pm\rangle_{28}\Big]. \quad (17)$$

Then Bob performs a BSM on atoms 2 and 8. If the measured result is $|\psi^+\rangle_{28}$, the whole state will collapse into

$$|\psi\rangle_{13569} = 1/2\big(a^2b|ggege\rangle_{13569} + a^2b|eeggg\rangle_{13569}\big). \quad (18)$$

At the same time, Charlie performs a C-NOT gate operation between atoms 3 and 6 with atom 3 used as target qubit and atom 6 used as control qubit. So the state $|\psi\rangle_{13569}$ becomes

$$|\psi\rangle'_{13569} = 1/2\big(a^2b|ggege\rangle_{13569} + a^2b|eegeg\rangle_{13569}\big), \quad (19)$$

which can be rewritten as

$$|\psi\rangle'_{13569} = 1\Big/2\sqrt{2}a^2b\big(|gee\rangle_{13569} \mp \mathrm{i}|egg\rangle_{13569}\big)|\phi^{\pm}\rangle_{36}. \quad (20)$$

Then Charlie performs a BSM on atoms 3 and 6. If the measurement result is $|\phi^+\rangle_{36}$, the whole state will collapse into

$$|\psi\rangle'_{159} = \sqrt{2}\Big/4a^2b(|gee\rangle_{159} - \mathrm{i}|egg\rangle_{159}). \quad (21)$$

Charlie sends atom 1 through a classical field and adjusts the phase and amplitude. The transitions $|g\rangle_1 \rightarrow |e\rangle_1$, $|e\rangle_1 \rightarrow |g\rangle_1$ can be realized. So we obtain the final state

$$|\varphi\rangle_{159} = \sqrt{2}\Big/4a^2b(|eee\rangle_{159} + \mathrm{i}|ggg\rangle_{159}). \quad (22)$$

This is the maximally GHZ classical entangled state. Three users share this final maximally entangled state. If Alice, Bob and Charlie select $|\phi^{\pm}\rangle_{47}$, $|\psi^{\pm}\rangle_{28}$ and $|\phi^{\pm}\rangle_{36}$ as measurement states, respectively, they can also gain the maximally entangled state by the above procedures. So the unknown quantum states are concentrated into one maximally entangled state with a successful probability $P = |a^2b|^2$.

4. Conclusions

In conclusion, we have presented entanglement concentration schemes for an unknown atomic nonmaximally entangled GHZ state via cavity QED techniques. In the schemes, we use the large-detuned interaction between two driven atoms and a quantized cavity mode, so they are insensitive to both the cavity decay and thermal field. In our schemes the initial pure nonmaximally entangled states are unknown, thus the concentration process can deal with the unknown input states successfully.

References

[1] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootter W K 1993 *Phys. Rev. Lett.* **70** 1895
Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[2] Song K H, Zhang W J and Guo G C 2002 *Chin. Phys.* **11** 218

[3] Zhan Y B 2004 *Chin. Phys.* **13** 1801

[4] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
Mattle K, Weinfurter H, Kwiat P G and Zeilinger A 1996 *Phys. Rev. Lett.* **76** 4656

[5] Buzěk V and Hillery M 1996 *Phys. Rev.* A **54** 1844

[6] Ekert A 1991 *Phys. Rev. Lett.* **67** 661
Bennett C H, Brassard G and Mermin N D 1992 *Phys. Rev. Lett.* **68** 557

[7] Hillery M, Buzěk V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829

[8] Bennett C H, Brassard G, Popescu S, Schumacher B, Smoin J A and Wootters W K 1996 *Phys. Rev. Lett.* **76** 722

[9] Bose S, Vedral V and Knight P L 1999 *Phys. Rev.* A **60** 194

[10] Kwiat P G, Barraza-Lopez S, Stefanov A and Gisin N 2001 *Nature* **409** 1014

[11] Zhao Z, Yang T, Chen Y A, Zhang A N and Pan J W 2003 *Phys. Rev. Lett.* **90** 207901

[12] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

[13] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024

[14] Sleator T and Weinfurter H 1995 *Phys. Rev. Lett.* **74** 4087

[15] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
Zheng S B and Guo G C 2001 *Phys. Rev.* A **63** 044302

[16] Gu Y J 2001 *Chin. Phys.* **10** 490

[17] Cao Z L, Yang M and Guo G C 2003 *Phys. Lett.* A **308** 349

[18] Cao Z L, Zhang L H and Yang M 2005 quant-ph/0510044

[19] Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737

[20] Yang M and Cao Z L 2004 quant-ph/0411195