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Teleportation of an unknown bipartite state via non-maximally entangled two-particle state*

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In this paper a new scheme for teleporting an unknown entangled state of two particles is proposed. To weaken the requirement for the quantum channel, without loss of generality, two communicators only share a non-maximally entangled two-particle state. Teleportation can be probabilistically realized if sender performs Bell-state measurements and Hadamard transformation and receiver introduces two auxiliary particles, operates C-not operation, single-qubit measurements and appropriate unitary transformations. The probability of successful teleportation is determined by the smaller one among the coefficients' absolute values of the quantum channel.

Keywords: teleportation, Bell-state measurements, Hadamard transformation, C-not operation
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1. Introduction

Quantum teleportation is one of the most striking features emerging from the quantum entanglement which is inherent in quantum mechanics. It is the process that an unknown quantum state is destroyed at a sending place (Alice) while a perfect replica appears at a remote place (Bob) via the combined action of quantum and classical channels.[1] No information about the unknown state is ever revealed during the teleportation process. Entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure(the entangled state itself). Since the first quantum teleportation scheme was proposed by Bennett *et al*,[1] there has been great attention both in theory and in experiment in recent years due to its important applications in quantum calculation[2,3] and quantum communication.[4−8] Teleportation has been demonstrated in experiments for the polarization state of a photon,[9] the state of a trapped ion[10] and, in the continuous variable regime, for the set of coherent states of a single mode radiation field[11] and so on.

Quantum teleportation of two qubits has recently been studied for pure and noisy quantum channels.[12,13] It is closely related to quantum computation as two-qubit teleportation and one-qubit unitary operations are sufficient to implement the universal gates required for quantum computation.[14] For teleporting quantum states, a number of quantum channels are used, such as the Einstein–Podolsky–Rosen (EPR) pair of two-particle,[1] the GHZ state of three particles[15] and the squeezed vacuum states[16] and so on. In this paper we focus our attention to the teleportation of an unknown entangled state of two particles. In Ref.[17], an unknown entangled state of two particles is teleported by using a Bell state. However a mixed state can be purified to a Bell state with zero probability.[18−20] In Ref.[21–23], two or more shared states are used as quantum channels. To weaken the requirement for quantum channel, i.e. to devise teleportation protocol that works with a reduced amount of entanglement shared between the two particles and without loss of generality, we propose a scheme for teleporting an unknown entangled state of two particles by using a non-maximally entangled two-particle state as quantum channel. The scheme requires Bell-state measurements (BMs) and Hadamard transformation at the sender's location and two auxiliary particles, two-qubit C-not operation, single-qubit measurements and appropriate unitary transformations at the receiver's side. In the scheme, an unknown entangled state of two particles can be

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perfectly teleported probabilistically from sender to receiver. The probability of successful teleportation is determined by the smaller one among the coefficients' absolute values of the partially entangled two-particle state (quantum channel). This proposed scheme is feasible by using today's technologies.

2. Teleportation of an unknown entangled state of two particles

Suppose that sender Alice wants to transmit an unknown entangled state of two particles $|\varphi\rangle_{34}$ to receiver Bob, who are spatially separated. $|\varphi\rangle_{34}$ is the following state:

$$|\varphi\rangle_{34} = x|00\rangle_{34} + y|11\rangle_{34}, \tag{1}$$

where x and y are unknown, except that $|x|^2+|y|^2 = 1$.

We suppose that the non-maximally entangled two-particle state shared by Alice and Bob can be described as

$$|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}, \tag{2}$$

where the coefficients α and β are real, $|\alpha|^2+|\beta|^2 = 1$. Since α and β are unknown probabilistic amplitudes of the state, the simplest situation is $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$, corresponding to the maximally entangled state. Generally speaking, the coefficients α and β are different, and $|\alpha| \neq |\beta|$. Without lose of generality, we may assume $|\alpha| \leq |\beta|$. Particles 1, 3, 4 belong to Alice and Bob owns particle 2. For other combinations of the inequality, similar results can be obtained.

The state of the whole system $|\psi\rangle_{1234} = |\phi\rangle_{12} \otimes |\varphi\rangle_{34}$ composed of particles 3,4 and quantum channel is given as

$$\begin{aligned}|\psi\rangle_{1234} =& |\phi\rangle_{12} \otimes |\varphi\rangle_{34} \\ =& (\alpha|00\rangle_{12} + \beta|11\rangle_{12})(x|00\rangle_{34} + y|11\rangle_{34}) \\ =& x\alpha|0000\rangle_{1324} + y\alpha|0101\rangle_{1324} \\ &+ x\beta|1010\rangle_{1324} + y\beta|1111\rangle_{1324}.\end{aligned} \tag{3}$$

The proposed teleportation scheme works in more details in the following process.

(1) Firstly, Alice performs BMs on particles 1 and 3. After Alice's measurements, the state of whole system in Eq.(3) will collapse into one of the following four results:

$$_{13}\langle\phi^+|\psi\rangle_{1234} = \frac{1}{\sqrt{2}}(x\alpha|00\rangle_{24} + y\beta|11\rangle_{24}), \tag{4}$$

$$_{13}\langle\phi^-|\psi\rangle_{1234} = \frac{1}{\sqrt{2}}(x\alpha|00\rangle_{24} - y\beta|11\rangle_{24}), \tag{5}$$

$$_{13}\langle\psi^+|\psi\rangle_{1234} = \frac{1}{\sqrt{2}}(y\alpha|01\rangle_{24} + x\beta|10\rangle_{24}), \tag{6}$$

$$_{13}\langle\psi^-|\psi\rangle_{1234} = \frac{1}{\sqrt{2}}(y\alpha|01\rangle_{24} - x\beta|10\rangle_{24}). \tag{7}$$

The Bell states in which the particles 1 and 3 are measured are

$$|\varphi^\pm\rangle_{13} = \frac{1}{\sqrt{2}}(|00\rangle_{13} \pm |11\rangle_{13}), \tag{8}$$

$$|\psi^\pm\rangle_{13} = \frac{1}{\sqrt{2}}(|01\rangle_{13} \pm |10\rangle_{13}). \tag{9}$$

Due to Alice's measurements, the entanglement among particles 3 and 4 disappears, and the new entanglement among particles 2 and 4 is set up, i.e. the entanglement swapping occurs.

(2) Alice carries out Hadamard (H) operation on particle 4.

H operation is in the following matrix form:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{10}$$

and

$$H|0\rangle_4 = \frac{1}{\sqrt{2}}(|0\rangle_4 + |1\rangle_4), \quad H|1\rangle_4 = \frac{1}{\sqrt{2}}(|0\rangle_4 - |1\rangle_4). \tag{11}$$

(3) Bob introduces an auxiliary two-state particle 5 with the initial state $|0\rangle_5$, and performs two-qubit C-not transformation ($C_{\rm NOT}$) on particles 2 and 5. $C_{\rm NOT}$ can be written as in matrix

$$C_{\rm NOT} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{12}$$

and

$$\begin{aligned} &C_{\rm NOT}|00\rangle_{25} = |00\rangle_{25}, \quad C_{\rm NOT}|01\rangle_{25} = |01\rangle_{25}, \\ &C_{\rm NOT}|10\rangle_{25} = |11\rangle_{25}, \quad C_{\rm NOT}|11\rangle_{25} = |10\rangle_{25}. \end{aligned} \tag{13}$$

H operation and C-not transformation transform these states $_{13}\langle\phi^+|\psi\rangle_{1234}|0\rangle_5$, $_{13}\langle\phi^-|\psi\rangle_{1234}|0\rangle_5$, $_{13}\langle\psi^+|\psi\rangle_{1234}|0\rangle_5$ and $_{13}\langle\psi^-|\psi\rangle_{1234}|0\rangle_5$ into the follow-

ing four states, respectively

$$\frac{1}{2}[(x\alpha|00\rangle_{25}+y\beta|11\rangle_{25})|0\rangle_4 + (x\alpha|00\rangle_{25}-y\beta|11\rangle_{25})|1\rangle_4], \quad (14)$$

$$\frac{1}{2}[(x\alpha|00\rangle_{25}-y\beta|11\rangle_{25})|0\rangle_4 + (x\alpha|00\rangle_{25}+y\beta|11\rangle_{25})|1\rangle_4], \quad (15)$$

$$\frac{1}{2}[(y\alpha|00\rangle_{25}+x\beta|11\rangle_{25})|0\rangle_4 - (y\alpha|00\rangle_{25}-x\beta|11\rangle_{25})|1\rangle_4], \quad (16)$$

$$\frac{1}{2}[(y\alpha|00\rangle_{25}-x\beta|11\rangle_{25})|0\rangle_4 - (y\alpha|00\rangle_{25}+x\beta|11\rangle_{25})|1\rangle_4]. \quad (17)$$

(4) Alice measures particle 4 on the basis $\{|0\rangle,|1\rangle\}$. After Alice's measurements, these states in Eqs.(14)–(17) will collapse into one of the following four states:

$$\frac{1}{2}(x\alpha|00\rangle_{25}+y\beta|11\rangle_{25}), \quad (18)$$

$$\frac{1}{2}(x\alpha|00\rangle_{25}-y\beta|11\rangle_{25}), \quad (19)$$

$$\frac{1}{2}(y\alpha|00\rangle_{25}+x\beta|11\rangle_{25}), \quad (20)$$

$$\frac{1}{2}(y\alpha|00\rangle_{25}-x\beta|11\rangle_{25}). \quad (21)$$

Alice notifies Bob the outcomes of both her measurements (BMs on particles 1,3 and single particle measurements on particle 4) via classical channel. Overall three bits of classical information are transmitted.

(5) Bob introduces another auxiliary two-state particle 6 with the initial state $|0\rangle_6$ and performs a collective unitary transformation U_1 on particles 2,5 and 6 under the basis $\{|00\rangle_{56},|10\rangle_{56},|01\rangle_{56},|11\rangle_{56}\}$, namely

$$U_1=\begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \quad (22)$$

where $A_i(i=1,2)$ is a 2×2 matrix and can be expressed as

$$A_1=\mathrm{diag}(a_0,a_1), \quad (23)$$

$$A_2=\mathrm{diag}(\sqrt{1-a_0^2},\sqrt{1-a_1^2}), \quad (24)$$

where $a_j(j=0,1$ and $|a_j|\le 1)$ depends on the state of particles 2,5 and 6.

We may take, according to the states in Eqs.(18)–(21)

$$a_0=1,\quad a_1=\pm\frac{\alpha}{\beta}, \quad (25)$$

where $a_0=1, a_1=\dfrac{\alpha}{\beta}$ corresponds to the states in Eqs.(18) and (20), and $a_0=1, a_1=-\dfrac{\alpha}{\beta}$ corresponds to the states in Eqs.(19) and (21).

The unitary transformation U_1 transforms

$$\frac{1}{2}(x\alpha|00\rangle_{25}+y\beta|11\rangle_{25})\otimes|0\rangle_6,$$

$$\frac{1}{2}(x\alpha|00\rangle_{25}-y\beta|11\rangle_{25})\otimes|0\rangle_6,$$

$$\frac{1}{2}(y\alpha|00\rangle_{25}+x\beta|11\rangle_{25})\otimes|0\rangle_6$$

and

$$\frac{1}{2}(y\alpha|00\rangle_{25}-x\beta|11\rangle_{25})\otimes|0\rangle_6$$

into the following four states, respectively:

$$\frac{1}{2}\alpha(x|00\rangle_{25}+y|11\rangle_{25})|0\rangle_6 + \frac{1}{2}\sqrt{\beta^2-\alpha^2}y|11\rangle_{25}|1\rangle_6, \quad (26)$$

$$\frac{1}{2}\alpha(x|00\rangle_{25}+y|11\rangle_{25})|0\rangle_6 - \frac{1}{2}\sqrt{\beta^2-\alpha^2}y|11\rangle_{25}|1\rangle_6, \quad (27)$$

$$\frac{1}{2}\alpha(y|00\rangle_{25}+x|11\rangle_{25})|0\rangle_6 + \frac{1}{2}\sqrt{\beta^2-\alpha^2}x|11\rangle_{25}|1\rangle_6, \quad (28)$$

$$\frac{1}{2}\alpha(y|00\rangle_{25}+x|11\rangle_{25})|0\rangle_6 - \frac{1}{2}\sqrt{\beta^2-\alpha^2}x|11\rangle_{25}|1\rangle_6. \quad (29)$$

(6) Then Bob measures particle 6 on the basis $\{|0\rangle,|1\rangle\}$. If his measurement outcome is $|1\rangle_6$, the teleportation fails. If the outcome $|0\rangle_6$ is obtained, the teleportation is successful. The states in Eqs.(26) and (27) will collapse into

$$\frac{1}{2}\alpha(x|00\rangle_{25}+y|11\rangle_{25}), \quad (30)$$

and the states in Eqs.(28) and (29) will collapse into

$$\frac{1}{2}\alpha(y|00\rangle_{25}+x|11\rangle_{25}). \quad (31)$$

Bob needs to establish a correspondence so that the coefficients x and y can correspond to $|00\rangle_{25}$ and $|11\rangle_{25}$ respectively. This may be realized by performing appropriate unitary transformation U_2 on particles 2 and 5 according to Alice and Bob's measurement outcomes. When collapsed state is in Eq.(30), Bob does not need to adopt any operation. However, unitary transformation $U_2=[|1\rangle\langle 0|+|0\rangle\langle 1|]_2\otimes[|1\rangle\langle 0|+|0\rangle\langle 1|]_5$ will be performed on particles 2 and 5 if the collapsed state is in Eq.(31).

Quantum channel of mixed state can never provide teleportation with fidelity 1. The probability of successful teleportation in this scheme may be expressed by the probabilistic amplitude of state in Eq.(30) or Eq.(31) as $(\frac{1}{2}\alpha)^2 = \frac{1}{4}|\alpha|^2$, where $|\alpha|$ is the smaller one among the two coefficients' absolute values $|\alpha|$ and $|\beta|$. Adding up all the contributions, we obtain the optimal probability of successful teleportation $P = \frac{1}{4}|\alpha|^2 \times 8 = 2|\alpha|^2$.

For the maximally entangled quantum channel, $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$, the total successful probability reaches one.

3. Conclusions

In summary, we have proposed a scheme to realize quantum teleportation of an unknown entangled state of two particles via a non-maximally entangled state as quantum channel. It is shown that, for such non-maximally entangled quantum channel, the teleportation can be perfectly realized with a certain probability by sender's BMs and Hadamard transformation and receiver's introducing two auxiliary particles, two-qubit C-not operation, single-qubit measurements and appropriate unitary transformations. Compared with previous proposals, i.e. using two EPR pairs or tripartite entangled states as quantum channels, our scheme requires a reduced amount of entanglement to achieve the same task. It is found that the probability of success is determined by the smaller one among the two coefficients' absolute values $|\alpha|$ and $|\beta|$. When $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$, the total probability equals one.

Finally, we indicate that any linear combination of the form

$$|\varphi\rangle_{34} = x|m_1 n_1\rangle_{34} + y|m_2 n_2\rangle_{34}, \qquad (32)$$

with $m_k, n_k = 0, 1 (k = 1, 2)$ can be teleported by the same protocol, because all these states can be brought to expression (1) by appropriate unitary transformations.

This proposed scheme is feasible by using today's technologies. The partially entangled two-particle state can be created by parametric down-conversion. The early proposals for reliable BMs are those which discriminate between the four polarization-entangled two-photon Bell states by using entanglement in additional degree of freedom[24] or using atomic coherence.[25] Hadamard transformation, C-not operation, single-qubit measurements and U_1 and U_2 transformations can be realized in experiments.

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