Vol 15 No 4, April 2006
1009-1963/2006/15(04)/0681-06
Chinese Physics
© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Quantum entanglement and quantum nonlocality for N-photon entangled states*

Sun Yan-Hua(孙艳华)[a)b)] and Kuang Le-Man(匡乐满)[a)†]

[a)] *Department of Physics, Hunan Normal University, Changsha* 410081, *China*

[b)] *Haikou College of Economic and Vocational Technology, Haikou* 570203, *China*

(Received 15 August 2005; revised manuscript received 9 September 2005)

Quantum entanglement and quantum nonlocality of N-photon entangled states $|\psi_{Nm}\rangle = C_m[\cos\gamma|N-m\rangle_1|m\rangle_2 + \mathrm{e}^{\mathrm{i}\theta_m}\sin\gamma|m\rangle_1|N-m\rangle_2]$ and their superpositions are studied. We point out that the relative phase θ_m affects the quantum nonlocality but not the quantum entanglement for the state $|\psi_{Nm}\rangle$. We show that quantum nonlocality can be controlled and manipulated by adjusting the state parameters of $|\psi_{Nm}\rangle$, superposition coefficients, and the azimuthal angles of the Bell operator. We also show that the violation of the Bell inequality can reach its maximal value under certain conditions. It is found that quantum superpositions based on $|\psi_{Nm}\rangle$ can increase the amount of entanglement, and give more ways to reach the maximal violation of the Bell inequality.

Keywords: quantum entanglement, quantum nonlocality, Bell inequality
PACC: 0365, 4250

1. Introduction

Quantum entanglement and quantum nonlocality are two striking aspects of quantum mechanics.[1−5] They are introduced by Einstein, Podolsky, and Rosen (EPR) in their famous paper.[1] The relationship between them has attracted much attention. They play an essential role in the modern understanding of quantum phenomena, and in quantum information transmission and processing. Bell proposed a remarkable inequality imposed by a local hidden variable theory,[6] which enables a quantitative test on quantum nonlocality. The quantum nonlocality test can be performed on an entangled system composed of two coherent systems. This entangled system can be used as a quantum entangled channel for quantum information transfer. Numerous theoretical studies and experimental demonstrations have been carried out to understand nonlocal properties of quantum states. Various versions of Bell's inequality[5,7,8] has been proposed. Gisin and Peres found pairs of observables whose correlations violate Bell's inequality for a discrete N-dimensional entangled state.[9] Banaszek and Wódkiewicz studied Bell's inequality for continuous-variable states in terms of Wigner representation in phase space based upon parity measurement and displacement operation. Recently, Chen *et al* studied Bell's inequality of continuous-variable states[10] using their newly defined pseudospin Bell operators, and they showed that the EPR state can maximally violate Bell's inequality in their framework.

Recently, much attention has been paid to N-photon entangled states and their superposition states. They are widely used to realize quantum lithography,[11−13] super-resolving phase measurements,[14] and quantum teleportation.[15] In a previous paper,[16] we investigated their quantum decoherence. The purpose of the present paper is to study quantum entanglement and quantum nonlocality of the following N-photon entangled states and their superpositions:

$$|\psi_{Nm}\rangle = C_m[\cos\gamma|N-m\rangle_1|m\rangle_2 + \mathrm{e}^{\mathrm{i}\theta_m}\sin\gamma|m\rangle_1|N-m\rangle_2], \quad (1)$$

where m takes the values from zero to N, the normal-

*Project supported by the National Fundamental Research Program (Grant No 2001CB309310), the National Natural Science Foundation of China (Grant Nos 10325523, 90203018 and 10075018), the Foundation of the Ministry of Education of China, and the Educational Committee of Hunan Province (Grant Nos 200248 and 02A026).
†Corresponding author. E-mail: lmkuang@hunnu.edu.cn

ization factor is given by $A_m^{-2} = 1 + \cos\theta_m \sin 2\gamma\delta_{N,2m}$ with γ and θ_m being the entanglement angle and the relative phase, respectively. We will calculate the von Neumann entropy and study the Bell's inequality for quantum states $|\psi_{Nm}\rangle$ and their superposition states. We will show that quantum superpositions for the two modes may increase the amount of entanglement, and N-photon entangled states can maximally violate Bell's inequality. This paper is organized as follows. In Sec. 2, we study the quantum entanglement of N-photon entangled states and their superposition states through analysing their von Neumann entropy. Quantum nonlocality of N-photon entangled states and their superposition states is investigated through discussing the violation of the Bell's inequality in the pseudospin Bell-operator formalism developed by Chen *et al*[10] in Section 3. The last section gives a summary and conclusions.

2. Quantum entanglement for N-photon entangled states

In this section, we study properties of quantum entanglement of the N-photon entangled states given by Eq.(1). In order to do this, we consider the nontrivial case of $N \neq 2m$, in which the normalization constant $C_m = 1$. The degree of the entanglement can be described by the von Neumann entropy defined by

$$E(\rho_1) = -\mathrm{Tr}_1(\rho_1 \log \rho_1), \tag{2}$$

where ρ_1 is the reduced density operator of the first mode.

For the N-photon entangled states defined by Eq. (1) we find the von Neumann entropy to be

$$E_1 = -\cos^2\gamma \log\cos^2\gamma - \sin^2\gamma \log\sin^2\gamma, \tag{3}$$

which indicates that quantum entanglement of the N-photon entangled states given by Eq.(1) is independent of the superposition phases θ_m and the total photon number N of the two modes, and changes periodically with the change of the entanglement angle γ. In particular, the amount of entanglement reaches the maximal value of $E_1 = 1$ when the entanglement angle takes values of $\gamma = k\pi + \pi/4$ with k being integers. In Fig.1 we plot the change of the von Neumann entropy against the entanglement angle.

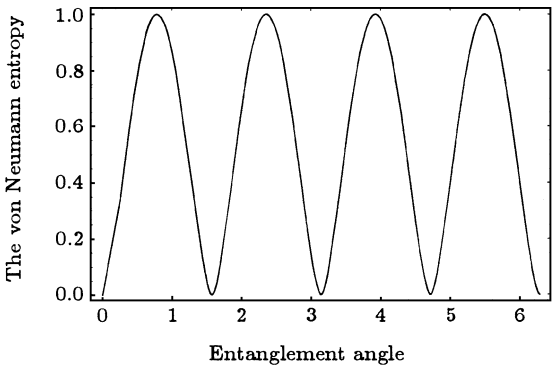


Fig.1. The von Neumann entropy of the N-photon entangled state is plotted against the entanglement angle γ.

We then study the quantum entanglement of superposition states based on the N-photon entangled states given by Eq.(1). Firstly, we consider a two-state superposition state defined by

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_{30}\rangle + |\psi_{31}\rangle), \tag{4}$$

which leads to the von Neumann entropy

$$E_2 = 1 - \cos^2\gamma \log\cos^2\gamma - \sin^2\gamma \log\sin^2\gamma, \tag{5}$$

which implies that the amount of entanglement of the two-component superposition state only depends on the entanglement angle γ of its basis. Especially, comparing Eq.(5) with (2) we find that the difference of the amount of entanglement between the two-component superposition state (4) and the basis state (1) is a positive constant, i.e., $E_2 - E_1 = 1$. This indicates that the amount of entanglement of the superposition state is always larger than that of the basis state for an arbitrary entanglement angle γ. In other words, starting with basis states defined by (1) one can construct quantum superposition states with larger amount of entanglement than that of the basis states. Hence, we may conclude that quantum superpositions of two modes may increase the amount of entanglement.

In order to further demonstrate the above idea of quantum superposition increasing the quantum entanglement, in what follows we take into account a multi-component quantum superposition state consisting of N bases with a fixed photon number N, but with different distributions m,

$$|\psi_N\rangle = A \sum_{m=0}^{N} \alpha_m |\psi_{Nm}\rangle, \tag{6}$$

where A is a normalization factor. It is straightforward to express the N-component superposition state

as the following number-sum Bell state,

$$|\Psi_N\rangle = \sum_{m=0}^{N} d_m |N-m\rangle_1 |m\rangle_2, \quad (7)$$

which is the eigenstate of the number-sum Bell operators $\hat{N} = \hat{N}_1 + \hat{N}_2$ with $\hat{N}_i$ being the number operators of the respective modes. The coefficients in Eq.(7) are given by

$$d_m = A(\alpha_m C_m \cos\gamma + \alpha_{N-m} C_{N-m} e^{i\theta_{N-m}} \sin\gamma), \quad (8)$$

where the normalization of the superposition state (8) implies that d_m satisfy the condition $\sum_{m=0}^{N} |d_m|^2 = 1$.

In order to calculate the von Neumann entropy of the superposition state (7), we need the reduced density operator

$$\rho_1 = \sum_{m=0}^{N} |d_m|^2 |N-m\rangle_1 \langle N-m|, \quad (9)$$

which leads to the following von Neumann entropy:

$$E_N = -\sum_{m=0}^{N} |d_m|^2 \log |d_m|^2. \quad (10)$$

For the sake of simplicity and without loss of generality, we consider the situation of equal-weight superposition in which we choose the superposition coefficients $\alpha_m = 1/\sqrt{N+1}$ and the relative phases $\theta_m = 2\pi m/N$. In this case, we have

$$|d_m|^2 = A^2 |C_m|^2 \{1 + \cos[2\pi(N-m)/N] \sin 2\gamma\}, \quad (11)$$

where the normalization constant is given by

$$A^{-2} = \sum_{m=0}^{N} |C_m|^2 \{1 + \cos[2\pi(N-m)/N] \sin 2\gamma\}. \quad (12)$$

Then the von Neumann entropy of the superposition state can be directly obtained through substituting (11) and (12) into (10). From Eqs.(10)–(12) we can find the maximal value of the von Neumann entropy of the superposition state to be $E_{N,\max} = \log_2(N+1)$ when $\gamma = k\pi/2$ with k being an arbitrary integer. This implies that the maximal amount of entanglement of the N-component superposition state only depends on the total photon number N, and increases with increasing N. In order to clearly see the influence of the number of components N and the entanglement angle γ, in Fig. 2 we plot the von Neumann entropy of the superposition states when $N = 1, 2, 3$, and 4, respectively. From Fig.2 we can see that the entanglement amount increases with increasing number of components, i.e., the photon number N, and changes periodically with respect to the entanglement angle γ.

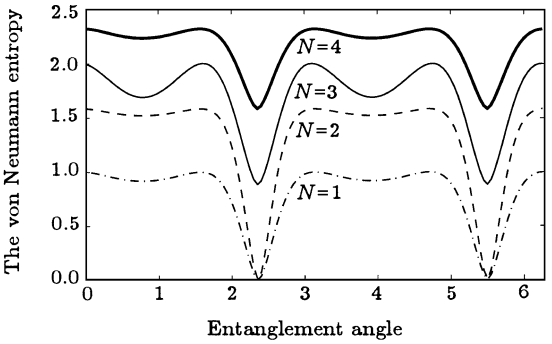


Fig.2. The von Neumann entropy of multi-component superposition states based on $|\psi_{N\,m}\rangle$ is plotted against the entanglement angle γ for $N = 1, 2, 3$, and 4, respectively.

3. Quantum nonlocality for N-photon entangled states

In this section, we study quantum nonlocality of N-photon entangled states and their superposition states through discussing the violation of Bell's inequality in the pseudospin Bell-operator formalism developed by Chen and co-workers.[10] Let us begin with a brief review of the pseudospin-operator formalism.[10] For a single-mode boson field, the pseudospin operators can be defined in terms of project operators in a Fock space in the following form:

$$S_z = \sum_{n=0}^{\infty} [|2n+1\rangle\langle 2n+1| - |2n\rangle\langle 2n|],$$
$$S_- = \sum_{n=0}^{\infty} [|2n\rangle\langle 2n+1|] = (S_{j+})^{\dagger}, \quad (13)$$

where $|n\rangle$ are the usual Fock states of the boson mode. The operator $S_z = -(-1)^{\hat{N}}$ with $\hat{N}$ being the number operator and $(-1)^N$ being the parity operator, and S_+ and S_- being the 'parity-flip' operators which satisfy the commutation relations of the $su(2)$ Lie algebra

$$[S_z, S_\pm] = \pm 2 S_\pm, \quad [S_+, S_-] = S_z. \quad (14)$$

For an arbitrary unit vector ending on the surface of a unit sphere $\boldsymbol{a} = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$, we have the following dot product:

$$\boldsymbol{a} \cdot \boldsymbol{S} = S_z \cos\theta_a + \sin\theta_a (e^{i\varphi_a} S_- + e^{-i\varphi_a} S_+). \quad (15)$$

Then for a two-mode boson field, the Bell operator due to Clauser, Horne, Shimony, and Holt (CHSH)

can be defined by

$$B = (\boldsymbol{a}\cdot\boldsymbol{S}_1)\otimes(\boldsymbol{b}\cdot\boldsymbol{S}_2) + (\boldsymbol{a}\cdot\boldsymbol{S}_1)\otimes(\boldsymbol{b}'\cdot\boldsymbol{S}_2) + (\boldsymbol{a}'\cdot\boldsymbol{S}_1)\otimes(\boldsymbol{b}\cdot\boldsymbol{S}_2) - (\boldsymbol{a}'\cdot\boldsymbol{S}_1)\otimes(\boldsymbol{b}'\cdot\boldsymbol{S}_2), \quad (16)$$

where $\boldsymbol{a}'$, $\boldsymbol{b}$, and $\boldsymbol{b}'$ are three unit vectors similarly defined as $\boldsymbol{a}$, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are defined in Eq.(13).

As well known, local hidden variable theories impose the Bell-CHSH inequality $|\langle B\rangle| \leq 2$ where $\langle B\rangle$ is the mean value of the Bell operator with respect to a given quantum state. However, in the quantum theory it is found that $|\langle B\rangle| \leq 2\sqrt{2}$, which implies that the Bell-CHSH inequality is violated. In particular, when $|\langle B\rangle| = 2\sqrt{2}$ for a given quantum state, we say that the Bell-CHSH inequality is maximally violated by the quantum state.

Quantum nonlocality of a quantum state can be described by the violation of the Bell-CHSH inequality. The expectation value of the Bell operator with respect to a quantum state $|\psi\rangle$ can be expressed in terms of the correlation functions as

$$\langle B\rangle = E(\theta_a,\theta_b) + E(\theta_a,\theta_{b'}) + E(\theta_{a'},\theta_b) - E(\theta_{a'},\theta_{b'}), \quad (17)$$

where the correlation functions are defined by

$$E(\theta_a,\theta_b) = \langle\psi|S^{(1)}_{\theta_a}\otimes S^{(2)}_{\theta_b}|\psi\rangle, \quad (18)$$

with

$$S^{(j)}_{\theta_a} = S_{jz}\cos\theta_a + S_{jx}\sin\theta_a. \quad (19)$$

We now investigate the quantum nonlocality of the N-photon entangled state given by Eq.(1). For this multi-photon entangled state we find the correlation function to be

$$E_{Nm}(\theta_a,\theta_b) = C_m^2\{[(-1)^N + K(\theta_m,\gamma)\delta_{N,2m}]\times\cos\theta_a\cos\theta_b + \delta_{N,2m\pm1}\times K(\theta_m,\gamma)\sin\theta_a\sin\theta_b\}, \quad (20)$$

in which we have introduced the following effective state parameter

$$K(\theta_m,\gamma) = \cos\theta_m\sin 2\gamma \quad (21)$$

to describe the effect of the basis state defined by (1) on the correlation functions.

Making use of the correlation function (20), from Eq.(17) we can get the expectation value of the Bell operator for arbitrary values of all azimuthal angles θ_a, θ_b, $\theta_{a'}$ and $\theta_{b'}$. As a concrete example, we consider the situation of $\theta_a = 0$, $\theta_{a'} = \pi/2$ and $\theta_b = -\theta_{b'}$. In this case, from Eqs.(17), (20) and (21) we can obtain the expectation value of the Bell operator given by

$$\langle B\rangle = 2C_m^2\{[(-1)^N + K(\theta_m,\gamma)\delta_{N,2m}]\cos\theta_b + \delta_{N,2m\pm1}K(\theta_m,\gamma)\sin\theta_b\}. \quad (22)$$

In order to observe the quantum nonlocality of the N-photon entangled state (1), we consider two different cases of $N = 2m$ and $N = 2m \pm 1$, respectively. When $N = 2m$, the quantum state given by Eq.(1) is disentangled, and reduces to

$$|\psi_{Nm}\rangle = C_m(\cos\gamma + \mathrm{e}^{\mathrm{i}\theta_m}\sin\gamma)|m\rangle_1|m\rangle_2. \quad (23)$$

Making use of Eqs.(17), (18) and (20), we obtain the expectation value of the Bell operator with respect to the disentangled state (23) $\langle B\rangle = 2\cos\theta_b$, which means that $|\langle B\rangle| \leq 2$. Hence, the unentangled state (23) cannot produce a violation of Bell's inequality.

On the other hand, when $N = 2m \pm 1$, the expectation value of the Bell operator with respect to the state (1) is given by

$$\langle B\rangle = 2[K(\theta_m,\gamma)\sin\theta_b - \cos\theta_b], \quad (24)$$

which indicates that for a fixed entanglement angle γ and a fixed relative phase θ_m, when $\theta_b = -\tan^{-1}K$, the expectation value of the Bell operator $\langle B\rangle$ reaches its maximum given by

$$\langle B\rangle_{max} = 2\sqrt{1 + K^2(\theta_m,\gamma)}, \quad (25)$$

which implies that $\langle B\rangle_{max} \geq 2$ due to $|K(\theta_m,\gamma)| \leq 1$. Thus, the N-photon entangled state always violates the Bell's inequality if $K(\theta_m,\gamma) \neq 0$. From Eqs.(21) and (24) we can see that the degree of violation of the Bell's inequalities depends upon both the entangling angle γ and the relative phase θ_m, and it changes periodically with respect to both γ and θ_m. In particular, we note that the relative phase θ_m seriously affects the mean value of the Bell operator although it does not affect the quantum entanglement of the quantum state given by (1). It is easy to find that when $|\psi_{Nm}\rangle$ reaches maximal entanglement, i.e., $\gamma = \pi/4$, and $\theta_m = 0$, we have $K(\theta_m,\gamma) = 1$. Under these conditions, we can reach the maximal violation of Bell's inequality with $\langle B\rangle_{\max} = 2\sqrt{2}$.

In Fig.3 we plot the degree of the violation of the Bell's inequality for N-photon entangled state against γ and θ_m. From Fig.3 we can see that the mean value of the Bell operator with respect to the multi-photon entangled state (1) changes periodically with both the

entanglement angle and the relative phase, and the Bell's inequality is always violated for arbitrary values of the entanglement angle and the relative phase except $\gamma = k\pi/2$ and $\theta = (2k+1)\pi/2$ with k an integer.

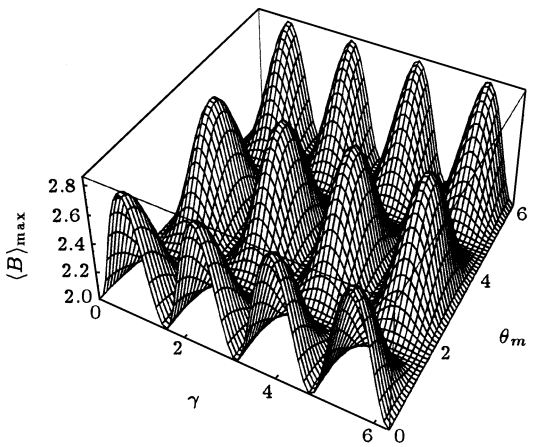


Fig.3. The degree of the maximal violation of the Bell's inequality for N-photon entangled state $\langle B\rangle_{\max}$ plotted against the entanglement angle γ and the relative phase θ_m.

Finally, we consider the quantum nonlocality of quantum superposition states which is formed using the multi-photon entangled state (1). In order to calculate the expectation value of the Bell operator with respect to the superposition states based on entangled states given by (1), we need the following correlation function

$$E_{mm'} = \langle\psi_{Nm}|S^{(1)}_{\theta_a}\otimes S^{(2)}_{\theta_b}|\psi_{Nm'}\rangle, \tag{26}$$

which is given by the following expression:

$$\begin{aligned} E_{mm'} =& C_m C^*_{m'}\cos\theta_a\cos\theta_b \\ &\times\{(-1)^N\delta_{m,m'}\delta_{N,m+m'}\cos\gamma\sin\gamma \\ &\times[\cos^2\gamma + \mathrm{e}^{\mathrm{i}(\theta_{m'}-\theta_m)}\sin^2\gamma] \\ &+(-1)^{m+m'}(\mathrm{e}^{\mathrm{i}\theta_{m'}}+\mathrm{e}^{-\mathrm{i}\theta_m})\} \\ &+ C_m C^*_{m'}\sin\theta_a\sin\theta_b \\ &\times\{[\cos^2\gamma+\mathrm{e}^{\mathrm{i}(\theta_{m'}-\theta_m)}\sin^2\gamma]\delta_{m,m'\pm1} \\ &+\cos\gamma\sin\gamma(\mathrm{e}^{\mathrm{i}\theta_{m'}}+\mathrm{e}^{-\mathrm{i}\theta_m})\delta_{N,m+m'\pm1}\}. \end{aligned} \tag{27}$$

As a simple example of analysing the quantum nonlocality of superposition states, we consider the following two-component superposition state:

$$|\psi_3\rangle = C(\alpha_0|\psi_{30}\rangle + \alpha_1|\psi_{31}\rangle), \tag{28}$$

where C is the normalization constant. Making use of Eq. (27), we can obtain the expectation value of the Bell operator for this superposition state,

$$\begin{aligned} \langle B\rangle =& -2\cos\theta_b + 4\sin\theta_b(|\alpha_0|^2+|\alpha_1|^2)^{-1} \\ &\times \mathrm{Re}[\alpha^*_0\alpha_1(\cos^2\gamma + \mathrm{e}^{\mathrm{i}(\theta_1-\theta_0)}\sin^2\gamma)], \end{aligned} \tag{29}$$

which indicates that $\langle B\rangle$ depends upon not only the superposition coefficients α_0, α_1, the azimuthal angles θ_a, θ_b, but also the state parameters of the basis states given by Eq.(1), θ_0, θ_1 and γ.

In order to observe the maximal violation of the Bell inequality, we can choose $\alpha_0 = \alpha_1 = 1$, the Bell function $\langle B\rangle$ given by Eq.(29) then becomes

$$\begin{aligned} \langle B\rangle =& 2\sin\theta_b[\cos^2\gamma + \cos(\theta_1-\theta_0)\sin^2\gamma] \\ &-2\cos\theta_b. \end{aligned} \tag{30}$$

Making use of Eq.(30), we can show that the maximal violation of the Bell inequality can be reached by controlling the azimuthal angle θ_b, the state parameters of the basis states given by Eq.(1), θ_0, θ_1 and γ. In fact, when $\theta_1 - \theta_0 = 2k\pi$ with k being an arbitrary integer, we can arrive at the following Bell function:

$$\langle B\rangle = 2(\sin\theta_b - \cos\theta_b), \tag{31}$$

which means that the values of the Bell function of $\langle B\rangle$ are independent of the entangling parameter of the basis state defined in (1), and we can reach the maximal violation of the Bell inequality $\langle B\rangle_{\max} = 2\sqrt{2}$ when $\theta_b = -\pi/4$.

On the other hand, when the phase difference is $\theta_1 - \theta_0 = (2k+1)\pi$ with k being an arbitrary integer, the Bell function given by Eq.(30) becomes

$$\langle B\rangle = 2[\sin\theta_b\cos(2\gamma) - \cos\theta_b], \tag{32}$$

from which we can see that when the entanglement angle satisfies $\gamma = k\pi$ with k being an arbitrary integer, Eq.(32) becomes Eq.(31), hence the maximal violation of the Bell inequality is reached with $\theta_b = -\pi/4$. However, when the entanglement angle satisfies $\gamma = (2k+1)\pi/2$ with k being an arbitrary integer, the Bell function (32) reduces to

$$\langle B\rangle = -2(\sin\theta_b + \cos\theta_b), \tag{33}$$

which implies that the violation of the Bell inequality reaches its maximal value $\langle B\rangle_{\max} = 2\sqrt{2}$ when $\theta_b = \pi/4$. Therefore, we can conclude that superposition states based on entangled states (1) can provide more ways of reaching the maximal violation of the Bell's inequality.

4. Conclusion

In summary, we have studied quantum entanglement and quantum nonlocality of N-photon entangled states for the two modes defined in Eq.(1) and their superpositions through investigating the von Neumann entropy and the violation of the Bell's inequality in the pseudospin Bell-operator formalism. For the multi-photon entangled states defined in Eq.(1) we have indicated that the von Neumann entropy is independent of the relative phase θ_m. Hence, quantum entanglement of these states only depends on the entanglement angle. However, the quantum nonlocality of these quantum states exhibits different dependence upon the state parameters. We have shown that both the entanglement angle and the relative phase seriously affect the violation of the Bell's inequality. And we have indicated that under certain conditions the maximal violation of the Bell's inequality can be reached.

It is worthwhile to mention that multi-component superposition states made from N-photon entangled states for the two modes defined in Eq.(1) exhibit some interesting characteristics in their quantum entanglement and quantum nonlocality. Firstly, we have found that these multi-component superposition states have larger amount of entanglement than that of the basis states defined by (1). Hence, quantum superpositions for the two modes can increase the amount of entanglement. This indicates the possibility of obtaining entangled states with a larger amount of entanglement starting from entangled states with a smaller amount of entanglement. Secondly, we have found that for these quantum superposition states there are more ways to make corresponding Bell's inequality reach the maximal violation, and shown that quantum nonlocality can be controlled and manipulated by adjusting the state parameters of $|\psi_{Nm}\rangle$, superposition coefficients, and the azimuthal angles of the Bell operator. We hope that the results obtained in the present paper would find their applications in quantum information processing[17−21] and the test of quantum nonlocality.[22,23]

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] You J, Li J H and Xie X T 2005 *Chin. Phys.* **14** 1329

[3] Tan H T, Xia H X and Li G X 2005 *Chin. Phys.* **14** 1382

[4] Fang M F, Zhou Q P and Zhou P 1999 *Chin. Phys.* **8** 401

[5] Cai X, Huang G M and Li G X 2005 *Chin. Phys.* **14** 223

[6] Bell S 1964 *Physics* (Long Island City, N.Y.) **1** 195

[7] Clauser J F, Horne M A, Shimony A and Holt R A 1969 *Phys. Rev. Lett.* **23** 880

[8] Clauser J F and Horne M A 1974 *Phys. Rev.* D **10** 526

[9] Gisin N and Peres A 1992 *Phys. Lett.* A **162** 15

[10] Chen Z B, Pan J W, Hou G and Zhang Y D 2002 *Phys. Rev. Lett.* **88** 040406

[11] Wang Y H and Kuang L M 2003*J. Opt. B: Quantum Semiclass. Opt.* **5** 405

[12] Kok P, Boto A N, Abrams D S, Williams C P, Braunstein S L and Dowling J P 2001 *Phys. Rev.* A **63** 063407

[13] Björk G, Sánchez-Soto L L and Söderholm J 2001 *Phys. Rev. Lett.* **86** 4516

[14] Mitchell M W, Lundeen J S and Steinberg A M 2004 *Nature* **429** 161

[15] Cochrane P T, Milburn G J and Munro W J 2000 *Phys. Rev.* A **62** 062307

[16] Sun Y H, Zhu X and Kuang L M 2005 *Chin. Phys. Lett.* **22** 1833

[17] Nielsen M and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridg, UK: Cambridge University Press)

[18] Chen Z B, Hou G and Zhang Y D 2002 *Phys. Rev.* A **65** 032317

[19] Lu J, Zhou L and Kuang L M 2004 *Phys. Lett.* A **330** 48

[20] Zhou L and Kuang L M 2003 *Phys. Lett.* A **315** 426

[21] Chen Z B Hou G and Zhang Y 2002*Phys. Rev.* A **65** 032317

[22] Jeong H, Son W, Kim M S, Ahm D and Brukner Č 2003 *Phys. Rev.* A **67** 012106

[23] Pitowsky I and Svozil K 2001 *Phys. Rev.* A **64** 014102