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Optimal conclusive teleportation of a d-dimensional two-particle unknown quantum state*

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A conclusive teleportation protocol of a d-dimensional two-particle unknown quantum state using three d-dimensional particles in an arbitrary pure state is proposed. A sender teleports the unknown state conclusively to a receiver by using the positive operator valued measure(POVM) and introducing an ancillary qudit to perform the generalized Bell basis measurement. We calculate the optimal teleportation fidelity. We also discuss and analyse the reason why the information on the teleported state is lost in the course of the protocol.

Keywords: teleportation, POVM, generalized Bell basis, teleportation fidelity
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1. Introduction

Entanglement is a key ingredient of quantum techniques for information processing. One of the striking consequences of quantum entanglement is the existence of the procedure called quantum teleportation.[1] Some teleportation protocols have been proposed.[2,3] Teleportation protocols can also be used to transmit quantum operations.[4,5]

Mor and Horodecki[6] proposed a conclusive teleportation protocol for a quantum state in two-dimensional Hilbert space by employing a positive operator valued measurement (POVM)[7] as the joint measurement at the sending station. Bandyopadhyay[8] and Li *et al*[9] have also proposed protocols to implement the conclusive teleportation in two-dimensional Hilbert space separately. Quantum information theory has been extensively developed for a two-level spin-1/2 system because this simple model provides a natural extension of a binomial bit to a quantum bit, namely, qubit, and gives a comprehensive understanding of fundamental quantum theory. It is only recently that d-dimensional quantum systems have attracted a considerable research effort. Quantum cloning in d-dimensions was studied by Zanardi[10] and quantum teleportation by Zubairy,[11] and Stenholm and Bardro.[12] Rungta *et al* called the d-dimensional quantum system the qudit and investigated its entanglement and separability.[13] The optimal conclusive teleportation of a d-dimensional quantum pure state was studied.[14−16] The qudit was extensively studied as a finite-dimensional version of a continuous variable state by Gottesman *et al*.[17]

In this paper we study the teleportation of a quantum state of two particles belonging to $d \times d$-dimensional Hilbert space. Our protocol considers the use of a non-maximally entangled pure state of three qudits as a quantum channel. We relate the quantum teleportation with the problem of quantum state discrimination, and show that the success of this scheme has a fundamental limit determined by how a set of non-orthogonal linearly-independent quantum states can be unambiguously distinguished. We also discuss the information on the teleported state which is lost in the course of the protocol using a non-maximally entangled state and the POVM's measurement by the sender. We show how much information on the teleported state is lost by evaluating the teleportation fidelity.

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The structure of the rest of this paper is as follows: In Section 2, a conclusive teleportation protocol of a d-dimensional two-particle unknown quantum state using three d-dimensional particles in an arbitrary pure state is proposed. The reason of information loss is also analysed. In Section 3,we conclude our present study.

2. Conclusive teleportation

Suppose Alice has an unknown quantum state of two particles, given by

$$|\varphi\rangle_{12} = \sum_{k=0}^{d-1} \alpha_k |k\rangle_1 |k\rangle_2, \tag{1}$$

where α_k is positive real value and $\sum_{k=0}^{d-1} \alpha_k^2 = 1$.
Alice shares the state of the quantum channel with Bob, given by

$$|\Phi\rangle_{345} = \sum_{k=0}^{d-1} \lambda_k |kkk\rangle_{345}, \tag{2}$$

where λ_k is positive real value and $\sum_{k=0}^{d-1} \lambda_k^2 = 1$.
Alice has qudit 3 and Bob has qudits 4 and 5. Alice wants to teleport the unknown state to Bob. The combined state is given by

$$\begin{aligned} |\Psi\rangle_{\text{total}} =& |\varphi\rangle_{12} \otimes |\Phi\rangle_{345} \\ =& \sum_{k=0}^{d-1} \alpha_k |k\rangle_1 |k\rangle_2 \otimes \sum_{k=0}^{d-1} \lambda_k |kkk\rangle_{345} \\ =& \sum_{k=0}^{d-1} \sum_{i=0}^{d-1} \alpha_k \lambda_i |k\rangle_1 |k\rangle_2 |iii\rangle_{345}. \end{aligned} \tag{3}$$

Let X and Z be $d \times d$ matrices such that $X|j\rangle = |(j+1) \bmod d\rangle$, and $Z|j\rangle = w^j |j\rangle$ with $w = \exp\{-2i\pi/d\}$. We can introduce d^2 linear-independent $d \times d$ - matrices $U_{st} = X^t Z^s$, which satisfy

$$U_{st} U_{s't'} = w^{st' - ts'} U_{s't'} U_{st}, \quad \text{tr}(U_{st}) = d\delta_{s0}\delta_{t0}. \tag{4}$$

One can also check whether $\{U_{st}\}$ satisfy the condition of bases of the unitary operators in Ref.[18], i.e.

$$\text{tr}(U_{st} U_{s't'}^{+}) = d\delta_{tt'}\delta_{ss'},$$

$$U_{st} U_{st}^{+} = I_{d\times d}, \tag{5}$$

where $I_{d\times d}$ is the $d \times d$ identity matrix. $\{U_{st}\}$ form a complete basis of $d \times d$ -matrices, namely, for any $d \times d$ matrix W, Wcan be expressed as

$$W = \frac{1}{n} \sum_{s,t} \text{tr}(U_{st}^{+} W) U_{st}. \tag{6}$$

From $\{U_{st}\}$, we can introduce the generalized Bell-states,

$$|\varphi_{st}^m\rangle = I \otimes U_{st}^* |\varphi^m\rangle = \frac{1}{\sqrt{d}} \sum_{i,j} (U_{st})_{ij}^* |ij\rangle, \tag{7}$$

where $|\varphi^m\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle$.

For any $|\varphi\rangle = \sum_{i=0}^{d-1} \lambda_i |ii\rangle$, λ_i is positive real value and

$$\sum_{i=0}^{d-1} \lambda_i^2 = 1. \tag{8}$$

In the conclusive teleportation protocol, the basis set of entangled states is obtained by using the same local unitary operators $\{U_{st}\}$ on the state $|\varphi\rangle$:

$$|\varphi_{st}\rangle = I \otimes U_{st}^* |\varphi\rangle, \tag{9}$$

where $s = 0, 1, \cdots$, and $d-1; t = 0, 1, \cdots$, and $d-1$.Note that the state vectors in Eq.(9) are linearly independent and form a basis set. The basis states $|\varphi_{st}\rangle$ are not necessarily orthonormal. Only when the channel is maximally entangled with $\lambda_i = 1/\sqrt{d}$ can the set of the basis states $|\varphi_{st}\rangle$ become $|\varphi_{st}^m\rangle$,which represents the von-Neumann orthogonal measurement. It is possible to write the state $|\varphi_{st}\rangle$ in the orthonormal basis $|ij\rangle$:

$$|\varphi_{st}\rangle = \sum_{i,j=0}^{d-1} (\Gamma_{st})_{ij} |ij\rangle, \tag{10}$$

where the matrix Γ_{ij}^{st} is calculated from Eqs.(8) and (9) as

$$(\Gamma_{st})_{ij} = (U_{st})_{ij}^* \lambda_j. \tag{11}$$

Because the basis states $|\varphi_{st}\rangle$ are linearly independent,the matrix Γ_{ij}^{st} is non-singular, and its inverse matrix is given by

$$(\Gamma_{st}^{-1})_{ij} = \frac{1}{d} (U_{st})_{ij} \lambda_j^{-1}. \tag{12}$$

Using Eqs.(10) and (12) yields

$$|ij\rangle = \sum_{st} (\Gamma_{st}^{-1})_{ij} |\varphi_{st}\rangle. \tag{13}$$

The completeness for the set of orthonormal bases $|ij\rangle$, by using Eq.(13), will change to

$$I_{d\times d} = \sum_{i,j=0}^{d-1} |ij\rangle\langle ij| = \sum_{i,j,s,t} (\Gamma_{st}^{-1})_{ij} |\varphi_{st}\rangle\langle ij|. \tag{14}$$

Alice introduces an ancillary qudit a in the state $\frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |j\rangle$ and performs the joint generalized Bell basis measurement. After Alice performs the joint generalized Bell basis measurement on qubit a and 1, the

combined state will change to

$$\begin{aligned}
&\langle\varphi_{st}^{m}|a\rangle\otimes|\Psi\rangle_{\text{total}}\\
=&\frac{1}{\sqrt{d}}\sum_{i,j=0}^{d-1}(U_{st})_{ij\,a1}\langle ij|\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle_a\\
&\otimes\sum_{k=0}^{d-1}\sum_{i=0}^{d-1}\alpha_k\lambda_i|k\rangle_1|k\rangle_2|iii\rangle_{345}\\
=&\frac{1}{d}\sum_{i,j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_j\lambda_k(U_{st})_{ij}|j\rangle_2|kkk\rangle_{345}. \qquad (15)
\end{aligned}$$

Alice obtains the measurement outcome s and t and performs the unitary operation

$$(U_{st})_{ij}|i\rangle_2\longrightarrow|i\rangle_2, \qquad (16)$$

then the state in Eq.(15) will change to

$$|\Psi\rangle_{2345}=\frac{1}{d}\sum_{i,j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_j\lambda_k|j\rangle_2|kkk\rangle_{345}, \qquad (17)$$

thus the state in Eq.(17) will change to

$$\begin{aligned}
|\Psi\rangle_{2345}=&\frac{1}{d}\sum_{i,j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_j\lambda_k|j\rangle_2|kkk\rangle_{345}\\
=&\Big[\sum_{i,j,s,t}(\Gamma_{st}^{-1})_{ij}|\varphi_{st}\rangle_{23}\langle ij|\Big]\\
&\times\frac{1}{d}\sum_{i,j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_j\lambda_k|j\rangle_2|kkk\rangle_{345}\\
=&\frac{1}{d^2}\sum_{i,j,s,t}(U_{st})_{ij}\alpha_i|\varphi_{st}\rangle_{23}|jj\rangle_{45}\\
=&\frac{1}{d^2}\sum_{st}|\varphi_{st}\rangle_{23}\otimes\sum_{i,j}(U_{st})_{ij}\alpha_i|jj\rangle_{45}, \qquad (18)
\end{aligned}$$

where $s=0,1,\cdots$, and $d-1$; $t=0,1,\cdots$, and $d-1$.In order to discriminate non-orthogonal $\{|\varphi_{st}\rangle_{23}\}$, we adopt the method of discriminating non-orthogonal quantum states in Ref.[19]. Suppose that Alice adopts a set of POVM operators $\{M_\alpha\}$ with $n>d^2$, which are designed to satisfy the condition that $p(\alpha|\beta)=\langle\varphi_\beta|M_\alpha|\varphi_\beta\rangle$ does not vanish only when $\alpha=\beta$ for $\alpha\le d^2$ and that $p(\alpha|\beta)$ is allowed not to have to vanish for $d^2<\alpha\le n$. This shows that when the measurement outcome is due to any of M_α for $\alpha\le d^2$, Alice can conclusively determine which non-orthogonal state the system is in, and Bob performs the corresponding unitary operation on his particles to obtain the perfect teleportation fidelity. On the other hand, the measurement bears an inconclusive result when the outcome is of M_α for $\alpha>d^2$. In the conclusive teleportation, d^2 operators are used to identify non-orthogonal states while the other operators consisting of the measurement set represent some inconclusive mixtures. In the inconclusive teleportation, Bob randomly chooses a unitary transform T_α and obtains a reduced teleportation fidelity. Then the final state is $\frac{1}{d^2}T_\alpha\sum_{i,j}(U_{st})_{ij}\alpha_i|jj\rangle_{45}$.

Similar to those in Ref.[19], the conclusive measurement operators are represented by general projectors as

$$M_\alpha=\lambda|\widetilde{\varphi}_{st}\rangle\langle\widetilde{\varphi}_{st}|,$$

$$\langle\varphi_{st}|\widetilde{\varphi}_{s't'}\rangle=\delta_{ss'}\delta_{tt'}, \qquad (19)$$

for $s=0,1,\cdots$, and $d-1$; $t=0,1,\cdots$, and $d-1$; $\alpha=td+s+1$.
And the inconclusive measurement operators are represented by

$$\begin{aligned}
M_{d^2+1}=&I_{d\times d}-\sum_{\alpha=1}^{d^2}M_\alpha\\
=&\sum_{i,j=0}^{d-1}\left(1-\frac{\lambda}{\lambda_j^2d}\right)|ij\rangle\langle ij|. \qquad (20)
\end{aligned}$$

Here the derivation of M_{d^2+1} can be seen in Ref.[19]. We can easily see that M_{d^2+1} is a matrix of rank d^2. So M_{d^2+1} can be decomposed into d^2 rank matrix operators. i.e.

$$\begin{aligned}
M_{d^2+1}=&\left(1-\frac{\lambda}{\lambda_0^2d}\right)|00\rangle\langle 00|\\
&+\left(1-\frac{\lambda}{\lambda_1^2d}\right)|01\rangle\langle 01|+\cdots\\
&+\left(1-\frac{\lambda}{\lambda_{d-1}^2d}\right)|0(d-1)\rangle\langle 0(d-1)|\\
&+\left(1-\frac{\lambda}{\lambda_0^2d}\right)|10\rangle\langle 10|+\left(1-\frac{\lambda}{\lambda_1^2d}\right)|11\rangle\langle 11|+\cdots\\
&+\left(1-\frac{\lambda}{\lambda_{d-1}^2d}\right)|1(d-1)\rangle\langle 1(d-1)|+\cdots\\
&+\left(1-\frac{\lambda}{\lambda_0^2d}\right)|(d-1)0\rangle\langle(d-1)0|+\left(1-\frac{\lambda}{\lambda_1^2d}\right)\\
&\times|(d-1)1\rangle\langle(d-1)1|+\cdots+\left(1-\frac{\lambda}{\lambda_{d-1}^2d}\right)\\
&\times|(d-1)(d-1)\rangle\langle(d-1)(d-1)|. \qquad (21)
\end{aligned}$$

From Eqs.(19) and (20), we can see that $0<\lambda\le\lambda_j^2d$ for all $j=0,1,\cdots$, and $d-1$. Suppose that λ_k is the smallest for all $j=0,1,\cdots$, and $d-1$, then POVM with the optimal when $\lambda=\lambda_k^2d$.

Now let us calculate the optimal average

fidelity[19] as follows:

$$\begin{aligned}\overline{F_{\rm optimal}} =&\overline{F_{\rm con}}+\overline{F_{\rm incon}}\\ =&\frac{1}{V}\int {\rm d}\Omega d^2\Big(\sum_{\alpha}\langle\Psi\rangle_{2345}|M_{\alpha}|\Psi\rangle_{2345}\Big)f_{\alpha}(\Omega)\\ =&\frac{1}{V}\int {\rm d}\Omega d^2\Big(\sum_{\alpha=1}^{d^2}(\langle\Psi\rangle_{2345}|M_{\alpha}|\Psi\rangle_{2345})\cdot 1\\ &+\sum_{i,j=1}^{d-1}\langle\Psi\rangle_{2345}|M_{i,j}|\Psi\rangle_{2345}f_{i,j}(\Omega)\Big)\\ =&\lambda+\frac{1}{V}\int {\rm d}\Omega d^2\sum_{i,j=1}^{d-1}\langle\Psi\rangle_{2345}|M_{i,j}|\Psi\rangle_{2345}\\ &\times|_{45}\langle\varphi|T_{\alpha}\sum_{i,j}(U_{st})_{ij}\alpha_i|jj\rangle_{45}|^2. \end{aligned}\tag{22}$$

After a lengthy calculation, we obtain

$$\overline{F_{\rm optimal}}\leq\lambda+1-\sum_{j=0}^{d-1}\frac{\lambda}{\lambda_j^2 d^2}.\tag{23}$$

We can see easily that $\overline{F_{\rm optimal}}<1$, which shows that the information on the unknown state is not transferred to the receiver completely. In addition, due to the non-orthogonality of the POVM, some information cannot be transferred to the receiver. We can show it by calculating the Shannon entropy of the information obtained by the receiver.

From Eq.(19),we can see that M_{d^2+1} is a matrix of rank d^2. Suppose that the outcome of a generalized measurement is ij, then the prior probability for that result, if the input is $|\varphi_{\alpha}\rangle$, will be

$$\begin{aligned}p_{ij\alpha}=&p_{\alpha}\Big(1-\frac{\lambda}{\lambda_j^2 d}\Big)\langle\varphi_{\alpha}|ij\rangle\langle ij|\varphi_{\alpha}\rangle\\ =&p_{\alpha}\Big(1-\frac{\lambda}{\lambda_j^2 d}\Big)|\langle ij|\sum_{i,j=0}^{d-1}(U_{\alpha})_{ij}^{*}\lambda_j|ij\rangle|^2\\ =&p_{\alpha}\Big(\lambda_j^2-\frac{\lambda}{d}\Big)|(U_{\alpha})_{ij}|^2,\end{aligned}\tag{24}$$

where $p_{\alpha}$is the probability of obtaining $|\varphi_{\alpha}\rangle$ and can be calculated out from Eq.(18). By Bayes' theorem, the posterior probability for input $|\varphi_{\alpha}\rangle$upon observing output ij is

$$Q_{\alpha ij}=\frac{p_{ij\alpha}}{\sum\limits_{\alpha=1}^{d^2}p_{ij\alpha}}.\tag{25}$$

The observer's final ignorant level, after receiving output ij, is given by the Shannon entropy,

$$H_{ij}=-\sum_{\alpha=1}^{d^2}Q_{\alpha ij}\ln Q_{\alpha ij},\tag{26}$$

which is often less than the initial entropy,

$$H_{\rm init}=-\sum_{\alpha=1}^{d^2}p_{\alpha}\ln p_{\alpha}.\tag{27}$$

Equations (25) and (26) imply that some quantum information is left in the inconclusive event.

Now we can give an example of two-dimensional conclusive teleportation to show the feasibility of this method. Suppose the unknown state $|\varphi\rangle_{12}=\alpha|00\rangle_{12}+\beta|11\rangle_{12}$. The state of the quantum channel $|ch\rangle_{345}=\cos\theta_c|000\rangle_{345}+\sin\theta_c|111\rangle_{345}$. In the conclusive teleportation protocol, the basis set of entangled states is obtained by using the local unitary operators $\{I,\sigma_x,\sigma_y,\sigma_z\}$ on the state $|\varphi\rangle=\cos\theta_c|00\rangle+\sin\theta_c|11\rangle$, as follows:

$$|\varphi_{\alpha}\rangle=(U_{\alpha}\otimes I)|\varphi\rangle,$$

$$U_{\alpha}=I,\sigma_x,{\rm i}\sigma_y,\text{and}\ \ \sigma_z;\alpha=1,2,3,4.$$

The combined state is $|\Phi\rangle_{12345}=|\varphi\rangle_{12}\otimes|ch\rangle_{345}$. After Alice performs the Bell basis measurement on her particles 1 and an ancillary qubit $|a\rangle$ initiated by the state$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, the state will change to

$$\begin{aligned}\langle\Phi^{\pm}|a\rangle\otimes|\Phi\rangle_{12345}=&\langle\Phi^{\pm}|a\rangle\otimes|\varphi\rangle_{12}\otimes|ch\rangle_{345}\\ =&\frac{1}{4}\Big[|\varphi_1\rangle(\alpha|00\rangle\pm\beta|11\rangle)\\ &+|\varphi_2\rangle(\alpha|11\rangle\pm\beta|00\rangle)\\ &+|\varphi_3\rangle(\alpha|11\rangle\mu\beta|00\rangle)\\ &+|\varphi_4\rangle(\alpha|00\rangle\mu\beta|11\rangle)\Big],\end{aligned}\tag{28}$$

$$\begin{aligned}\langle\Psi^{\pm}|a\rangle\otimes|\Phi\rangle_{12345}=&\langle\Psi^{\pm}|a\rangle\otimes|\varphi\rangle_{12}\otimes|ch\rangle_{345}\\ =&\frac{1}{4}\Big[|\varphi_1\rangle(\alpha|00\rangle\pm\beta|11\rangle)\\ &+|\varphi_2\rangle(\alpha|11\rangle\pm\beta|00\rangle)\\ &+|\varphi_3\rangle(\alpha|11\rangle\mu\beta|00\rangle)\\ &+|\varphi_4\rangle(\alpha|00\rangle\mu\beta|11\rangle)\Big].\end{aligned}\tag{29}$$

By discriminating non-orthogonal $|\varphi_{\alpha}\rangle$ optimally, we can now calculate the optimal teleportation fidelity

$$\begin{aligned}\overline{F_{\rm optimal}}=&\overline{F_{\rm con}}+\overline{F_{\rm incon}}\\ =&\sum_{i=1}^{4}4\times\frac{1}{16}\lambda\times 1+\sum_{i=1}^{4}4\times\frac{1}{16}(1-\lambda)\times\frac{1}{2}\\ =&\frac{1+\lambda}{2}\\ =&\frac{1+2\min\{|\alpha|^2,|\beta|^2\}}{2},\end{aligned}\tag{30}$$

when $\min\{|\alpha|^2,|\beta|^2\}\longrightarrow\frac{1}{2}$, and $\overline{F_{\rm optimal}}\longrightarrow 1$.

3. Conclusions

A conclusive teleportation protocol of a d-dimensional two-particle unknown quantum state using three d-dimensional particles in an arbitrary pure state is proposed. A sender teleports the unknown state conclusively to a receiver by using the positive operator valued measure(POVM) and introducing an ancillary qudit to perform the generalized Bell basis measurement. We calculate the optimal teleportation fidelity. We also discuss and analyse the reason why the information on the teleported state is lost in the course of the protocol using a non-maximally entangled state and POVM's measurement by the sender.

In this paper, we only consider the case of entangled pure state as a quantum channel. It is also interesting to discuss the case of mixed entangled state as a quantum channel and study its teleportation fidelity and optimal teleportation.

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