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One- and two-photon absorption-amplification response in a four-level atomic system with vacuum-induced coherence*

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This paper theoretically studies the effects of the vacuum-induced coherence on one- and two-photon absorption in a four-level atomic medium. It finds that the one- and two-photon absorption and amplification properties are quite sensitive to the vacuum-induced coherence. It is also shown that the one- and two-photon absorption spectra can be dramatically affected by modulating the relative phase of the applied fields. With the proper choice of the relative phase, the amplification without inversion for the probe field can be realized.

Keywords: one-photon absorption, two-photon absorption, vacuum-induced coherence, probe amplification

PACC: 4250

1. Introduction

Effects of atomic coherence and quantum interference currently become an important and intensive tool in resonant nonlinear optical physics.[1−33] The wide applications of these effects in nonlinear optical regime have led to high-efficiency generation of short-wave length coherent radiation at pump intensities approaching the single-photon level,[1−5] nonlinear spectra at very low light intensity,[6−11] ultraslow optical solitons,[12,13] and frequency conversion without stringent phase-matching requirements.[14−17] In particular, nonlinear optics assisted by electromagnetically induced transparency (EIT) under few-photon optical field due to extremely large Kerr nonlinearities and refractive-index enhancement without absorption has been observed,[18−20] and it opens up the possibilities for a new regime of quantum nonlinear optics. For example, Harris *et al* proposed the use of EIT to suppress absorption of the short-wavelength light generated in a four-wave mixing (FWM) scheme and showed that the FWM efficiency could be greatly enhanced.[1] Agarwal and Harshawardhan showed that in a four-level Y-type system, two-photon absorption could be selectively suppressed or enhanced.[6] In a recent experiment, Yan *et al* demonstrated that EIT in a standard Λ-type configuration could be used to suppress both single-photon and two-photon absorptions simultaneously.[18] Later on, Wu *et al* investigated and discussed a FWM scheme in a five-level atomic system and hyper-Raman scattering (HRS) in resonant coherent media by using EIT, which led to suppressing both two-photon and three-photon absorptions in both FWM and HRS schemes and enabling the FWM to proceed through real, resonant intermediate states without absorption loss.[2,3] Quite recently, Wu *et al* again analysed a lifetime-broadened four-state FWM scheme in the ultraslow propagation regime and put forward a new type of induced transparency resulted from multiphoton destructive interference.[25] On the other hand, the effects of the spontaneously generated coherence (SGC) and/or vacuum-induced coherence (VIC) on absorption and amplification together with coherent population trapping (CPT) have been extensively investigated recently.[34−42] Menon and Agarwal reported the influences of the spontaneously gen-

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erated coherence on the pump-probe response of a Λ-type three-level atomic system and showed that such coherence could preserve both EIT and CPT phenomena.[39] Li *et al* investigated quantum interference between decay channels of a V-type three-level atom in a multilayer dielectric medium and found that in the anisotropic vacuum, quantum interference could appear even if the two dipole moments were orthogonal to each other.[40] Very recently, Xu *et al* investigated the effects of this SGC on the transient-absorption process and found that the transient gain or absorption properties could be greatly affected by the SGC.[41]

In this paper, we investigate the response of one- and two-photon absorption in a four-level atomic system with VIC effect arising from the cross coupling between two spontaneous decay channels due to the interaction with the vacuum of the radiation field. At the same time, we also analyse in detail the influence of the relative phase of the applied fields on one- and two-photon absorption spectra. Section 2 establishes the model, i.e., the Hamiltonian of the system and the evolution equations of the atomic operators assuming the rotating wave approximation. Section 3 is devoted to presenting the numerical results and to discussing the influence of VIC effect on the one- and two-photon absorption spectra. Finally, Section 4 is a brief discussion and summary.

2. Theoretical model and density-matrix equations of motion

Let us first consider a four-level atom interacting with three laser fields, as shown in Fig. 1. The atomic states are denoted by $|0\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$, where states $|1\rangle$ and $|2\rangle$ are closely spaced. A probe laser field $E_{\rm p}$ (angular frequency $\omega_{\rm p}$ and Rabi frequency $2\Omega_{\rm p}$) is applied to the transition $|0\rangle \leftrightarrow |2\rangle$ to serve as the first step of the two-photon excitation of atomic state $|3\rangle$. A signal laser field $E_{\rm s}$ (angular frequency $\omega_{\rm s}$ and Rabi frequency $2\Omega_{\rm s}$, coupled to the transition $|2\rangle \leftrightarrow |3\rangle$) completes the two-photon excitation of upper atomic state $|3\rangle$. A control laser field $E_{\rm c}$ (angular frequency $\omega_{\rm c}$ and Rabi frequency $2\Omega_{\rm c}$) couples states $|0\rangle$ and $|1\rangle$ to produce dynamically induced coherence for the one- and two-photon transparencies. The quantities $2\gamma_{10}$, $2\gamma_{20}$ and $2\gamma_{32}$ are the spontaneous decay rates from levels $|1\rangle$ to $|0\rangle$, levels $|2\rangle$ to $|0\rangle$ and levels $|3\rangle$ to $|2\rangle$, respectively.

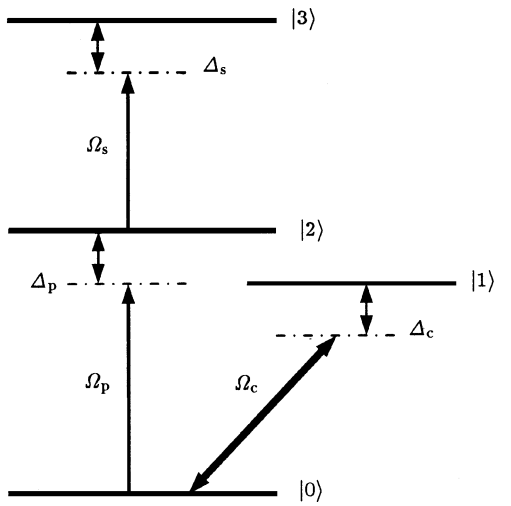


Fig.1. Schematic diagram of a four-level atomic system under study. Here, $2\Omega_{\rm p}$, $2\Omega_{\rm c}$, and $2\Omega_{\rm s}$ are Rabi frequencies of probe, control and signal laser fields, respectively. $\Delta_{\rm p}$, $\Delta_{\rm c}$, and $\Delta_{\rm s}$ are the corresponding atom–field detunings.

Under the electro-dipole interaction and rotating-wave approximations, the semiclassical Hamiltonian describing the atom-field interaction for the system under study can be written in the interaction picture as (taking $\hbar = 1$)

$$\begin{aligned} H_{\rm int} =& \Delta_{\rm c}|1\rangle\langle 1| + \Delta_{\rm p}|2\rangle\langle 2| + (\Delta_{\rm p} + \Delta_{\rm s})|3\rangle\langle 3| \\ & - (\Omega_{\rm p}|2\rangle\langle 0| + \Omega_{\rm c}|1\rangle\langle 0| + \Omega_{\rm s}|3\rangle\langle 2| + {\rm H.c.}), \quad (1) \end{aligned}$$

where the symbol 'H.c.' means Hermitian conjugation. The quantities Ω_n (n=p, c, s) denote one-half Rabi frequencies for the relevant laser-driven transitions, i.e., $\Omega_{\rm p} = \mu_{20}E_{\rm p}/(2\hbar)$, $\Omega_{\rm c} = \mu_{10}E_{\rm c}/(2\hbar)$, $\Omega_{\rm s} = \mu_{32}E_{\rm s}/(2\hbar)$, and the dipole moments $\mu_{ij} = \boldsymbol{\mu}_{ij} \cdot \hat{e}_{\rm L}$ with $\hat{e}_{\rm L}$ being the polarization unit vector of the laser fields. $\Delta_{\rm p} = (\epsilon_2 - \epsilon_0)/\hbar - \omega_{\rm p}$, $\Delta_{\rm c} = (\epsilon_1 - \epsilon_0)/\hbar - \omega_{\rm c}$, and $\Delta_{\rm s} = (\epsilon_3 - \epsilon_2)/\hbar - \omega_{\rm s}$ are the detunings of the laser-driven fields from the corresponding optical transitions, where ϵ_j ($j = 0-3$) is the energy of the atomic states $|j\rangle$ (we have taken $\epsilon_0 = 0$ as the energy origin for the sake of simplicity).

By using the Liouville equation of motion for the density matrix and then the Weisskopf–Wigner theory of spontaneous emission,[43] we can easily obtain the following density matrix equations of motion:

$$
\begin{aligned}
\dot{\rho}_{11} &= \mathrm{i}\Omega_{\mathrm{c}}\rho_{01} - \mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{10} - 2\gamma_{10}\rho_{11} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta(\rho_{12}+\rho_{21}),\\
\dot{\rho}_{22} &= \mathrm{i}\Omega_{\mathrm{p}}\rho_{02} - \mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{20} + \mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{32} - \mathrm{i}\Omega_{\mathrm{s}}\rho_{23} + 2\gamma_{32}\rho_{33} - 2\gamma_{20}\rho_{22} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta(\rho_{12}+\rho_{21}),\\
\dot{\rho}_{33} &= \mathrm{i}\Omega_{\mathrm{s}}\rho_{23} - \mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{32} - 2\gamma_{23}\rho_{33},\\
\dot{\rho}_{01} &= -(\gamma_{10} - \mathrm{i}\Delta_{\mathrm{c}})\rho_{01} + \mathrm{i}\Omega_{\mathrm{c}}^{*}(\rho_{11}-\rho_{00}) + \mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{21} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\rho_{02},\\
\dot{\rho}_{02} &= -(\gamma_{20} - \mathrm{i}\Delta_{\mathrm{p}})\rho_{02} + \mathrm{i}\Omega_{\mathrm{p}}^{*}(\rho_{22}-\rho_{00}) + \mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{12} - \mathrm{i}\Omega_{\mathrm{s}}\rho_{03} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\rho_{01},\\
\dot{\rho}_{03} &= -[\gamma_{32} - \mathrm{i}(\Delta_{\mathrm{p}}+\Delta_{\mathrm{s}})]\rho_{03} + \mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{23} + \mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{13} - \mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{02},\\
\dot{\rho}_{12} &= -[\gamma_{10}+\gamma_{20} - \mathrm{i}(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}})]\rho_{12} + \mathrm{i}\Omega_{\mathrm{c}}\rho_{02} - \mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{10} - \mathrm{i}\Omega_{\mathrm{s}}\rho_{13} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta(\rho_{11}+\rho_{22}),\\
\dot{\rho}_{13} &= -[\gamma_{10}+\gamma_{32} - \mathrm{i}(\Delta_{\mathrm{p}}+\Delta_{\mathrm{s}}-\Delta_{\mathrm{c}})]\rho_{13} + \mathrm{i}\Omega_{\mathrm{c}}\rho_{03} - \mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{12} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\rho_{23},\\
\dot{\rho}_{23} &= -(\gamma_{20}+\gamma_{32} - \mathrm{i}\Delta_{\mathrm{s}})\rho_{23} + \mathrm{i}\Omega_{\mathrm{s}}^{*}(\rho_{33}-\rho_{22}) + \mathrm{i}\Omega_{\mathrm{p}}\rho_{03} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\rho_{13},\\
\rho_{00} &+ \rho_{11} + \rho_{22} + \rho_{33} = 1,
\end{aligned}
\tag{2}
$$

where the dots denote the derivative with respect to time t, and the terms including $\sqrt{\gamma_{20}\gamma_{10}}\cos\theta\rho_{ij}$ represent the quantum interference effect resulting from the cross coupling between two decay pathways $|1\rangle \leftrightarrow |0\rangle$ and $|2\rangle \leftrightarrow |0\rangle$, i.e., VIC effect. θ is the angle between the two induced dipole moments $\boldsymbol{\mu}_{10}$ and $\boldsymbol{\mu}_{20}$. If the above angles subject to $\theta = 0$ (a pair of parallel dipole moments), the system exhibits maximum VIC effect, while if the angles $\theta = \pi/2$ (a pair of orthogonal dipole moments), no VIC effect occurs. Alternatively, it is worthwhile to point out that only for small energy spacing between the two higher levels $|1\rangle$ and $|2\rangle$ the VIC effect is remarkable; as for large energy spacing, the oscillatory terms will average out to zero and such an effect will vanish. The existence of the VIC effect makes the system quite sensitive to phases of the probe and control fields, thus the Rabi frequencies have to be treated as complex-valued parameters. We use ϕ_{p} and ϕ_{c} to denote relevant phases of the probe and control fields, then we reexpress $\Omega_{\mathrm{p}} = \Omega_{\mathrm{p0}}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{p}}}$, $\Omega_{\mathrm{c}} = \Omega_{\mathrm{c0}}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{c}}}$ where Ω_{p0} and Ω_{c0} are real parameters, and other density matrix elements $\sigma_{ii} = \rho_{ii}$ $(i = 0-3)$, $\sigma_{32} = \rho_{32}$, $\sigma_{20} = \rho_{20}\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{p}}}$, $\sigma_{10} = \rho_{10}\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{c}}}$, $\sigma_{30} = \rho_{30}\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{p}}}$, $\sigma_{21} = \rho_{21}\mathrm{e}^{\mathrm{i}\Phi}$, $\sigma_{31} = \rho_{31}\mathrm{e}^{\mathrm{i}\Phi}$, where $\Phi = \phi_{\mathrm{c}} - \phi_{\mathrm{p}}$ is the relative phase between the probe and control fields. Again, we assume that Rabi frequency Ω_{s} of the pump laser field is real-valued parameter. Then from Eq.(2) we can obtain the corresponding density matrix equations about σ_{ij} as follows:

$$
\begin{aligned}
\dot{\sigma}_{11} &= \mathrm{i}\Omega_{\mathrm{c0}}(\sigma_{01}-\sigma_{10}) - 2\gamma_{10}\sigma_{11} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta(\mathrm{e}^{\mathrm{i}\Phi}\sigma_{12} + \mathrm{e}^{-\mathrm{i}\Phi}\sigma_{21}),\\
\dot{\sigma}_{22} &= \mathrm{i}\Omega_{\mathrm{p0}}(\sigma_{02}-\sigma_{20}) + \mathrm{i}\Omega_{\mathrm{s}}(\sigma_{32}-\sigma_{23}) + 2\gamma_{32}\sigma_{33} - 2\gamma_{20}\sigma_{22} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta(\mathrm{e}^{\mathrm{i}\Phi}\sigma_{12} + \mathrm{e}^{-\mathrm{i}\Phi}\sigma_{21}),\\
\dot{\sigma}_{33} &= \mathrm{i}\Omega_{\mathrm{s}}(\sigma_{23}-\sigma_{32}) - 2\gamma_{23}\sigma_{33},\\
\dot{\sigma}_{01} &= -(\gamma_{10} - \mathrm{i}\Delta_{\mathrm{c}})\sigma_{01} + \mathrm{i}\Omega_{\mathrm{c0}}(\sigma_{11}-\sigma_{00}) + \mathrm{i}\Omega_{\mathrm{p0}}\sigma_{21} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\mathrm{e}^{\mathrm{i}\Phi}\sigma_{02},\\
\dot{\sigma}_{02} &= -(\gamma_{20} - \mathrm{i}\Delta_{\mathrm{p}})\sigma_{02} + \mathrm{i}\Omega_{\mathrm{p0}}(\sigma_{22}-\sigma_{00}) + \mathrm{i}\Omega_{\mathrm{c0}}\sigma_{12} - \mathrm{i}\Omega_{\mathrm{s}}\sigma_{03} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\mathrm{e}^{-\mathrm{i}\Phi}\sigma_{01},\\
\dot{\sigma}_{03} &= -[\gamma_{32} - \mathrm{i}(\Delta_{\mathrm{p}}+\Delta_{\mathrm{s}})]\sigma_{03} + \mathrm{i}\Omega_{\mathrm{p0}}\sigma_{23} + \mathrm{i}\Omega_{\mathrm{c0}}\sigma_{13} - \mathrm{i}\Omega_{\mathrm{s}}\sigma_{02},\\
\dot{\sigma}_{12} &= -[\gamma_{10}+\gamma_{20} - \mathrm{i}(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}})]\sigma_{12} + \mathrm{i}\Omega_{\mathrm{c0}}\sigma_{02} - \mathrm{i}\Omega_{\mathrm{p0}}\sigma_{10} - \mathrm{i}\Omega_{\mathrm{s}}\sigma_{13} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\mathrm{e}^{-\mathrm{i}\Phi}(\sigma_{11}+\sigma_{22}),\\
\dot{\sigma}_{13} &= -[\gamma_{10}+\gamma_{32} - \mathrm{i}(\Delta_{\mathrm{p}}+\Delta_{\mathrm{s}}-\Delta_{\mathrm{c0}})]\sigma_{13} + \mathrm{i}\Omega_{\mathrm{c0}}\sigma_{03} - \mathrm{i}\Omega_{\mathrm{s}}\sigma_{12} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\mathrm{e}^{-\mathrm{i}\Phi}\sigma_{23},\\
\dot{\sigma}_{23} &= -(\gamma_{20}+\gamma_{32} - \mathrm{i}\Delta_{\mathrm{s}})\sigma_{23} + \mathrm{i}\Omega_{\mathrm{s}}(\sigma_{33}-\sigma_{22}) + \mathrm{i}\Omega_{\mathrm{p0}}\sigma_{03} - \sqrt{\gamma_{20}\gamma_{10}}\cos\theta\mathrm{e}^{\mathrm{i}\Phi}\sigma_{13},\\
\sigma_{00} &+ \sigma_{11} + \sigma_{22} + \sigma_{33} = 1,
\end{aligned}
\tag{3}
$$

Closure of this atomic system requires that $\sigma_{00}+\sigma_{11}+\sigma_{22}+\sigma_{33}=1$ and $\sigma_{ij}=\sigma_{ji}^{*}$. As it is well-known, the one-photon absorption (also named as probe absorption) for the pathway $|0\rangle \rightarrow |2\rangle$ is proportional to the imaginary part of σ_{02}. If $\mathrm{Im}(\sigma_{02})>0$, the probe field will be amplified. In contrast, the probe field will be absorbed. The two-photon absorption for the pathway $|0\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ is proportional to the population distribution in the excited state $|3\rangle$, i.e., σ_{33}.[6] In the following, we begin with investigating the response of one- and two-photon absorption-amplification by solving the time-dependent density matrix equations (3) numerically via a nice mathematica code. Note that, in this paper, parameters $\Omega_{\mathrm{p,c,s}}$, γ_{10}, γ_{20}, γ_{32} and $\Delta_{\mathrm{p,c,s}}$ are in units of γ.

3. Analysis for one- and two-photon absorption and amplification responses

First of all, we will analyse how the VIC effect modifies the transient one-photon absorption spectra and the temporal dynamics of populations. In Fig.2, we plot the time evolution of the one-photon absorption or gain coefficient $\mathrm{Im}(\sigma_{02})$ for not considering the VIC effect ($\theta=\pi/2$) and considering the VIC effect ($\theta=\pi/3$) under the initial conditions that $\sigma_{00}(0)=1$, and other $\sigma_{ij}(0)=0$ $(i,j=0-3)$. It is clearly shown from Fig.2 that, when no VIC effect exists in this system ($\theta=\pi/2$, solid curve), the probe field shows oscillatory behaviour versus time, i.e., the probe field exhibits periodic absorption and amplification, and at last reaches a steady-state value (the steady-state value equals to zero approximately). While when the VIC effect occurs ($\theta=\pi/3$), the behaviour of the one-photon transient absorption or gain is greatly affected by the relative phase Φ, and it exhibits different features with different values of the relative phase Φ. With $\Phi=0$ (dotted curve), transient gain appears only during a very short time in the first cycle (see Fig. 2(b)), then the transient gain disappears, the probe field exhibits periodic absorption, up to a steady-state value (the steady-state value is negative). On the contrary, with $\Phi=\pi$ (dashed curve), the transient absorption appears only during a very short time at the beginning in the first and second cycles, then the transient absorption disappears, the probe field exhibits periodic amplification, and eventually arrives at a considerable large steady-state value (the steady-state value is positive). Under such two cases, the probe field no longer exhibits period amplification and absorption as in the case of $\theta=\pi/2$. Furthermore, the amplitude of the oscillation is much larger than that of $\theta=\pi/2$, which means that we can get the large transient gain just by choosing the proper value of the relative phase Φ, such as $\Phi=\pi$. Note that from Fig. 3, one has $\sigma_{00}>\sigma_{22}$. Therefore, there is no population inversion for the probe transition $|0\rangle \rightarrow |2\rangle$, and a proper choice of the relative phase Φ between the probe and control fields is the necessary condition of probe amplification without population inversion.

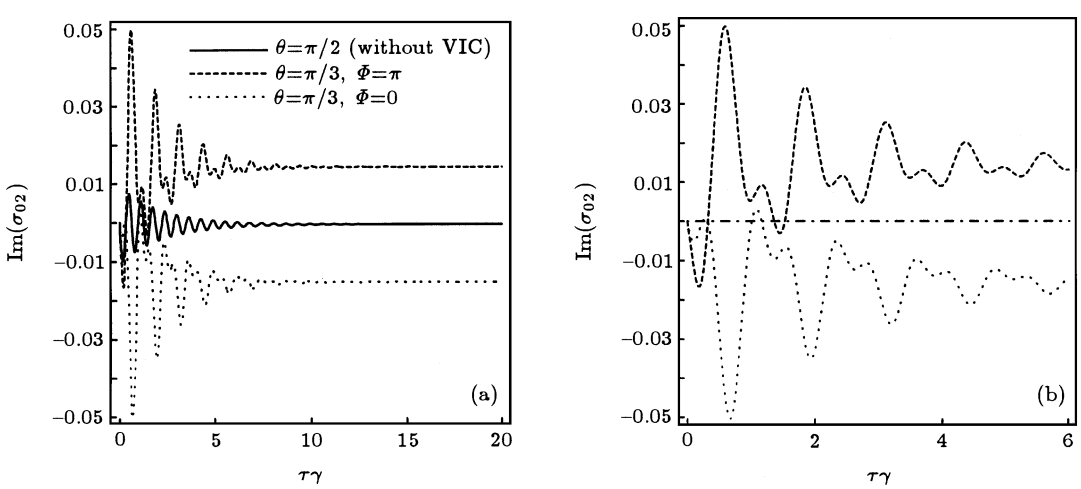


Fig.2. (a) Plots of one-photon absorption for the pathway $|0\rangle \rightarrow |2\rangle$ as a function of normalized time in a four-level atomic system with or without VIC effect under the initial conditions that $\sigma_{00}(0)=1$, and other $\sigma_{ij}(0)=0$ $(i,j=0-3)$. Other fitting parameters: $\Delta_{\mathrm{c}}=\Delta_{\mathrm{s}}=\Delta_{\mathrm{p}}=0$, $\gamma_{10}=\gamma_{20}=0.3\gamma$, $\gamma_{32}=0.1\gamma$, $\Omega_{\mathrm{c0}}=5\gamma$, $\Omega_{\mathrm{p0}}=0.1\gamma$, and $\Omega_{\mathrm{s}}=0.1\gamma$. (b) The same quantities in a short-time region for clarity.

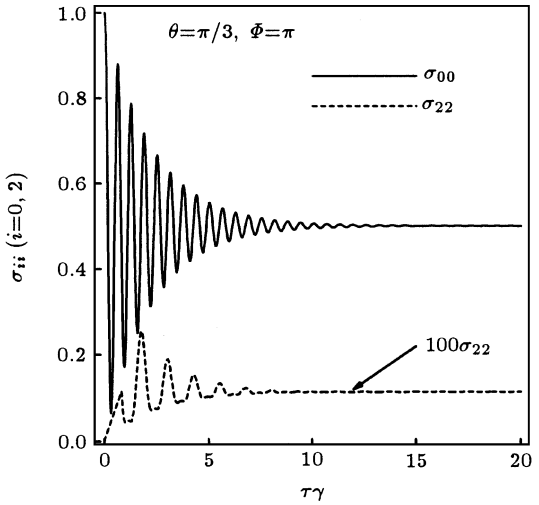


Fig.3. Plots of population distributions as a function of normalized time in a four-level atomic system with VIC effect under the initial conditions that $\sigma_{00}(0) = 1$, and other $\sigma_{ij}(0) = 0$ $(i, j = 0 - 3)$. Other fitting parameters: $\Delta_c = \Delta_s = \Delta_p = 0$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$. Dashed curve has been magnified by a factor 100.

In Fig.4, we plot the time evolution of the two-photon absorption coefficient σ_{33} with or without VIC effect under the same initial condition as above. It is shown from Fig.4 that, when no VIC effect exists ($\theta = \pi/2$, dash-dotted curve), the two-photon process $|0\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ no longer exhibits oscillation behaviour in the process of the entire time evolution, the two-photon absorption is identical to a zero steady-state value. While when the VIC effect occurs ($\theta = \pi/3$), the behaviour of the two-photon absorption is greatly changed, and it also exhibits different

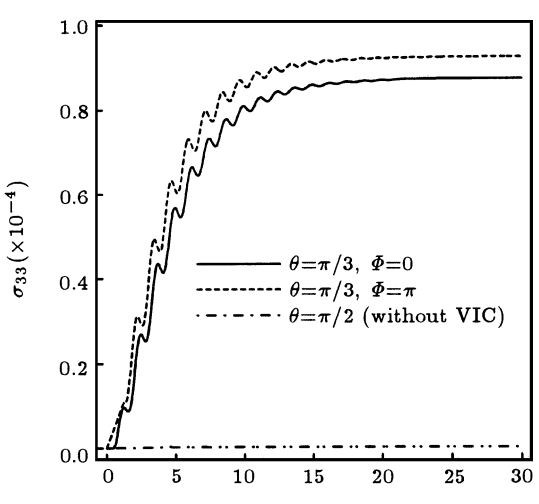


Fig.4. Plots of two-photon absorption for the pathway $|0\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ as a function of normalized time in a four-level atomic system with or without VIC effect under the initial conditions that $\sigma_{00}(0) = 1$, and other $\sigma_{ij}(0) = 0$ $(i, j = 0 - 3)$. Other fitting parameters: $\Delta_c = \Delta_s = \Delta_p = 0$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$.

different features with values of the relative phase Φ. With $\Phi = 0$ (solid curve), the two-photon absorption exhibits rapidly oscillation, and then reaches a positive steady-state value. With $\Phi = \pi$ (dashed curve), the behaviour of the two-photon absorption is similar to the situation of $\Phi = 0$, except that the amplitude of the two-photon absorption is larger than that with $\Phi = 0$.

In Fig.5(a), in the presence of VIC effect we plot one-photon absorption or gain coefficient $\mathrm{Im}(\sigma_{02})$ for the pathway $|0\rangle \rightarrow |2\rangle$ as a function of probe detuning Δ_p with different relative phase Φ in the steady-state limit. When the relative phase Φ is adjusted from 0 to $3\pi/2$, one-photon absorption or gain coefficient has a conversion from negative value to positive value, the probe amplification can be realized near the center line of the probe transition $\Delta_p = 0$. It is shown that the spectra of the weak-probe field can be changed from absorption to amplification by modulating the relative phase Φ, as a result we can efficiently control the behaviour of the absorption-gain spectra by an appropriate choice of the phase Φ. By comparing the curves in Fig.5(a), it is shown that, for different relative phase Φ, the exhibition of the probe amplification is different. Specifically, for the case that $\Phi = \pi/2$ (dashed curve), the amplification can be observed near the probe detuning $\Delta_p = -5\gamma$, which means that the probe field is far-off-resonant with atomic transition frequency. However, the case of $\Phi = 3\pi/2$ (dotted curve) is just opposite to that of $\Phi = \pi/2$ and they are symmetric. Obviously, we can obtain the dotted curve from the dashed curve simply by replacing Δ_p with $-\Delta_p$. For the case $\Phi = \pi$ (dash-dotted curve), the amplification can be observed for the probe detuning in the range $-4.7\gamma < \Delta_p < 4.7\gamma$. For the case $\Phi = 0$ (solid curve), the absorption appears for the probe detuning in the range $-5.25\gamma < \Delta_p < 5.25\gamma$. For the purpose of comparison, in the absence of VIC effect we plot one-photon gain-absorption coefficient $\mathrm{Im}(\sigma_{02})$ for the pathway $|0\rangle \rightarrow |2\rangle$ as a function of probe detuning Δ_p in the steady-state limit in Fig.5(b). We find that, due to the existence of the strong control field Ω_c, one-photon absorption spectra (i.e., probe absorption spectra) take on the two-peak structure, the double peaks are located at $\Delta_p = \pm\Omega_c$ corresponding to the energy separation of the dressed states $|+\rangle = 1/\sqrt{2}(|1\rangle + |0\rangle)$ and $|-\rangle = 1/\sqrt{2}(|1\rangle - |0\rangle)$ due to the dynamic Autler–Townes splitting. The probe field is transparent for a wide range of the probe detuning.

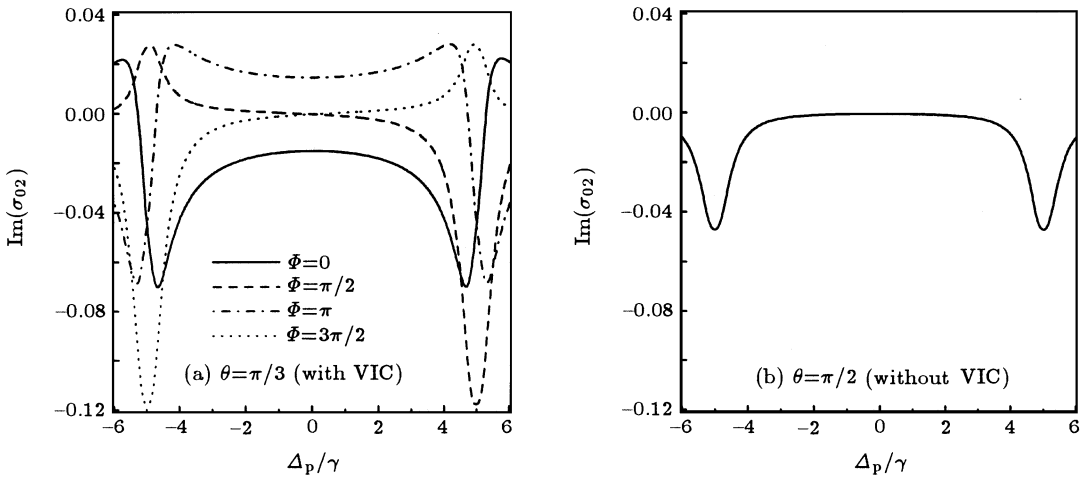


Fig.5. Plots of one-photon absorption for the pathway $|0\rangle \to |2\rangle$ as a function of normalized probe detuning in a four-level atomic system with (a) or without (b) VIC effect in the steady-state limit. Other fitting parameters: $\Delta_c = \Delta_s = 0$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$.

In Fig.6(a), in the presence of VIC effect we plot two-photon absorption coefficient σ_{33} for the pathway $|0\rangle \to |2\rangle \to |3\rangle$ as a function of probe detuning Δ_p with different relative phase Φ in the steady-state limit. As such, adjusting the relative phase Φ from 0 to $3\pi/2$, we find that two-photon absorption coefficient always has a vanishing value at center line of the probe transition $\Delta_p = 0$. The peak values of the two-photon absorption are always located at $\Delta_p = \pm\Omega_c$. By comparing the curves in Fig.6(a), it is shown that, for different relative phase, the peak values of the two-photon absorption are different. Specifically, for the case of $\Phi = \pi/2$ (dashed curve), one peak value at the probe detuning $\Delta_p = -5\gamma$ is maximal, the other at the probe detuning $\Delta_p = 5\gamma$ is minimal. However, the case of $\Phi = 3\pi/2$ (dotted curve) is just opposite to that of $\Phi = \pi/2$ and they are symmetric. It is straightforward to show from Fig.6(a) that we can obtain the dotted curve from the dashed curve just by replacing Δ_p with $-\Delta_p$. For the cases of both $\Phi = \pi$ (dash-dotted curve) and $\Phi = 0$ (solid curve), the peak values at the probe detuning $\Delta_p = \pm 5\gamma$ are symmetric respectively. For comparison, in the absence of VIC effect we plot two-photon absorption coefficient σ_{33} for the pathway $|0\rangle \to |2\rangle \to |3\rangle$ as a function of probe detuning Δ_p in the steady-state limit. We observe that two-photon absorption spectra also exhibit two-peak structure, the double peaks with equal amplitude are located at $\Delta_p = \pm\Omega_c$ corresponding to the energy separation of the dressed states $|+\rangle = 1/\sqrt{2}(|1\rangle + |0\rangle)$ and $|-\rangle = 1/\sqrt{2}(|1\rangle - |0\rangle)$. The reason for the above results is as follows. The suppression or enhancement

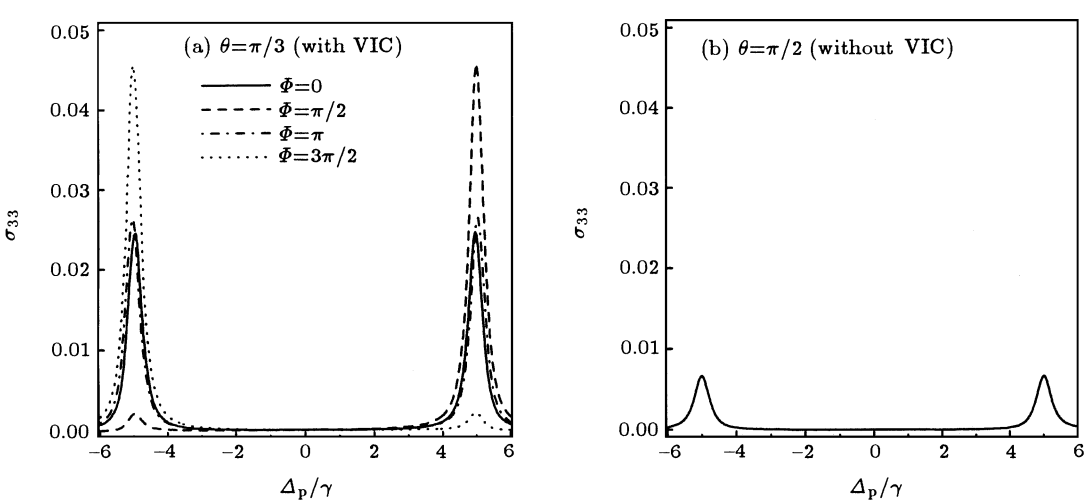


Fig.6. Plots of two-photon absorption for the pathway $|0\rangle \to |2\rangle \to |3\rangle$ as a function of normalized probe detuning in a four-level atomic system with (a) or without (b) VIC effect in the steady-state limit. Other fitting parameters: $\Delta_c = \Delta_s = 0$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$.

of the one- and two-photon absorption induced by the external coherent field Ω_c can be understood by the dressed states created by the strong control field: $|+\rangle$ and $|-\rangle$ with the corresponding eigenvalues $E_+ = \Omega_c$ and $E_- = -\Omega_c$. The transparency against one-photon absorption corresponding to the pathway $|0\rangle \to |2\rangle$ is induced by the destructive interference between two channels: $|0\rangle \to |+\rangle$ and $|0\rangle \to |-\rangle$, and suppression or enhancement of two-photon absorption for the pathway $|0\rangle \to |2\rangle \to |3\rangle$ is induced by the destructive or constructive interference between two channels: $|0\rangle \to |+\rangle \to |3\rangle$ and $|0\rangle \to |-\rangle \to |3\rangle$ together with the interference from spontaneous emission.

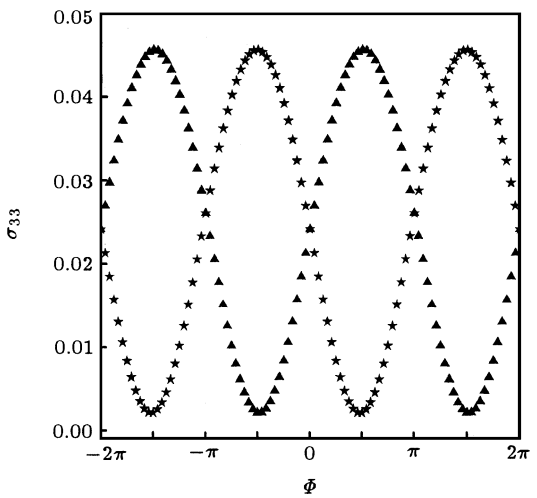


Fig.7. Plots of two-photon absorption for the pathway $|0\rangle \to |2\rangle \to |3\rangle$ as a function of relative phase Φ in a four-level atomic system with VIC effect in the steady-state limit. Triangles (▲): $\Delta_p = 5\gamma$; Stars ($\star$): $\Delta_p = -5\gamma$. Other fitting parameters: $\Delta_c = \Delta_s = 0$, $\theta = \pi/3$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$.

Now we turn our attention to the influence of the relative phase between the probe and control fields on absorption spectra. When $\Delta_c = \Delta_s = \Delta_p = 0$, one-photon absorption or gain versus the relative phase is plotted in Fig.8. From the figure, it can be seen clearly that the probe amplification between the transition $|0\rangle \to |2\rangle$ can be observed when $\cos\Phi < 0$, while when $\cos\Phi > 0$, the probe field is absorbed. Large amplification can be obtained by choosing the relative phase appropriately, e.g., $\Phi = 2k\pi + \pi$ (k is integer). These are consistent with the results obtained from Fig.5(a). Two-photon absorption at two peak-values $\Delta_p = \pm 5\gamma$ versus the relative phase is also plotted in Fig.7. For the case of $\Delta_p = 5\gamma$, the minimum of two-photon absorption can be achieved by choosing the relative phase appropriately, e.g., $\Phi = 2k\pi - \pi/2$ (k is integer). However, for the case of $\Delta_p = -5\gamma$, the minimum of two-photon absorption can be achieved by choosing the relative phase appropriately, e.g., $\Phi = 2k\pi + \pi/2$ (k is integer). Obviously, we can obtain the triangle (▲) curve corresponding to $\Delta_p = 5\gamma$ from the star ($\star$) curve corresponding to $\Delta_p = -5\gamma$ just by replacing σ_{33} with $-\sigma_{33}$. These are in good agreement with the results obtained from Fig.6(a). Alternatively, it is clear that one- and two- photon absorption or gain spectra at a fixed probe detuning is a periodical function of the relative phase Φ with the period of 2π.

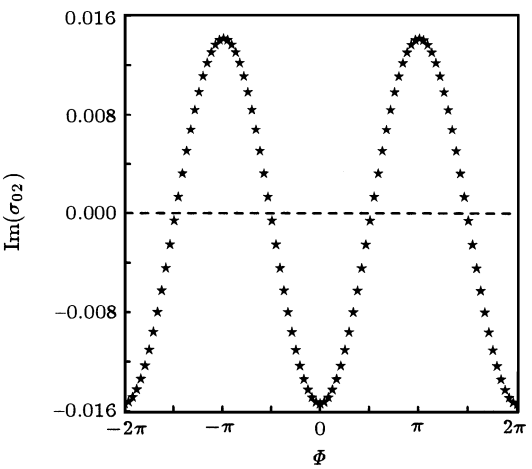


Fig.8. Plots of one-photon absorption for the pathway $|0\rangle \to |2\rangle$ as a function of relative phase Φ in a four-level atomic system with VIC effect in the steady-state limit. Other fitting parameters: $\Delta_c = \Delta_s = \Delta_p = 0$, $\theta = \pi/3$, $\gamma_{10} = \gamma_{20} = 0.3\gamma$, $\gamma_{32} = 0.1\gamma$, $\Omega_{c0} = 5\gamma$, $\Omega_{p0} = 0.1\gamma$, and $\Omega_s = 0.1\gamma$.

4. Discussion and summary

We now give a brief discussion on the experimental realization of our proposed scheme. There is a principled difficulty in the realization of atomic interference via spontaneous emission for atomic transitions in free space. In order to realize such an interference, Kocharovskaya *et al* proposed to make use of the technique of photonic band-gap materials and semiconductor quantum dots.[42] According to their idea, we conclude that the pseudophotonic band-gap structures have a special purpose, they allow the propagation of electromagnetic waves with a certain polarization and strongly forbid those with orthogonal polarizations. Therefore, the spontaneous decay with forbidden polarization is also forbidden and this allows us to remove the cancellation of different contributions resulting from different polarizations of spontaneous emission occurring in free space, which is just what we need. Moreover, in the recent years with great

progress in the techniques and reforming equipment, many efforts have been made on the fabrication of the semiconductor microstructure, which can allow for the creation of artificial quantum systems with desirable properties similar to atomic optical properties, based on which we can fabricate a quantum dot with sufficiently close upper levels to implement our four-level scheme.

To summarize, we have numerically analysed and discussed the response of one- and two-photon absorption in a four-level atomic medium with or without VIC effect. The results show that the one- and two-photon absorption and gain properties are quite sensitive to the VIC effect. In addition, it is further shown that the absorption-gain spectra can be dramatically varied by modulating the relative phase of the applied fields. We also give a brief discussion on the experimental realization of the proposed scheme.

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