Vol 15 No 3, March 2006
1009-1963/2006/15(03)/0562-06
Chinese Physics
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Interaction of solitary waves in magnetized warm dusty plasmas with dust charging effects*

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(Received 29 June 2005; revised manuscript received 12 October 2005)

In consideration of adiabatic dust charge variation, the combined effect of the external magnetized field and the dust temperature on head-on collision of the three-dimensional dust acoustic solitary waves is investigated. By using the extended Poincaré–Lighthill–Kuo method, the phase shifts and the trajectories of two solitons after the collision are obtained. The effects of the magnitude and the obliqueness of the external magnetic field and the dust temperature on the solitary wave collisions are discussed in detail.

Keywords: soliton collision, magnetized dusty plasmas, dust charging effect
PACC: 5235S, 5235

1. Introduction

Solitary waves, an important nonlinear topic because of their existence in space and astrophysical plasmas, have been extensively studied both theoretically and experimentally. Micrometre or sub-micrometre sized dust grains are important ingredient of space plasma which has significant effect on wave properties. Hence, study of solitary waves in dusty plasmas has been received considerable attention.[1−11] In recent years, the investigation of solitary waves also has been extended to non-uniform,[1−3] nonplanar,[4,5] and dissipative[6,7] dusty plasmas and has made considerable success in clarifying many aspects of the characteristics of solitary waves. The excitation, propagation, and interaction of solitary waves are important issues in theoretical studies. However, there are very few investigations about the interactions of solitary waves in dusty plasmas. We know that one of the striking properties of solitons is their asymptotic preservation of form when they undergo a head-on collision. What interests us most is the phase shifts and the trajectories of the two solitons after a collision. Huang and Velarde[12] discussed the head-on collision of two concentric cylindrical ion-acoustic solitary waves. The properties of the head-on collision between two one-dimensional[13] and non-planar[14] dust acoustic solitary waves are also given by Xue. However, all these few studies on soliton collision are focused on un-magnetized, one-dimensional or cylindrical/spherical symmetry dusty plasmas. It is known that the external magnetic field would have significant effect on solitary wave propagation. We can expect that the external magnetic field would also have significant effect on solitary interaction. Hence, in this paper, in consideration of adiabatic ($\tau_{\rm ch}/\tau_{\rm d} = 0$) dust charge variation, where $\tau_{\rm ch}$ is the charging time scale and τ_d is the hydrodynamical time scale, we investigate the combined effect of the external magnetized field and the dust temperature on head-on collision of the three-dimensional dust acoustic solitary waves. By using the extended Poincaré–Lighthill–Kuo (PLK) method, the phase shifts and the trajectories of the two solitons after collision are obtained. It is discussed in detail how the magnitude and the obliqueness of the external magnetic field and the dust temperature modify the properties of dust acoustic solitary wave collision.

2. Governing equations

We consider a three-component dusty plasma sys-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10347006 and 10475066), and by the Natural Science Foundation of Gansu Province, China (Grant No 3ZS051-A25-013).
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tem, which consists of extremely massive, micron-sized, negatively charged hot dust fluid, and Boltzmann distributed electrons and ions, in the presence of an external static magnetic field directed along z-axis ($B_0 \| z_0$, where z_0 is a unit vector along z direction). Charge neutrality at equilibrium requires that $n_{i0} = Z_{d0} n_{d0} + n_{e0}$, where n_{i0}, n_{d0} and n_{e0} are the unperturbed ion, dust, and electron densities, respectively, and Z_{d0} is the number of electrons residing on a dust grain. The dust acoustic wave dynamics can be described by the following three-dimensional set of continuity, momentum, pressure, and Poisson's equations:

$$\begin{aligned} &\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d V_d) = 0, \\ &\frac{\partial V_d}{\partial t} + V_d \cdot (\bigtriangledown V_d) + \frac{\sigma_d}{n_d} \nabla p_d \\ &= Z_d [\nabla \phi + \omega_{cd} (V_d \times z_0)], \\ &\frac{\partial p_d}{\partial t} + V_d \cdot \bigtriangledown p_d + \gamma p_d \nabla \cdot V_d = 0, \\ &\nabla^2 \phi = Z_d n_d + n_e - n_i, \end{aligned} \tag{1}$$

where $V_d = (u, v, w)$, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$. V_d is the dust fluid velocity normalized to the dust acoustic speed $C_d = \sqrt{Z_{d0} T_i / m_d}$ with T_i being the ion-temperature (in energy units) and m_d being the mass of negatively charged dust particles; n_d is the dust grain number density normalized to n_{d0}; $\sigma_i = T_i / T_e$, with T_e being the electron-temperature (in energy units); $\mu = n_{e0}/n_{i0}$, n_i is the ion number density normalized to n_{i0}; $\sigma_d = T_d/(Z_{d0} T_i)$ with T_d being the dust-temperature (in energy unit); ϕ is the electrostatic potential normalized to T_i/e with e being the magnitude of the electron charge. The time and space variables are normalized by the dust plasma period $\omega_{pd}^{-1} = \sqrt{m_d / 4\pi n_{d0} Z_{d0}^2 e^2}$ and the Debye length $\lambda_D = C_d/\omega_{pd}$, respectively. $\omega_{cd} = (Z_{d0} e B_0)/m_d$ is the dust cyclotron frequency normalized to ω_{pd}. p_d is the dust pressure normalized to $n_{d0} Z_{d0} T_d$. $\gamma = 5/3$ is the adiabatic ratio of specific heats.

The Boltzmann distributed electrons and ions are directly given by

$$n_i = \frac{1}{1-\mu} \exp(-\phi), \quad n_e = \frac{\mu}{1-\mu} \exp(\sigma_i \phi). \tag{2}$$

Dust charge variation Q_d comes in through the charge current balance equation[15]

$$\frac{\partial Q_d}{\partial t} + u_d \frac{\partial Q_d}{\partial x} = I_e + I_i, \tag{3}$$

which is valid for grain charging, arising from plasma currents due to electrons and ions reaching the grain surface. We assume that the dust grain radius (a) $\ll$ the electron gyroradius (ρ_e). The charging characteristics are not significantly influenced by the existence of external magnetic field, since for $a \ll \rho_e$, the curvature effect of the trajectory of an electron (ion) impinging on a dust grain of radius a can be neglected.[16,17] Hence, when the streaming velocities of the electrons and ions are much smaller than their respective thermal velocities, the expressions for the electron and ion currents for spherical grains of radius R are given by $I_e = -\pi R^2 e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \exp\left(\frac{e\Phi}{T_e}\right)$, $I_i = \pi R^2 e \left(\frac{8T_i}{\pi m_i}\right)^{1/2} n_i \left(1 - \frac{e\Phi}{T_i}\right)$, where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. It is well known that the characteristic time for dust motion is of the order of tens of milliseconds for micrometre-sized grains, while the dust charging time is typically of the order of 10^{-8}s; therefore, the motion of dust is not so fast that the contribution from the electron current to dust is balanced for the ions. It follows that $\mathrm{d}Q_d/\mathrm{d}t \ll I_e, I_i$ and the current balance equation reads $I_e + I_i = 0$, and we obtain

$$\alpha(1-\psi)\exp(-s\phi) - \mu \exp[\sigma_i(\phi + \psi)] = 0, \tag{4}$$

where $\alpha = \sqrt{\sigma_i/\mu_i}$, $\mu_i = m_i/m_e$, and $\psi = e\Phi/T_i$. Because the dust charges $Q_d = a\Phi$, i.e., $-eZ_d = aT_i\psi/e$, we obtain the normalized dust charges (also denoted by Z_d)

$$Z_d = \frac{\psi}{\psi_0}, \tag{5}$$

where $\psi_0 = \psi(\phi = 0)$ is the dust surface floating potential corresponding to the unperturbed plasma potential, which is determined by

$$\alpha(1 - \psi_0) - \mu \exp(\sigma_i \psi_0) = 0. \tag{6}$$

Now, according to Eq.(5) and after expanding $\psi = \psi(0 + \phi)$ near $\phi = 0$, Eq.(6) becomes

$$Z_d = 1 + \gamma_1 \phi + \gamma_2 \phi^2 + \cdots, \tag{7}$$

where

$$\gamma_1 = \frac{1}{\psi_0}\frac{\mathrm{d}\psi(\phi)}{\mathrm{d}\phi}\mid_{\phi=0} = -\frac{1}{\psi_0}\frac{(1+\sigma_\mathrm{i})(1-\psi_0)}{1+\sigma_\mathrm{i}(1-\psi_0)},$$

$$\gamma_2 = \frac{1}{2\psi_0}\frac{\mathrm{d}^2\psi(\phi)}{\mathrm{d}\phi^2}\mid_{\phi=0}$$

$$= \frac{1}{2\psi_0}\frac{(1-\sigma_\mathrm{i}^2)(1-\psi_0)+2[1-\sigma_\mathrm{i}^2(1-\psi_0)]\psi'(0)-\sigma_\mathrm{i}^2(1-\psi_0)[\psi'(0)]^2}{(1+\sigma_\mathrm{i})(1-\psi_0)}.$$

Here $\psi'(0) = \frac{\mathrm{d}\psi(\phi)}{\mathrm{d}\phi}$.The variables γ_1, γ_2, which are related to the physical parameters, describe the effects of dust charge variation. Setting $\gamma_1 = \gamma_2 = 0$, we obtain the constant dust charge case, i.e., $Z_\mathrm{d} = 1$.

3. The dynamics of soliton's head-on collision and discussion

We assume that in the plasma, two solitons, A and B, which are, asymptotically, far apart in their initial states and travel toward each other obliquely under the external magnetic field with an angle of θ. After some time they interact, collide, and then depart. We also assume that the solitons have small amplitudes $\sim \epsilon$ (where ϵ is a small formal perturbation parameter) and the interaction between solitons are weak. Hence, we expect that the collision will be quasi-elastic, so that it will only cause shifts of the post-collision trajectories (phase shift). In order to analyse the effect of collision, we employ an extended PLK perturbation method.[18,12,13] According to this method, we introduce stretched coordinates,

$$\xi = \epsilon(l_x x + l_y y + l_z z - \lambda t) + \epsilon^2 P_0(\eta,\tau) + \epsilon^3 P_1(\xi,\eta,\tau) + \cdots, \quad (8)$$

$$\eta = \epsilon(l_x x + l_y y + l_z z + \lambda t) + \epsilon^2 Q_0(\xi,\tau) + \epsilon^3 Q_1(\xi,\eta,\tau) + \cdots, \quad (9)$$

$$\tau = \epsilon^3 t, \quad (10)$$

where ξ and η denote the trajectories of the two solitons travelling toward each other. The wave velocity λ and the variables of P_j and Q_j are to be determined. l_x, l_y, and l_z are the directional cosines of the wave vector $\boldsymbol{k}$ along x-, y-, and z-axes,, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. From Eqs.(8)–(10) we have

$$\frac{\partial}{\partial x_\mathrm{i}} = l_{x_\mathrm{i}}\epsilon\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right) + l_{x_\mathrm{i}}\epsilon^3\left(P_{0\eta}\frac{\partial}{\partial\xi}+Q_{0\xi}\frac{\partial}{\partial\eta}\right)+\cdots, \quad (11)$$

$$\frac{\partial}{\partial t} = \epsilon\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right) + \epsilon^3\left(\frac{\partial}{\partial\tau}+\lambda P_{0\eta}\frac{\partial}{\partial\xi}-\lambda Q_{0\xi}\frac{\partial}{\partial\eta}\right)+\cdots, \quad (12)$$

where $x_\mathrm{i} = x, y, z$, respectively. The dependent variables are expanded as

$$\begin{aligned} n_\mathrm{d} &= 1+\epsilon^2 n_1+\epsilon^3 n_2+\ldots \\ p_\mathrm{d} &= 1+\epsilon^2 p_1+\epsilon^3 p_2+\ldots \\ u &= \epsilon^3 u_1+\epsilon^4 u_2+\ldots \\ v &= \epsilon^3 v_1+\epsilon^4 v_2+\ldots \\ w &= \epsilon^2 w_1+\epsilon^3 w_2+\ldots \\ \phi &= \epsilon^2\phi_1+\epsilon^3\phi_2+\ldots \end{aligned} \quad (13)$$

Substituting Eqs.(2), (7), and Eqs.(11)–(13) into Eqs.(1) and collecting the terms in the different powers of ϵ, to lowest order in ϵ, we obtain the following relations

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_1 + l_z\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)u_1 = 0, \quad (14)$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)w_1 + l_z\sigma_\mathrm{d}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)p_1 = l_z\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_1, \quad (15)$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)p_1 + \gamma l_z\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)w_1 = 0, \quad (16)$$

$$l_x\sigma\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)p_1 = l_x\sigma_\mathrm{d}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_1 - \omega_\mathrm{cd}v_1, \quad (17)$$

$$l_y\sigma\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)p_1 = l_y\sigma_\mathrm{d}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi_1 + \omega_\mathrm{cd}u_1, \quad (18)$$

$$n_1 = -\chi\phi_1, \quad (19)$$

where $\chi = \gamma_1 + (1+\mu\sigma_\mathrm{i})/(1-\mu)$. The solutions of Eqs.(14)–(19) read

$$\phi_1 = \Phi_1(\xi,\tau) + \Phi_2(\eta,\tau), \quad (20)$$

$$n_1 = -\chi[\Phi_1(\xi,\tau) + \Phi_2(\eta,\tau)], \tag{21}$$

$$w_1 = -\lambda\chi[\Phi_1(\xi,\tau) - \Phi_2(\eta,\tau)]/l_z, \tag{22}$$

$$p_1 = -\chi\gamma[\Phi_1(\xi,\tau) + \Phi_2(\eta,\tau)], \tag{23}$$

and

$$u_1 = -\left(\frac{\lambda}{l_z}\right)^2 \frac{\chi l_y}{\omega_{\rm cd}}[\Phi_{1\xi}(\xi,\tau) + \Phi_{2\eta}(\eta,\tau)], \tag{24}$$

$$v_1 = \left(\frac{\lambda}{l_z}\right)^2 \frac{\chi l_x}{\omega_{\rm cd}}[\Phi_{1\xi}(\xi,\tau) + \Phi_{2\eta}(\eta,\tau)]. \tag{25}$$

Under the solvability condition, the wave velocity $\lambda^2 = l_z^2(1/\chi + \sigma_{\rm d}\gamma)$ is also obtained. The unknown functions of Φ_1 and Φ_2 will be determined at the higher order. Equations (20)–(23) imply that, at the leading order, we have two waves, $\Phi_1(\xi,\tau)$ and $\Phi_2(\eta,\tau)$, which are travelling toward each other. At the next order, the solutions also have the following shape:

$$\phi_2 = \Psi_1(\xi,\tau) + \Psi_2(\eta,\tau), \tag{26}$$

$$n_2 = -\chi[\Psi_1(\xi,\tau) + \Psi_2(\eta,\tau)], \tag{27}$$

$$w_2 = -\lambda\chi[\Psi_1(\xi,\tau) - \Psi_2(\eta,\tau)]/l_z, \tag{28}$$

$$p_2 = -\chi\gamma[\Psi_1(\xi,\tau) + \Psi_2(\eta,\tau)]; \tag{29}$$

and

$$\begin{aligned} u_2 =& -\left(\frac{\lambda}{l_z}\right)^2 \frac{\chi l_y}{\omega_{\rm cd}}[\Psi_{1\xi}(\xi,\tau) + \Psi_{2\eta}(\eta,\tau)] \\ &+ \frac{\lambda^3\chi l_x}{l_z^2\omega_{\rm cd}^2}[\Phi_{2\eta\eta}(\eta,\tau) - \Phi_{1\xi\xi}(\xi,\tau)], \end{aligned} \tag{30}$$

$$\begin{aligned} v_2 =& \left(\frac{\lambda}{l_z}\right)^2 \frac{\chi l_x}{\omega_{\rm cd}}[\Psi_{1\xi}(\xi,\tau) + \Psi_{2\eta}(\eta,\tau)] \\ &+ \frac{\lambda^3\chi l_y}{l_z^2\omega_{\rm cd}^2}[\Phi_{2\eta\eta}(\eta,\tau) - \Phi_{1\xi\xi}(\xi,\tau)], \end{aligned} \tag{31}$$

where Ψ_1 and Ψ_2 are to be determined.

To the next higher order, we can deduce

$$\begin{aligned} \frac{l_z}{\chi\lambda}\frac{\partial^2 w_3}{\partial\xi\partial\eta} =& \frac{1}{2\lambda}\frac{\partial}{\partial\xi}\left(\frac{\partial\Phi_1}{\partial\tau} - a\Phi_1\frac{\partial\Phi_1}{\partial\xi} + b\frac{\partial^3\Phi_1}{\partial\xi^3}\right) \\ &+ \frac{1}{2\lambda}\frac{\partial}{\partial\eta}\left(\frac{\partial\Phi_2}{\partial\tau} + a\Phi_2\frac{\partial\Phi_2}{\partial\eta} - b\frac{\partial^3\Phi_2}{\partial\eta^3}\right) \\ &+ \left(\frac{\partial P_0}{\partial\eta} + c\Phi_2\right)\frac{\partial^2\Phi_1}{\partial\xi^2} \\ &- \left(\frac{\partial Q_0}{\partial\xi} + c\Phi_1\right)\frac{\partial^2\Phi_2}{\partial\eta^2}; \end{aligned} \tag{32}$$

or

$$\begin{aligned} \frac{l_z}{\chi\lambda}w_3 =& \frac{1}{2\lambda}\int\left(\frac{\partial\Phi_1}{\partial\tau} - a\Phi_1\frac{\partial\Phi_1}{\partial\xi} + b\frac{\partial^3\Phi_1}{\partial\xi^3}\right){\rm d}\eta \\ &+ \frac{1}{2\lambda}\int\left(\frac{\partial\Phi_2}{\partial\tau} + a\Phi_2\frac{\partial\Phi_2}{\partial\eta} - b\frac{\partial^3\Phi_2}{\partial\eta^3}\right){\rm d}\xi \\ &+ \int\left(\frac{\partial P_0}{\partial\eta} + c\Phi_2\right)\frac{\partial^2\Phi_1}{\partial\xi^2}{\rm d}\xi{\rm d}\eta \\ &- \int\left(\frac{\partial Q_0}{\partial\xi} + c\Phi_1\right)\frac{\partial^2\Phi_2}{\partial\eta^2}{\rm d}\xi{\rm d}\eta, \end{aligned} \tag{33}$$

where

$$\begin{aligned} a =& \frac{l_z^2}{2\lambda\chi^2}\left[\chi\left(3\frac{1+\mu\sigma_{\rm i}}{1-\mu} + \sigma_{\rm d}\gamma(1+\gamma)\chi^2\right)\right. \\ &\left. + 2\gamma_2 + \frac{\mu\sigma_{\rm i}^2 - 1}{1-\mu}\right], \\ b =& \frac{l_z^2}{2\lambda\chi^2}\left[1 + \frac{\lambda^4(1-l_z^2)\chi^2}{l_z^4\omega_{\rm cd}^2}\right], \\ c =& \frac{l_z}{4\lambda^2\chi^2}\left\{\frac{\chi^2}{l_z}[\sigma_{\rm d}\gamma l_z^2(1-\gamma+\chi) + \chi\lambda^2]\right. \\ &\left. + \chi\gamma_1 l_z - 2l_z\gamma_2 - l_z\frac{\mu\sigma_{\rm i}^2 - 1}{1-\mu}\right\}. \end{aligned}$$

The first (second) term in Eq.(33) is proportional to $\eta(\xi)$ because the integrated function is independent of $\eta(\xi)$. Thus the first two terms of Eq.(33) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence we have

$$\frac{\partial\Phi_1}{\partial\tau} - a\Phi_1\frac{\partial\Phi_1}{\partial\xi} + b\frac{\partial^3\Phi_1}{\partial\xi^3} = 0, \tag{34}$$

$$\frac{\partial\Phi_2}{\partial\tau} + a\Phi_2\frac{\partial\Phi_2}{\partial\eta} - b\frac{\partial^3\Phi_2}{\partial\eta^3} = 0. \tag{35}$$

The third and fourth terms in Eq.(33) are not secular terms in this order, but they will become secular in the next order.[18] Hence we have

$$\frac{\partial P_0}{\partial\eta} = -c\Phi_2, \tag{36}$$

$$\frac{\partial Q_0}{\partial\xi} = -c\Phi_1. \tag{37}$$

Equations (34) and (35) are the two-side travelling wave KdV equations in the reference frames of ξ and η, respectively. Their corresponding solutions are

$$\Phi_1 = \Phi_A\sec^2\left[\left(\frac{a\Phi_A}{12b}\right)^{1/2}\left(\xi - \frac{1}{3}a\Phi_A\tau\right)\right], \tag{38}$$

$$\Phi_2 = \Phi_B\sec^2\left[\left(\frac{a\Phi_B}{12b}\right)^{1/2}\left(\eta - \frac{1}{3}a\Phi_B\tau\right)\right], \tag{39}$$

where Φ_A and Φ_B are the amplitudes of the two solitons A and B at the initial position. The leading

phase changes due to the collision can be calculated from Eqs.(36) and (37):

$$P_0 = -c\left(\frac{12b\Phi_B}{a}\right)^{1/2}\left\{\tanh\left[\left(\frac{a\Phi_B}{12b}\right)^{1/2} \times\left(\eta+\frac{1}{3}a\Phi_B\tau\right)\right]+1\right\}, \tag{40}$$

$$Q_0 = -c\left(\frac{12b\Phi_A}{a}\right)^{1/2}\left\{\tanh\left[\left(\frac{a\Phi_A}{12b}\right)^{1/2} \times\left(\xi-\frac{1}{3}a\Phi_A\tau\right)\right]-1\right\}. \tag{41}$$

Hence, up to $O(\epsilon^2)$, the trajectories of the two solitary waves for weak head-on interaction are

$$\xi = \epsilon(l_x x + l_y y + l_z z - \lambda t) - \epsilon^2 c\left(\frac{12b\Phi_B}{a}\right)^{1/2} \times\left\{\tanh\left[\left(\frac{a\Phi_B}{12b}\right)^{1/2} \times\left(\eta+\frac{1}{3}a\Phi_B\tau\right)\right]+1\right\}+O(\epsilon^3), \tag{42}$$

$$\eta = \epsilon(l_x x + l_y y + l_z z + \lambda t) - \epsilon^2 c\left(\frac{12b\Phi_A}{a}\right)^{1/2} \times\left\{\tanh\left[\left(\frac{a\Phi_A}{12b}\right)^{1/2} \times\left(\xi-\frac{1}{3}a\Phi_A\tau\right)\right]-1\right\}+O(\epsilon^3). \tag{43}$$

To obtain the phase shifts after a head-on collision of the two solitons, we assume that the solitons A and B are, asymptotically, far from each other at the initial time ($t=-\infty$), i.e., soliton A is at $\xi=0,\eta=-\infty$, and soliton B is at $\eta=0,\xi=+\infty$, respectively. After the collision ($t=+\infty$), the soliton A is far to the right of soliton B, i.e., soliton A is at $\xi=0,\eta=+\infty$, and soliton B is at $\eta=0,\xi=-\infty$. Using Eqs.(42) and (43) we obtain the corresponding phase shifts Δ_A and Δ_B as follows:

$$\Delta_A = \epsilon(x-\lambda t)|_{\xi=0,\eta=+\infty} - \epsilon(x-\lambda t)|_{\xi=0,\eta=-\infty} = 2\epsilon^2 c\left(\frac{12b\Phi_B}{a}\right)^{1/2}, \tag{44}$$

$$\Delta_B = \epsilon(x+\lambda t)|_{\eta=0,\xi=-\infty} - \epsilon(x+\lambda t)|_{\eta=0,\xi=+\infty} = -2\epsilon^2 c\left(\frac{12b\Phi_A}{a}\right)^{1/2}. \tag{45}$$

Since soliton A is travelling to the right and soliton B is travelling to the left, we see from Eqs.(41) and (42) that due to the collision each soliton has a positive phase shift in its travelling direction. The magnitudes of the phase shifts are related to the physical parameters, i.e., $\epsilon, \mu, \sigma_{\rm i}, \sigma_{\rm d}, l_z, \omega_{\rm cd}$, and $\Phi_A(\Phi_B)$. In this paper, we are interested in the discussion of the modification of phase shifts by the external magnetic field. The variation of phase shifts with propagation angle θ ($l_z=\cos(\theta)$) and $\omega_{\rm cd}$ are shown in Fig.1. In all results, all physical quantities are dimensionless and $\epsilon=0.1$, $\Phi_A=\Phi_B=0.1$ are used. Figure 1 indicates that, the phase shift increases with obliqueness θ but decreases with $\omega_{\rm cd}$. Phase shift increases quickly with θ when $\omega_{\rm cd}$ is small and changes smoothly with θ when $\omega_{\rm cd}$ is larger. When the obliqueness is small, the external magnetic field has a weak effect on soliton collision. On the other hand, when the obliqueness is larger, the external magnetic field modifies the soliton collision property significantly. The effect of dust temperature on phase shift is also shown in Fig.2. We can find that the effect of dust temperature on phase shift is weak, especially, for small obliqueness.

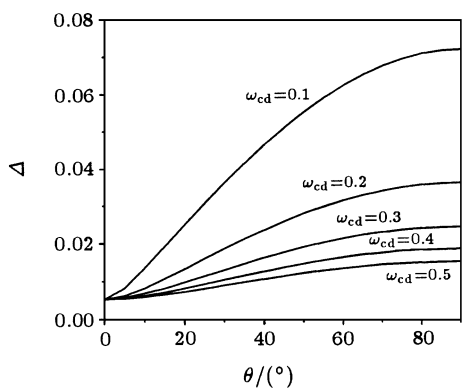


Fig.1. Phase shifts against propagation angle for different $\omega_{\rm cd}$ $\sigma_{\rm d}$=0.1, $\sigma_{\rm t}$=0.1, μ=0.25, ε=0.1, A=0.1.

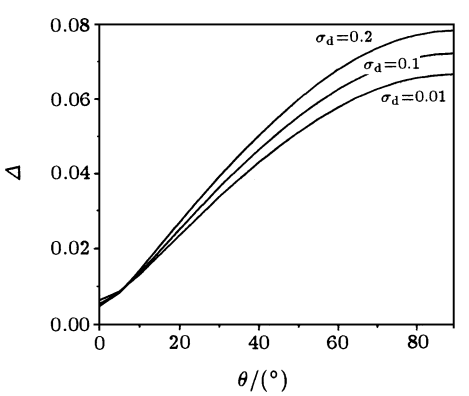


Fig.2. Phase shifts against propagation angle for different $\sigma_{\rm d}$ ($\sigma_{\rm t}$=0.1, μ=0.25, $\omega_{\rm cd}$=0.1, ε=0.1, A=0.1).

In summary, the phase shifts and the trajectories describing the collision of two dust acoustic solitary waves in a magnetized warm dusty plasma with adiabatic dust charge fluctuation are derived by the extended PLK method. It is shown that the properties

of soliton collision are influenced significantly by the obliqueness and magnitude of the external magnetic field. With reference to the present study, we can have a more profound understanding of the properties of dust acoustic solitary wave collision.

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