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# Supercontinuum spectra generation in the single-mode optical fibre with concave dispersion profile*

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In this paper, a new method is proposed to generate broad supercontinuum (SC) spectra in the single-mode optical fibre with concave dispersion profile. We numerically simulate pulse evolutions and discuss physics mechanism in detail for SC spectrum generation in the optical fibre with concave dispersion profile. Furthermore, general criteria are presented for specifying the shape of SC spectrum by introducing normalized parameters, which are related to the fibres and the initial pump pulses. The results show that the flat and broad SC spectra are indeed generated in our proposed optical fibre.

**Keywords:** supercontinuum spectrum, optical pulse compression, dispersion-flattened / dispersion-decreasing fibre, normalized parameters

**PACC:** 4230Q, 4225B, 4265

## 1. Introduction

Generation of supercontinuum (SC) pulse, which produces a wide optical spectrum in an optical medium excited by an intense optical pulse, has been studied in various optical media. Several applications of SC pulses have been realized not only in spectroscopic test, biologic medicine and optical measurement, but also in dense wavelength-division multiplexing (DWDM) transmission system.[1]

To date, several approaches have been reported for generating SC source in the fibres with various dispersion profiles. Lou *et al*[2] used a dispersion-decreasing fibre (DDF) to broaden the SC spectrum. Takushima *et al* showed that a dispersion-flattened fibre (DFF) with constant normal chromatic dispersion along the optical fibre can also generate SC pulses[3] and demonstrated experimentally by Futami *et al*[4] Okuno *et al* confirmed that a dispersion-flattened /dispersion-decreasing fibre (DFDF) can produce a much broader spectrum.[5−7] Mori *et al* obtained a flat and broad SC spectrum in a DFDF with a convex dispersion profile, which were in good agreement with experimental results.[8,9] In this paper, we propose a new approach to generate SC spectrum in an optical fibre with a concave dispersion profile. The numerical results show that the optical fibre with concave dispersion profile is much beneficial to generate broader SC spectra than the DFDF with convex dispersion profile.

## 2. Theoretical models

### 2.1. Total field equation in frequency domain

It is well known that the propagation of an ultra-short pulse in optical fibres obeys nonlinear Schrödinger (NLS) equation in the time domain,[10] and it can be solved by split-step Fourier-Transform method. In order to obtain the SC spectra accurately, we apply another method to solving NLS equation in the frequency domain since the SC bandwidth is much broad. This method is rather appropriate for the case of SC bandwidth over several hundred nanometres.

We defined the Fourier transform of the electric

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field envelope $A(z,\tau)$ as[10]

$$\tilde{A}(z,\Omega) = FT[A(z,\tau)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(\mathrm{i}\Omega\tau)A(z,\tau)\mathrm{d}\tau, \tag{1}$$

where $FT[A(z,\tau)]$ denotes Fourier transform. $\Omega = (\omega_0 - \omega)T_0$ is the normalized frequency. $\tau = t - z/V_g$, with $V_g$ being the group velocity corresponding to $\omega_0$. The evolution of optical pulse described by the total field equation of frequency domain in the optical fibre is expressed as follows:

$$\begin{aligned}\frac{\partial \tilde{A}_{\mathrm{total}}(z,\Omega)}{\partial z} =& -\mathrm{i}\Delta(\Omega)\tilde{A}_{\mathrm{total}}(z,\Omega) - \Gamma(\Omega)\tilde{A}_{\mathrm{total}}(z,\Omega) \\ & - \mathrm{i}\left(1+\frac{\Omega}{\omega_0}\right) Q_{\mathrm{ker}\,r} FT_{\Omega}\left[A_{\mathrm{total}}(z,\tau)\left|A_{\mathrm{total}}(z,\tau)\right|^2\right] \\ & - \mathrm{i}\left(1+\frac{\Omega}{\omega_0}\right) Q_{\mathrm{Raman}} FT_{\Omega}\left(A_{\mathrm{total}}(z,\tau) \times FT_{\tau}^{-1}\left\{S_{\mathrm{Raman}} FT_{\Omega}\left[\left|A_{\mathrm{total}}(z,\tau)\right|^2\right]\right\}\right). \end{aligned} \tag{2}$$

On the right hand side of Eq.(2), the first, second, third and fourth items represent dispersion effect, fibre loss, Kerr effect and stimulated Raman scattering (SRS), respectively. Other parameters are shown as follows:

$$\Delta(\Omega) = \sum_{k=2}^{\infty} \beta_0^{(k)} \frac{\Omega^k}{k!}, \quad \beta_0^{(k)} = \frac{\mathrm{d}^k \beta}{\mathrm{d}\omega^k}(\omega_0),$$

$$Q_{\mathrm{ker}\,r} = \frac{\omega_0}{c}\frac{3\chi^{(3)}_{xxxx}}{n_0}\frac{c\mu_0}{4n_0}\frac{P_{\mathrm{scale}}}{A_{\mathrm{eff}}},$$

$$Q_{\mathrm{Raman}} = \frac{\omega_0}{c} G_{\mathrm{Raman}} \frac{P_{\mathrm{scale}}}{A_{\mathrm{eff}}}. \tag{3}$$

$FT_{\tau}^{-1}$ represents anti-Fourier transformation. $\Delta(\Omega)$, $\chi^{(3)}_{xxxx}$ and $\mathrm{G_{Raman}}$ are fibre dispersions, Kerr polarization tensor and Raman response function, respectively. The effective area is $A_{\mathrm{eff}} = \pi\sigma^2$, where $\sigma$ represents optical spot dimension of the optical beam.

### 2.2. Dispersion characteristic of the optical fibre

Since the dispersion profile of an optical fibre plays a key role in SC generation, we use DFDF with a new concave dispersion profile (CADF) as transmission medium.[11] Assuming that the dispersion value $D(\lambda,z)$ of CADF can be expressed as

$$D(\lambda,z) = D_0\left(1-\frac{z}{L_0}\right) + K_2(\lambda-\lambda_0)^2 + K_4(\lambda-\lambda_0)^4, \tag{4}$$

where $D_0$ is the initial chromatic dispersion at a fibre input; effective length $L_0$ is defined as the propagation distance, before which the dispersion coefficients are positive for C-band wavelengths. $\lambda_0$ is the central wavelength of the pump pulse. $K_2 = \partial^2/\partial\lambda^2 D(\lambda_0, L_0)$ and $K_4 = \partial^4/\partial\lambda^4 D(\lambda_0, L_0)$ represent the second and the fourth order dispersion coefficients of the optical fibre, respectively. $K_2$ has a negative value in CADF, being the same as that in convex situation. Figure 1 illustrates the chromatic dispersion profile of CADF, with parameters $\lambda_0 = 1550$nm, $D_0 = 6$ps/(nm/km), $K_2 = -0.0001$ps/(km/nm$^3$), $K_4 = 8.0\times10^9$(ps/km/nm$^5$) and $L_0 = 0.6$km, $z = 1.3L_0$. The characteristics of the dispersion curve are described as follows:

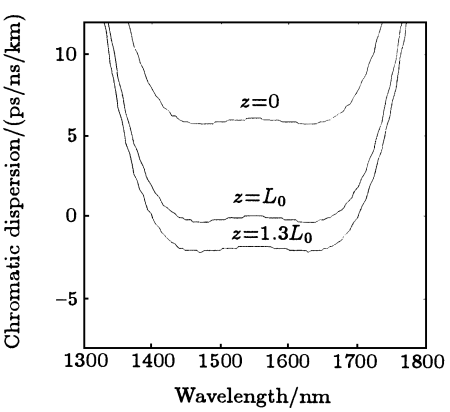



**Fig.1.** The profile of chromatic dispersion.

1. Chromatic dispersion decreases from a positive value (anomalous group velocity dispersion( GVD)) to a negative value (normal GVD) with propagation distance. It is positive over the whole C-band wavelengths when $z < L_0$, and partly negative when $z > L_0$ with two zero-dispersion wavelengths (ZDWs).

2. Dispersion curve is symmetrical to the central wavelength $\lambda_0$=1550nm. Blue and red shifted components will synchronously enter into the optical fibre from anomalous GVD region to normal GVD re-

gion when the optical pulse propagates along the fibre. That is the reason why the optical spectra are symmetrical in this situation.

3. The concaveness of the chromatic dispersion varies as a function of wavelength,[11] showing a new dispersion profile. It almost keeps the same value and has a much broad span in the vicinity of the central wavelength $\lambda_0$. The corresponding chromatic dispersion coefficient changes dramatically when the wavelength deviates from the central wavelength. Furthermore, the dispersion span continues to enlarge and gradually become flat as the propagation distance increases, which is in favour of generating a broad and flat SC spectrum.

## 3. Simulation results and analysis

For $\lambda_0 = \lambda_{\text{peak}}$, the slope of the fibre dispersion equals zero at the point of central wavelength, thus the higher-order dispersions must be considered.[12] It is found that the higher-order dispersions up to the 7th order are very important for optical pulse spectra broadening in CADF. The higher-order dispersions calculated using equation $\beta_k = \left.\frac{\mathrm{d}^k\beta}{\mathrm{d}\omega^k}\right|_{\omega=\omega_0}$ can be expressed as:

$$\beta_2(\lambda_0, z) = -\frac{\lambda_0^2}{2\pi c} D_0 \left(1 - \frac{z}{L_0}\right);$$

$$\beta_3(\lambda_0, z) = \frac{\lambda_0^3}{(2\pi c)^2} 2D_0 \left(1 - \frac{z}{L_0}\right);$$

$$\beta_4(\lambda_0, z) = -\frac{\lambda_0^4}{(2\pi c)^3} \left[6D_0 \left(1 - \frac{z}{L_0}\right) + 2k_2\lambda_0^2\right];$$

$$\beta_5(\lambda_0, z) = \frac{\lambda_0^5}{(2\pi c)^4} \left(24D_0 \left(1 - \frac{z}{L_0}\right) + 24k_2\lambda_0^2\right);$$

$$\beta_6(\lambda_0, z) = -\frac{\lambda_0^6}{(2\pi c)^5} \left(120D_0 \left(1 - \frac{z}{L_0}\right) + 240k_2\lambda_0^2 + 24K_4\lambda_0^4\right);$$

$$\beta_7(\lambda_0, z) = \frac{\lambda_0^7}{(2\pi c)^6} (720D_0 \left(1 - \frac{z}{L_0}\right) + 2400k_2\lambda_0^2 + 720K_4\lambda_0^4). \quad (5)$$

Provided that the input pulse can be expressed as

$$A(0, T) = \sqrt{P_0}\,\mathrm{sech}\left(\frac{T}{T_0}\right), \quad (6)$$

where $P_0$ is the peak power, $T_0$ is related to the full width at half maximum (FWHM) of the input pulse, we have $T_{\text{FWHM}} \approx 1.763T_0$. The optical pulse is a Fourier-transform-limited pulse with $T_0$=4ps. Other parameters selected in our numerical calculations are fibre loss $\Gamma(\Omega)$=0.2dB/km, nonlinear coefficient $\gamma = 2.43\mathrm{W}^{-1}\mathrm{km}^{-1}$, and effective peak power $\gamma p_0 = 5.86\mathrm{km}^{-1}$.

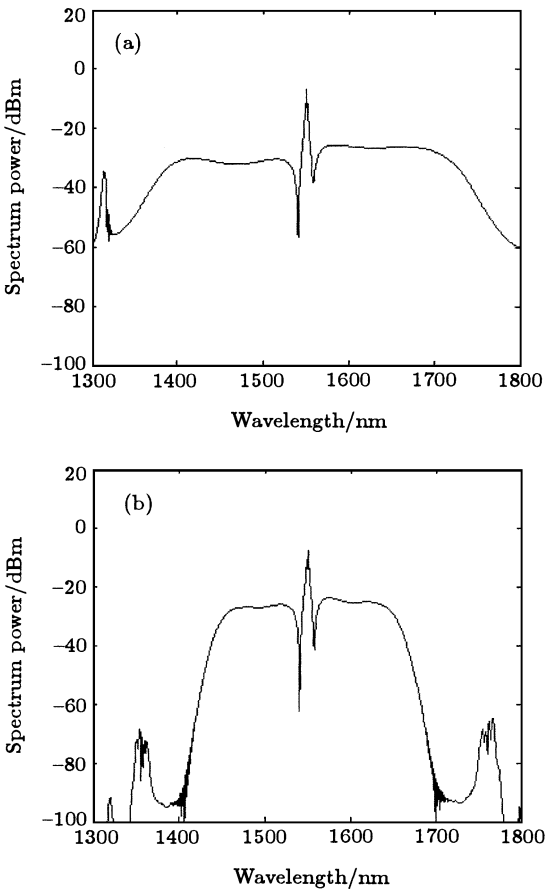



**Fig.2.** Generation of SC spectra in different optical fibres. Curve (a): in a fibre with a concave dispersion profile. Curve (b): in a fibre with a convex dispersion profile.

Figure 2 (a) displays that the SC spectrum can be generated in the fibre with concave dispersion profile. It is fairly flat over a few hundreds of nanometre. The power level on the longer-wavelength side is higher than that on the shorter-wavelength side. This results from stimulated Raman scattering effect, which is in agreement with that in Ref.[9]. The existence of a power peak at $\lambda_0$=1550nm is a character in the SC generation of all the dispersion-decreasing optical fibres. The bandwidth at 27dB below the spectral peak (–27dB bandwidth) is 352nm. In order to compare SC generation in the fibre with the convex dispersion profile, Fig.2(b) gives the optical spectrum obtained for the same parameters except $K_2$=0. The bandwidth is calculated to be about 214 nm (–27dB bandwidth). It

is clear that the optical fibre with a concave dispersion profile can produce a much broader SC spectrum than that with a convex dispersion profile.

The physical mechanism of SC spectrum generation in CADF can be well explained as follows. SC spectrum generation consists of two stages: adiabatic soliton compression in the anomalous GVD region and SC spectrum formation in the normal GVD region.[13] The two stages are realized under different input conditions. Fig.3 shows the evolutions of pulse shapes along the optical fibre. In the anomalous GVD region, the pump pulse is compressed and the peak power is increased. Since CADF has no ZDW over the whole wavelength in anomalous GVD region, pump pulse will be heavily compressed. This results in stronger nonlinear effects (especially SPM effects). At the same time, high pump power will generate higher-order solitons. These solitons will be broadened and compressed periodically[14] and form many lower-order solitons. The compression factor reaches its maximal value at $z = L_0$. The compressed optical pulse obtains the highest peak power but with many small oscillatory peaks as shown in Fig.3. When propagation distance $z$ varies from $L_0$ to $1.3L_0$, blue-shifted and red-shifted components will transmit from anomalous to the normal dispersion region simultaneously. So, the optical spectrum is broadened symmetrically. Once $z$ goes beyond $L_0$, the soliton waves transform into dispersive waves rapidly due to the higher-order dispersion effects.[15] Because $D(\lambda, z)$ has two ZDWs in this region, the spectrum will be further broadened. Since the pump power is very high, multiple Stokes and anti-Stokes spectral components will be generated by stimulated Raman scattering effect. The spectrum near the two sides of the pump pulse is magnified and further broadened. However, the spectrum at central wavelength is too late to be broadened. A remnant spectrum always appears. Dispersive waves also exist on the two sides of the SC spectrum, whose intensity is much lower than that of the pump pulse.

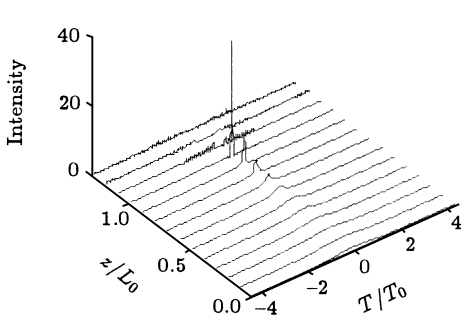



**Fig.3.** Evolution of pulse shapes.

## 4. Different normalized parameters for SC generation

Following the method used in Ref.[9], we obtain four normalized parameters to describe the necessary conditions for generating SC pulse with flat broad spectrum, which can be expressed as

$$\Delta_0 = \frac{\lambda_0^2}{2\pi c}\frac{D_0}{\gamma P_0 T_0^2};$$

$$\Delta_2 = \left(\frac{\lambda_0^2}{2\pi c}\right)^3 \frac{2K_2}{\gamma P_0 T_0^4};$$

$$\Delta_4 = \left(\frac{\lambda_0^2}{2\pi c}\right)^5 \frac{24K_4}{\gamma P_0 T_0^6};$$

$$\xi_0 = \gamma P_0 L_0. \tag{7}$$

In Eq.(7), the normalized parameters $\Delta_0$, $\Delta_2$, $\Delta_4$ and $\xi_0$ represent the fibre dispersion parameters. Functions between SC spectrum and $\Delta_0$, $\Delta_2$, $\Delta_4$, and $\xi_0$ can be reflected by parameters $D_0$, $K_2$, $K_4$ and $L_0$. Figure 4 shows the curves of SC bandwidth versus fibre dispersion parameters. Fibre loss and higher-nonlinear effects are ignored in the calculations.

Figure 4(a) shows the relationship between input dispersion $D_0$ and SC spectra when $0.14 \leq \Delta_0 \leq 0.38$, $\Delta_2$=$-2.14\times10^{-6}$, $\Delta_4$=$3.50\times10^{-10}$, and $\xi_0$=3.50. $D_0$ is the dispersion value of input optical fibre. As mentioned above, under the same conditions, when the optical pulse is heavily compressed in the anomalous GVD region, a broader SC bandwidth will be obtained after the optical pulse enters the normal GVD region. This can also be explained by the initial value when the pump pulse enters the anomalous GVD region. In order that the optical pulse can be well compressed in the anomalous GVD region, the dispersion and nonlinear effects must be balanced. SC bandwidth is found to change drastically when $D_0$ exceeds a certain threshold value. However, the dispersion gradually balances with the nonlinear effects when $D_0$ increases, resulting in further optical pulse compression. We can see that SC bandwidth is broadened after the optical pulse passes through the normal dispersion region. In Fig.4(a), SC bandwidth increases sharply with increasing $D_0$ when it exceeds its threshold 5ps/(nm/km). However, as $D_0$ reaches a certain high value, the optical pulse compression maintains and the optical pulse peak power keeps the same value.

Thus, SC spectrum does not change. The plateau of SC spectrum in Fig.4 (a) directly reflects the above results. Furthermore, when$D_0$ further increases, at about 9ps/ns/km, nonlinear effects cannot counteract the dispersion effects. Optical pulse broadens while SC bandwidth decreases.

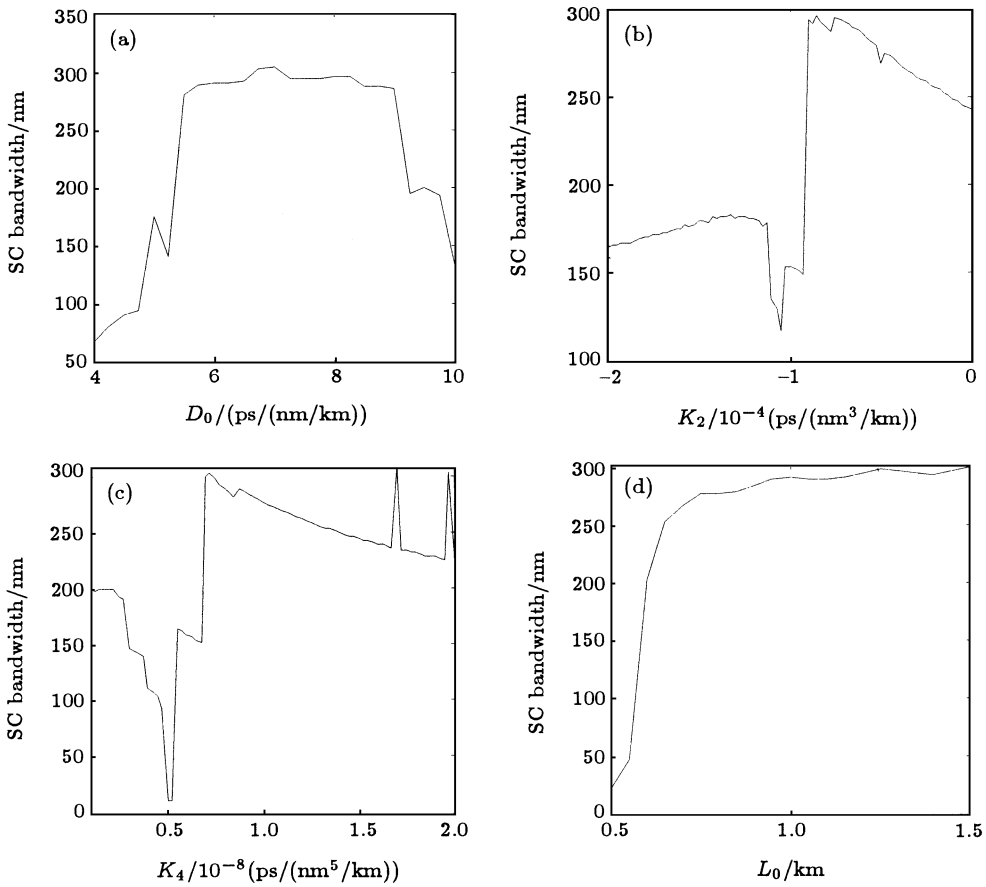



**Fig.4.** SC spectral width plotted as a function of normalized parameters. (a) SC bandwidth versus the initial dispersion $D_0$; (b) SC bandwidth as a function of the 2nd dispersion $K_2$; (c) SC bandwidth as a function of the 4nd dispersion $K_4$; (d) SC bandwidth versus the effective length $L_0$.

Figure 4(b) shows the SC spectra versus the 2nd dispersion $K_2$ when $-5.36 \times 10^{-6} \leq \Delta_2 \leq 0$, $\Delta_0$=0.222, $\Delta_4$=3.50×10$^{-10}$, and $\xi_0$=3.50. It is obvious that the curve of SC bandwidth versus $K_2$ can be divided into three parts. The SC bandwidth changes remarkably only when the value of $K_2$ is around $-1.0\times10^{-4}$ps/(nm$^3$/km). The SC bandwidth changes little in the other two parts. Usually, $K_2$ is a negative value, when $K_2$ is close to zero, the dispersion curve becomes concave. We have already proved that CADF will generate a broader SC spectrum than that in convex case. So, broader SC spectrum is obtained when $K_2$ is near zero. Nevertheless, when $K_2$ is equal to –1ps/(nm$^3$/km), the dispersion profile changes from convex to concave shape, from which we gain a dramatically different SC spectrum. It is further confirmed that CADF is more beneficial to SC pulse broadening than the fibre with convex dispersion profile.

Figure 4(c) illustrates the SC spectrum as a function of the 4th dispersion $K_4$ when $0 \leq \Delta_4 \leq 8.74 \times 10^{-10}$, $\Delta_0$=0.222, $\Delta_2$=–2.14×10$^{-6}$, and $\xi_0$=3.50. Figure 4(c) has the same variation trend for SC bandwidth as that of Fig.4(b). As in Eq.(4), $K_4$ stands for the characteristics at the bottom of concave dispersion curve, such as the span and flatness. The optical pulse can form a broad SC spectrum only when the dispersion span of CADF becomes flat and large at the bottom. However, when $K_4 = 0$, the dispersion curve becomes convex. Under this condition, SC spectrum is obtained with a narrow bandwidth. Therefore, the SC spectrum has a large bandwidth and ba-

sically keeps the value unchanged after $K_4$ exceeds 0.7ps/(nm$^5$/km) in this region. We also find some peak fluctuation, which is supposed to be formed by spectrum noise of the optical pulse.

Figure 4(d) illustrates the SC bandwidth versus the effective length $L_0$ when $2.92 \leq \xi_0 \leq 8.76$, $\Delta_0$=0.222, $\Delta_2$=$-2.14\times10^{-6}$, and $\Delta_4$=$3.50\times10^{-10}$. $L_0$ stands for the fibre length for the optical pulse propagating in the anomalous GVD region. The optical pulse can be well compressed when $L_0$ is selected appropriately. Our numerical results show that there exists a threshold value in SC spectrum generation. When $L_0$ is selected to be larger than the threshold, the shape of the SC spectrum is unchangeable. The bandwidths of the SC spectra are almost the same. Thus, when the optical pulse is well compressed in the optical fibre, the further transmission distance can not affect SC broadening.

# 5. Conclusion

In this paper, we propose a new DFDF fibre with concave dispersion profile (CADF) to generate SC spectrum. By numerically calculating nonlinear Schrödinger equation in frequency domain, we obtain a flat and broad SC spectrum in our proposed DFDF fibre. General criteria are presented for specifying the shape of the SC spectrum by introducing normalized parameters, which are related to the optical fibre and the initial pump pulse. Our calculation results show that normalized parameters are important for the generation of the SC spectra. Different normalized parameters can generate different SC spectra when the optical fibres have different fibre parameters. Effects of normalized parameters on SC spectra are also calculated in this paper. It is proven that the optical fibre with a concave dispersion profile is more beneficial to generate a broader SC spectrum.

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