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An implementation of cellular automaton model for single-line train working diagram

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According to the railway transportation system's characteristics, a new cellular automaton model for the single-line railway system is presented in this paper. Based on this model, several simulations were done to imitate the train operation under three working diagrams. From a different angle the results show how the organization of train operation impacts on the railway carrying capacity. By using the non-parallel train working diagram the influence of fast-train on slow-train is found to be the strongest. Many slow-trains have to wait in-between neighbouring stations to let the fast-train(s) pass through first. So the slow-train will advance like a wave propagating from the departure station to the arrival station. This also resembles the situation of a highway jammed traffic flow. Furthermore, the nonuniformity of travel times between the sections also greatly limits the railway carrying capacity. After converting the nonuniform sections into the sections with uniform travel times while the total travel time is kept unchanged, all three carrying capacities are improved greatly as shown by simulation. It also shows that the cellular automaton model is an effective and feasible way to investigate the railway transportation system.

Keywords: train working diagram, cellular automaton model, carrying capacity, single-line railway
PACC: 0550

1. Introduction

Railway is one of the major transportation means in China. The total length of railway operating lines has reached 7.3 million km by the end of 2003. However, it takes up only 6.13 percent of the whole passenger traffic volume and 14.17 percent of the total freight volume. Besides the unbalance of the resource distribution, the restrained carrying capacity of the railway is one of the key internal reasons to cause those low proportions. Usually, the carrying capacity of a railway section is firstly calculated from the carrying capacity of a parallel train working diagram. Then based on the calculation, the carrying capacity of a non-parallel train working diagram is achieved at last. Nowadays, manual adjustment is still a common method to reckon the carrying capacity of a single-line parallel train working diagram, where the calculation starts from the longest section and then was adjusted step by step during each train crossing scheme at station.[1] Yan designed an integer programming model and the corresponding algorithm to obtain the carrying capacity of a single-line parallel working diagram with trains in pairs but dispatched not in pursuit.[2] In recent years, more and more researches have been focusing on applying the cellular automaton (CA) model and theory to investigate the urban traffic flow.[3−7] Among these researchers the German scientists, Nagel and Schrekenberg, innovatively applied the CA theory to the traffic field and investigated in details one-dimensional traffic flow on the highway.[3] Their model is called NS model afterwards. Because the railway has its special characteristics, until now there have been few papers to report the application of CA theory to the railway. Wang *et al*[8] used the "class" concept of object-oriented method to describe the railway CA model, where only a general outline was presented without the computer simulation. In the present paper, we put forward a new CA model of the single-line train working diagram and some experiments with Pentium 4.0 were implemented to compare the single-line capacities un-

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der the three different working diagrams. Here, we select the single-line as the modelling object because 66.30 percent of the total railway line operated is in single line.

This paper is organized as follows: the NS model is reviewed in section 2 in order to compare it with the new CA model. Section 3 analyses the similarities and differences between the railway and the highway from a view of building CA model. According to the comparison, a new CA model of the single-line train working diagram is then proposed. In section 4, several simulations are implemented to compare the results of the three working diagrams of the single-line railway. A summary is given in section 5.

2. NS model

In 1992, Nagel and Schrekenberg introduced a very simple CA model (called the NS model) which provides a microscopic description of the vehicular motion using a set of update rules. Although it is one of the simplest traffic flow models, it is capable of reproducing important properties of a real traffic flow. In the NS model, the road is subdivided into uniform cells, each with a length of 7.5m, with 133veh/km being the density of jammed cars. Each cell i is either empty or occupied by only one vehicle with a discrete speed $V_i(t) \in \{0,\ldots, V_{\max}\}$. But in reality, the vehicle speed spreads widely. In addition, the unit of speed is cell/time-step in the model. The constant $V_{\max}$ is the permitted maximum speed.

The motion of the vehicles is described by the following rules (parallel dynamics):

Deceleration rule: (to avoid accidents)

$$V_i(t+1) = \min(V_i(t), gap_i(t));$$

Acceleration rule:

$$V_i(t+1) = \min(V_i(t)+1, V_{\max});$$

Randomization rule: (with a certain probability P)

$$V_i(t+1) = \max(V_i(t), 0);$$

Movement:

$$X_i(t+1) = X_i(t) + V_i(t+1),$$

where $X_i(t)$ denotes the cellular position of vehicle i at time t. $V_i(t)$ is the speed of vehicle i at time t. $gap_i(t)$ is the number of empty cells in front of the vehicle i at time t, with $gap_i(t) = X_{i-1}(t) - X_i(t) - 1$. A time step corresponds to $\Delta t \approx 1s$, the typical reaction time of a driver. At a maximum speed $V_{\max}$=5 cell/time-step the car can speed up to 135km/h in reality. Because drivers would vary their driving behaviour without any obvious reason, it is necessary to introduce the so-called breaking noise P, which is a deceleration probability to imitate them.

3. The CA model of single-line train working diagram

In the viewpoint of building the CA model, several similarities and differences between the railway and the highway must be described as follows:

3.1. Similarities and differences between the railway and the highway

3.1.1. Similarities

1) After the highway is subdivided into cells, each cell is occupied by just one vehicle at the same time. Similarly, every railway section also permits one train to run on it at a certain period.

2) One car must advance in one direction till the end of the road, that is, it cannot move back or turn around during its journey. So does a train.

3.1.2. Differences

1) All cells of a road are identical. But in this paper, a railway is celled into two kinds: station node cell and section line cell. Even for the line cells, the length of each cell may be different because a train needs different times to travel on sections in reality.

2) Single lane of a road prohibits cars from bidirectional running, whereas a railway single line allows a train to go from the leftmost station node and at the same time another train to come from the rightmost station node.

3) A car driver may accelerate or decelerate at will, but the velocity of a train cannot change at random.

4) A car arrives at an origin cell without any rule. A train departs at time scheduled by the train working diagram.

5) A car movement is affected not only by the number of idle cells in front of it, but also by its predecessor. The motion of a train is determined only by the empty section cell before it.

3.2. CA modelling

In the paper, the cells of a single-line railway consist of node cells and line cells. The capacity of one

node cell is unlimited, i.e. it can contain N trains simultaneously. Node cell is also regarded as one physical cell because a train needs one time step to pass it, and the travel time and the waiting time continue to add when it stops at the node cell. A line cell is further subdivided into numerous sub-cells according to its length. The length of each sub-cell takes one time step. A train speed has only two possible values: 0 or 1. Here, 0 represents a train's stopping at station and 1 a train's moving. Hence, the CA model of a single-line train working diagram is given as follows:

1) The speed of each train i at time t+1:

$$V_i(t+1) = \begin{cases} 0, & \text{if adjoining line cell in front} \\ & \text{of it is occupied;} \\ 1, & \text{if the line cell is empty.} \end{cases}$$

2) The position of train i at time t+1:

$$X_i(t+1) = X_i(t) + V_i(t+1).$$

4. Computer simulation and output analysis

4.1. Celled single line

The single line (Fig.1) to be celled includes 7 stations and 6 sections. So it is simply divided into 7 node cells and 6 line cells. In Fig.1, each small circle is a node cell with a labelled character above it, as M, a, b, c, d, e and N. The black segment and the white segment represent the sub-cells with the same size. According to different travel times of 6 sections, line cells labelled as Ma, ab, bc, cd, de, and eN contain 4, 4, 2, 1, 6 and 1 sub-cells respectively. The down direction is from the node cell M to N. Station M is the origin node for a down-train but the destination node for an up-train, while station N is the origin node for an up-train but the destination node for a down-train. Other stations are just passing nodes without any train appearing or vanishing. We can see that the length of these line cells is very nonuniform.

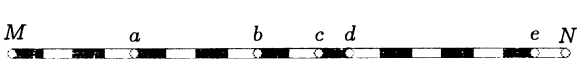


Fig.1. Celled single-line railway.

4.2. Three train working diagrams

There are three scheduled modes for trains to run on a railway in Fig.1. The three modes correspond to three train working diagrams. The first is a single-line working diagram scheduled with trains in pairs. Here, all trains are slow-trains with equal grades. Figure 2 shows its spatio-temporal plot. The second mode is a single-line non-parallel train working diagram, where the fast-trains run in the down direction and the slow-trains in the opposite direction. Only after one fast-train arrives at the end node N, could the next fast-train depart from the origin station M. For slow-trains, if the line cell eN is empty, a slow-train may go out. Figure 3 illustrates its spatio-temporal plot. The third one is a single-line train working diagram of trains dispatched in pursuit, where the fast-trains run in the down direction and no train departs from the node cell N. Only if the first section Ma is empty, could the next fast-train start from station M. Figure 4 displays its spatio-temporal plot.

4.3. Time steps for one simulation

One simulation lasts 500 time steps, thus the results are the sum of data obtained over 500 time steps. Once the simulation period is over, the trains still in the way do not continue their motion. They are counted in the number of departed but unfinished trains. The unit of travel time and waiting time is time-step too.

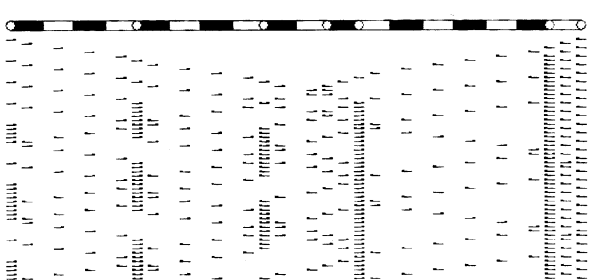


Fig.2. Spatio-temporal plot of single-line working diagram scheduled with trains in pairs.

4.4. Spatio-temporal plots

In Figs.2–4, the symbol " ▬ " indicates the fast-trains in the down direction (fast down-train), and " ▬ " the slow-trains in the up direction (slow up-trains) and " ▬ " the meeting of trains at the station. These figures show that the number of waiting trains is higher and the length of waiting times is longer at the two neighbouring nodes of a long line cell. This forms a bottleneck to heavily limit the carrying capacity of the whole railway. Train density before station d is double that after station d (Fig.4). Through the

computer simulation, it is also shown that after the frequency of departing trains reduces to one half, the waiting trains immediately disappear at station d. In

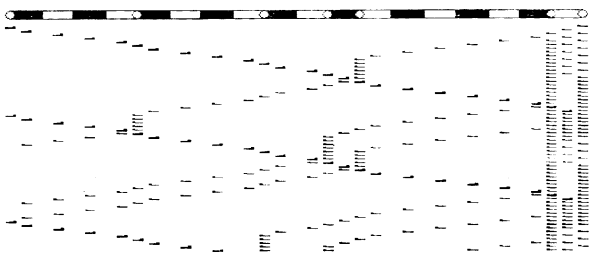


Fig.3. Spatio-temporal plot of single-line non-parallel train working diagram.

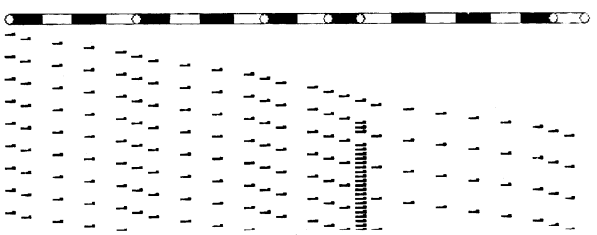


Fig.4. Spatio-temporal plot of single-line train working diagram of trains dispatched in pursuit.

Fig.3, the fast down-train causes many slow up-trains to wait in front of the station so as to let the former pass through. So the slow-train moves like a wave propagating from the origin station N to the arrival node M. This resembles the situation of traffic jam movement occurring in the highway traffic flow in the same direction.

4.5. The datasheet for the unbalanced line cells

Table 1 shows the datasheet for the three train working diagrams with unbalanced line cells. From Table 1, we can see that the percentage of trains arrived is higher under the third working diagram, i.e. its capacity utilization factor is better. However, because the travel time between neighbouring sections is extremely nonuniform, it still leads to many trains to wait at the station d that is one of node of the longest section de. Hence, it also reduces the total carrying capacity. Comparatively, the percentages of mode 1 and mode 2 are lower and equal to 27.15% and 25.90% respectively. But that of mode 1 is still higher than that of mode 2. This shows that the fast-train impacts slightly on slow-train in the opposite direction. As for waiting time, the percentage of waiting time under mode 1 is 90.84%, and that of mode 2 is 88.28%. In addition, since the length of line cell Ma is longer than that of eN, the number of trains dispatched from node cell M is fewer than that of node cell N.

Table 1. The datasheet for the three train working diagrams with unbalanced line cells.

Train Working Diagram		Trains	Trains	Sum of trains	Sum of trains	Travel	Waiting	Sum of	Sum of
Model	Direction	dispatched	arrived	dispatched	arrived	time	time	travel time	waiting time
1	Down	49	15	291	79	2471	2201	15525	14103
	Up	242	64			13054	11902		
2	Down	18	17	251	65	408	0	10854	9582
	Up	233	48			10446	9582		
3	Down	100	68	100	68	6188	4556	6188	4556
	Up	0	0			0	0		

4.6. The datasheet for the case of uniform line cells

Table 2 shows the datasheet for the three train working diagrams with uniform line cells.

Table 2. The datasheet for the three train working diagrams with uniform line cells.

Train Working Diagram		Trains	Trains	Sum of trains	Sum of trains	Travel	Waiting	Sum of	Sum of
Model	Direction	dispatched	arrived	dispatched	arrived	time	time	travel time	waiting time
1	Down	73	69	146	138	2417	1175	4834	2350
	Up	73	69			2417	1175		
2	Down	20	19	158	105	456	0	11509	9505
	Up	138	86			11053	9505		
3	Down	125	119	125	119	2856	0	2856	0
	Up	0	0			0	0		

In the simulation, the line cells are uniform, i.e. the lengths of all line cells are equal. This case does not occur in real railway. We do this just for explaining clearly the influence of nonuniformity of sections on the carrying capacity. The total number of line cells and the whole travel time are kept unchanged. Comparing Table 2 to Table 1, we could conclude that the carrying capacities of all the three modes are improved greatly. The percentage of mode 1 reaches 94.52% and mode 2 66.46%. Without considering trains still running in the way when simulation period is over, the percentage of mode 3 is 100.00%. Waiting times under mode 1 and 3 also diminish. Whereas, the waiting time of slow-train is still high under mode 2. Moreover, except for mode 3, the number of departure trains reduces, but the number of arrival trains evidently increases.

5. Conclusion

A new CA model for the single-line railway has been presented. Based on this model, several simulations have been done for the three working diagrams. From a different angle the results show that there exists a direct dependence of the railway carrying capacity on the organization of train operation and the uniformity of sections. Although the model is very simple and the capacity of node is unlimited, the results in this paper may be helpful for the future research and provide a tool to investigate the railway system.

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