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Entropy evolvement properties in a system of Schrödinger cat state light field interacting with two entangled atoms*

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The field entropy can be regarded as a measurement of the degree of entanglement between the light field and the atoms of a system which is composed of two-level atoms initially in an entangled state interacting with the Schrödinger cat state. The influences of the strength of light field and the phase angle between the two coherent states on the field entropy are discussed by using numerical calculations. The result shows that when the strength of light field is large enough the field entropy is not zero and the degrees of entanglement between the atoms and the three different states of the light fields are equal. When the strength of the light field is small, the degree of entanglement is maximum in a system of the two entangled atoms interacting with an odd coherent state; it is intermediate for a system of the two entangled atoms interacting with the Yurke–Stoler coherent state, and it is minimum in a system of the two entangled atoms interacting with an even coherent state.

Keywords: field entropy, entangled state, Schrödinger cat state
PACC: 4250, 0365

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics. A number of entanglement measures have been discussed in the literature, such as the von Neumann reduced entropy, the relative entropy of entanglement.[1] The quantum entropy has been used to study the dynamic characteristics of two-level atom interacting with light field.[2] It is shown that the quantum entropy is a useful physics quantity for measuring pure degree of entanglement state. Field entropy has been studied in many papers.[3−10] However, these works are limited to the cases of single atom being initially in a single state or two atoms being initially in a product state interacting with single mode field or two mode fields. Recently, two two-level atoms in entangled state has aroused huge research interest. The entangled state indicates the quantum nonlocal properties and is widely applied in the quantum computation, quantum cryptography, quantum teleportation and so on.[11−14] In order to be close to the fact of experiment, and lead to a better understanding of basic quantum phenomena in the environment, we have formulated the entanglement in a system of two entangled atoms interacting with a coherent light field in a Kerr medium, and have shown that the entanglement can be controlled by the coupling constant of dipole–dipole interaction between atoms and the coupling strength of the Kerr medium with the light field.[15]

The purpose of this paper is to study the entropy dynamics of the light field, we consider the situation of two two-level atoms being initially in an entangled state interacting with the Schrödinger cat state. The influences of the strength of light field and the phase angle between the two coherent states on the field entropy are discussed by using full quantum theory.

2. The model and its solution

We consider two two-level atoms A and B are simultaneously injected into a single mode light cavity, the dipole interaction between atoms can be neglected. There are same couplings of the two atoms as interacting with a field, under these conditions. The Hamil-

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tonian of the system can be written as ($\hbar = 1$)

$$\hat{H} = \omega \hat{a}^{+}\hat{a} + \omega_0 \sum_{j=\mathrm{A}}^{\mathrm{B}} \hat{S}_z^j + g \sum_{j=\mathrm{A}}^{\mathrm{B}} (\hat{a}^{+}\hat{S}_{-}^j + \hat{a}\hat{S}_{+}^j), \quad (1)$$

where $\hat{a}^{+}$ and $\hat{a}$ are the creation and annihilation operators of the monochromatic photons with frequency ω respectively, ω_0 denotes the atomic transition frequency, $\hat{S}_z^j$ and $\hat{S}_{\pm}^j$ (j=A, B) denote the inversion and transition operators of the jth atom respectively, g describes the coupling constant of the field and the atom.

If we consider that at time $t = 0$ the two atoms and the field are decoupled, and assume that the two atoms are initially in an entangled state:

$$|\varphi_a(0)\rangle = \frac{1}{\sqrt{2}}(|e_{\mathrm{A}}, g_{\mathrm{B}}\rangle + |g_{\mathrm{A}}, e_{\mathrm{B}}\rangle), \quad (2)$$

the field is initially in a superposition of two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, namely the Schrödinger cat state[16]

$$|\varphi_{\mathrm{f}}(0)\rangle = |\alpha, \varphi\rangle = \{2[1 + \cos\varphi \exp(-2|\alpha|^2)]\}^{-1/2} \times [|\alpha\rangle + \mathrm{e}^{\mathrm{i}\varphi}|-\alpha\rangle], \quad (3)$$

where

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = \sum_{n=0}^{\infty} F_n |n\rangle,$$

$$F_n = \exp(-|\alpha|^2/2)(|\alpha|^{2n}/n!)^{1/2}\exp(\mathrm{i}n\phi),$$

$$|-\alpha\rangle = \sum_{n=0}^{\infty} F_n' |n\rangle,$$

$$F_n' = \exp(-|\alpha|^2/2)(-|\alpha|^{2n}/n!)^{1/2}\exp(\mathrm{i}n\phi),$$

$$\hat{D}(\alpha) = \exp(\alpha\hat{a}^{+} - \alpha^{*}\hat{a}),$$

with $|0\rangle$ being the vacuum state of a harmonic oscillator and $|n\rangle$ the n-particle eigenstate. φ is the phase angle between the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, $\hat{D}(\alpha)$ is a displacement operator, $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\phi}$ is a complex parameter, ϕ and $|\alpha|$ represent the direction angle of the excitation and the strength of light field, respectively (for simplicity, we suppose $\phi = 0$), coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are 180° out of phase with respect to each other.

Obviously, if $\varphi = 0$, then Eq.(3) describes the even coherent state:[16]

$$|\alpha\rangle_{\mathrm{e}} = \{2[1 + \exp(-2|\alpha|^2)]\}^{-1/2}(|\alpha\rangle + |-\alpha\rangle);$$

when $\varphi = \pi$, then Eq.(3) is equal to the odd coherent state:

$$|\alpha\rangle_{\mathrm{o}} = \{2[1 - \exp(-2|\alpha|^2)]\}^{-1/2}(|\alpha\rangle - |-\alpha\rangle);$$

and, finally, for $\varphi = \pi/2$, the state (3) is equal to the Yurke–Stoler coherent state:[17]

$$|\alpha\rangle_{\mathrm{YS}} = (2)^{-1/2}(|\alpha\rangle + \mathrm{i}|-\alpha\rangle).$$

Then the initial state vector of the system is

$$|\varphi_{\mathrm{s}}(0)\rangle = \frac{1}{\sqrt{2}}(|e_{\mathrm{A}}, g_{\mathrm{B}}\rangle + |g_{\mathrm{A}}, e_{\mathrm{B}}\rangle) \otimes |\alpha, \varphi\rangle. \quad (4)$$

We only consider resonance situation, namely, transition frequency of atom and effective action resonance frequency of the field satisfy $\omega_{\mathrm{a}} = \omega$. Then, along with time evolution, the state vector of the system at any time $t > 0$ is given by

$$|\varphi_{\mathrm{s}}(t)\rangle = \sum_{n=0}^{\infty} [A_n(t)|e_{\mathrm{A}}, e_{\mathrm{B}}, n-1\rangle + B_n(t)|e_{\mathrm{A}}, g_{\mathrm{B}}, n\rangle + C_n(t)|g_{\mathrm{A}}, e_{\mathrm{B}}, n\rangle + D_n(t)|g_{\mathrm{A}}, g_{\mathrm{B}}, n+1\rangle]. \quad (5)$$

Based on the initial condition

$$A(0) = D(0) = 0, \quad B(0) = C(0) = \frac{1}{\sqrt{2}}, \quad (6)$$

the solution of Schrödinger equation in the interaction representation is given by

$$\begin{cases} A_n(t) = A(t)(F_n + F_n'\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2|\alpha|^2)]\}^{-1/2}, \\ B_n(t) = C_n(t) = B(t)(F_n + F_n'\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2|\alpha|^2)]\}^{-1/2}, \\ D_n(t) = D(t)(F_n + F_n'\mathrm{e}^{\mathrm{i}\varphi})\{2[1 + \cos\varphi\exp(-2|\alpha|^2)]\}^{-1/2}, \end{cases} \quad (7)$$

where

$$\begin{cases} A(t) = -\mathrm{i}\sqrt{\dfrac{n}{2n+1}}\sin[\sqrt{2(2n+1)}gt], \\ B(t) = C(t) = \dfrac{1}{\sqrt{2}}\cos[\sqrt{2(2n+1)}gt], \\ D(t) = -\mathrm{i}\sqrt{\dfrac{n+1}{2n+1}}\sin[\sqrt{2(2n+1)}gt]. \end{cases} \quad (8)$$

The properties of the quantum entanglement between two entangled atoms and Schrödinger cat state field will be discussed in the next section by using above results.

3. Field entropy

We use the von Neumann entropy as a measurement of the degree of entanglement between the field and the two atoms of the system under consideration. The von Neumann entropy S of a system is defined as[18]

$$S(t) = -\mathrm{Tr}(\hat{\rho}(t)\ln\hat{\rho}(t)), \tag{9}$$

where $\hat{\rho}$ is the density operator for a given quantum system. For a system in which both the entangled atoms and light field start from pure states, the entropy of the system is always zero, i.e., $S(t) = 0$. Therefore the entropies of two interacting subsystems are precisely equal, i.e., $S_{\mathrm{a}}(t) = S_{\mathrm{f}}(t)$. In the following, we only consider the time behaviour of the field entropy $S_{\mathrm{f}}(t)$. In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues of the reduced field density operator. From Eq.(5), we have the reduced field density operator

$$\begin{aligned}\hat{\rho}_{\mathrm{f}}(t) =& \mathrm{Tr}_{\mathrm{atom}}[\hat{\rho}(t)] \\ =& |\mu(t)\rangle\langle\mu(t)| + |\nu_1(t)\rangle\langle\nu_1(t)| \\ &+ |\nu_2(t)\rangle\langle\nu_2(t)| + |\tau(t)\rangle\langle\tau(t)| \\ =& |\mu(t)\rangle\langle\mu(t)| + |\nu(t)\rangle\langle\nu(t)| + |\tau(t)\rangle\langle\tau(t)|,\end{aligned} \tag{10}$$

where

$$\begin{cases} |\mu(t)\rangle = \sum\limits_{n=1}^{\infty} A_n(t)\,|n-1\rangle, \\ |\nu(t)\rangle = |\nu_1(t)\rangle + \mathrm{i}\,|\nu_2(t)\rangle = (1+\mathrm{i})\sum\limits_{n=0}^{\infty} B_n(t)\,|n\rangle, \\ |\tau(t)\rangle = \sum\limits_{n=0}^{\infty} D_n(t)\,|n+1\rangle, \end{cases} \tag{11}$$

with $|\nu_2(t)\rangle = |\nu_1(t)\rangle = \sum\limits_{n=0}^{\infty} C_n(t)\,|n\rangle$. We assume that the eigenstate of the reduced field density operator can be written in the following form:

$$|\pi_{\mathrm{f}}(t)\rangle = \lambda_1\,|\mu(t)\rangle + \lambda_2\,|\nu(t)\rangle + \lambda_3\,|\tau(t)\rangle. \tag{12}$$

If we apply the density matrix given by Eq.(10) to the state equation (12), we find that

$$\begin{aligned}\hat{\rho}_{\mathrm{f}}(t)\,|\pi_{\mathrm{f}}(t)\rangle =& \left(\langle\mu(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_1}\langle\mu(t)|\nu(t)\rangle + \frac{\lambda_3}{\lambda_1}\langle\mu(t)|\tau(t)\rangle\right)\lambda_1\,|\mu(t)\rangle \\ &+ \left(\langle\nu(t)|\nu(t)\rangle + \frac{\lambda_1}{\lambda_2}\langle\nu(t)|\mu(t)\rangle + \frac{\lambda_3}{\lambda_2}\langle\nu(t)|\tau(t)\rangle\right)\lambda_2\,|\nu(t)\rangle \\ &+ \left(\langle\tau(t)|\tau(t)\rangle + \frac{\lambda_1}{\lambda_3}\langle\tau(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_3}\langle\tau(t)|\nu(t)\rangle\right)\lambda_3\,|\tau(t)\rangle.\end{aligned} \tag{13}$$

Consequently, for $|\pi_{\mathrm{f}}(t)\rangle$ to be an eigenstate of $\hat{\rho}_{\mathrm{f}}(t)$ we must have the relation

$$\begin{aligned}\pi_{\mathrm{f}}(t) &= \left(\langle\mu(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_1}\langle\mu(t)|\nu(t)\rangle + \frac{\lambda_3}{\lambda_1}\langle\mu(t)|\tau(t)\rangle\right) \\ &= \left(\langle\nu(t)|\nu(t)\rangle + \frac{\lambda_1}{\lambda_2}\langle\nu(t)|\mu(t)\rangle + \frac{\lambda_3}{\lambda_2}\langle\nu(t)|\tau(t)\rangle\right) \\ &= \left(\langle\tau(t)|\tau(t)\rangle + \frac{\lambda_1}{\lambda_3}\langle\tau(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_3}\langle\tau(t)|\nu(t)\rangle\right),\end{aligned} \tag{14}$$

where $\pi_{\mathrm{f}}(t)$ is an eigenvalue of the density operator $\hat{\rho}_{\mathrm{f}}(t)$. Then the eigenvalues of the density matrix of the field are given by

$$\begin{cases} \pi_{\mathrm{f}}^1(t) = \dfrac{\theta_1}{3} - \dfrac{2}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}\cos\beta, \\ \pi_{\mathrm{f}}^2(t) = \dfrac{\theta_1}{3} + \dfrac{1}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}(\cos\beta + \sqrt{3}\sin\beta), \\ \pi_{\mathrm{f}}^3(t) = \dfrac{\theta_1}{3} + \dfrac{1}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}(\cos\beta - \sqrt{3}\sin\beta), \end{cases} \tag{15}$$

where

$$\begin{cases} \beta = \frac{1}{3}\arccos\left(\frac{\xi}{\varsigma}\right), \\ \xi = 9\theta_1\theta_2 - 2\theta_1^2 - 27\theta_3 \\ \varsigma = 2(\theta_1^2 - 3\theta_2)^{\frac{3}{2}}, \end{cases} , \tag{16}$$

with

$$\begin{aligned} \theta_1 =& -(\langle\mu(t)|\mu(t)\rangle + \langle\nu(t)|\nu(t)\rangle + \langle\tau(t)|\tau(t)\rangle), \\ \theta_2 =& \langle\mu(t)|\mu(t)\rangle\langle\tau(t)|\tau(t)\rangle \\ &+ \langle\mu(t)|\mu(t)\rangle\langle\nu(t)|\nu(t)\rangle \\ &+ \langle\nu(t)|\nu(t)\rangle\langle\tau(t)|\tau(t)\rangle - |\langle\mu(t)|\nu(t)\rangle|^2 \\ &- |\langle\mu(t)|\tau(t)\rangle|^2 - |\langle\nu(t)|\tau(t)\rangle|^2, \\ \theta_3 =& \langle\mu(t)|\mu(t)\rangle\langle\nu(t)|\nu(t)\rangle\langle\tau(t)|\tau(t)\rangle \\ &+ \langle\mu(t)|\nu(t)\rangle\langle\tau(t)|\mu(t)\rangle\langle\nu(t)|\tau(t)\rangle \\ &- \langle\mu(t)|\mu(t)\rangle|\langle\nu(t)|\tau(t)\rangle|^2 \\ &- \langle\nu(t)|\nu(t)\rangle|\langle\tau(t)|\mu(t)\rangle|^2 \\ &- \langle\tau(t)|\tau(t)\rangle|\langle\mu(t)|\nu(t)\rangle|^2. \end{aligned}$$

The field entropy $S_{\rm f}(t)$ may be expressed in terms of the eigenvalues $\pi_{\rm f}^i(t)$ (i=1, 2, 3) for the reduced field density operator as

$$\begin{aligned} S_{\rm f}(t) =& -\{\pi_{\rm f}^1(t)\ln[\pi_{\rm f}^1(t)] + \pi_{\rm f}^2(t)\ln[\pi_{\rm f}^2(t)] \\ &+ \pi_{\rm f}^3(t)\ln[\pi_{\rm f}^3(t)]\}. \end{aligned} \tag{17}$$

If $S_{\rm f}(t)$ takes its minimum value 0, the two coherent fields and the two atoms are in the disentangled state. If $S_{\rm f}(t)$ takes its maximum value 1, the two coherent fields and the two atoms are in the maximum entangled state.

Figures 1, 2 and 3 present the numerical results of the time evolution of the field entropy for two parameters φ and $|\alpha|$, respectively. It is shown that the field entropy is strongly dependent on φ, the phase angle between the two coherent states and the strength of the light field $|\alpha|$. In Figs.1–3, the ordinates represent (a) $S_{\rm f}+8$, (b) $S_{\rm f}+6$, (c) $S_{\rm f}+4$, (d) $S_{\rm f}+2$, (e) $S_{\rm f}$.

In Fig.1 is shown the influence of the strength of the light field on time evolution of the field entropy $S_{\rm f}(t)$ in a system in the Yurke–Stoler coherent state of the light field interacting with entangled atoms. The result tells us that for different $|\alpha|$, the time evolution of the field entropy $S_{\rm f}(t)$ is quite complicated. When $|\alpha|$ is small, the field entropy $S_{\rm f}(t)$ is the change of orderliness, the atoms and the light field are alternating between the entangled and disentangled states. With increasing $|\alpha|$, the field entropy $S_{\rm f}(t)$ is not the change of orderliness, but the increase of the degree of entanglement between the light field and the two atoms. When $|\alpha|$ is large enough, the atoms and the light field are always in the entangled state.

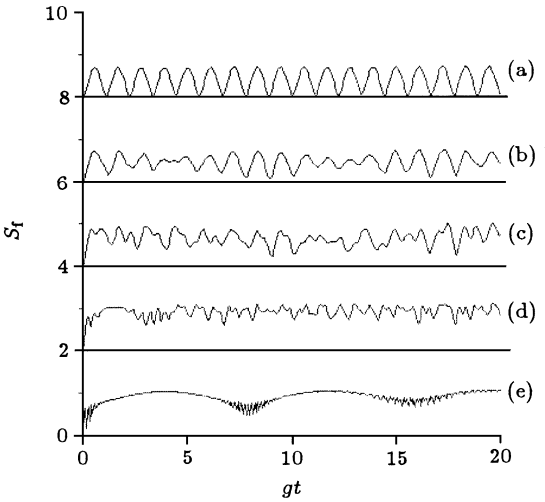


Fig.1. Time evolution of the field entropy for $\varphi = \pi/2$. (a) $|\alpha| = 0.1$, (b) $|\alpha| = 0.5$, (c) $|\alpha| = 1$, (d) $|\alpha| = 2$, (e) $|\alpha| = 5$.

In Fig.2 is shown the influence of the strength of the light field on the time evolution of the field entropy $S_{\rm f}(t)$ in a system in the odd coherent state of the light field interacting with entangled atoms. We can see that when $|\alpha|$ is small, the amplitude of vibration of the field entropy $S_{\rm f}(t)$ shows periodicity. It is obvious that the atoms and the light field are always changing alternately between the maximum entangled state and the disentangled state. With increasing $|\alpha|$, the field entropy $S_{\rm f}(t)$ increases gradually, and the degree of entanglement between the light field and the atoms is rising, the atoms and the light field are always in the entangled state.

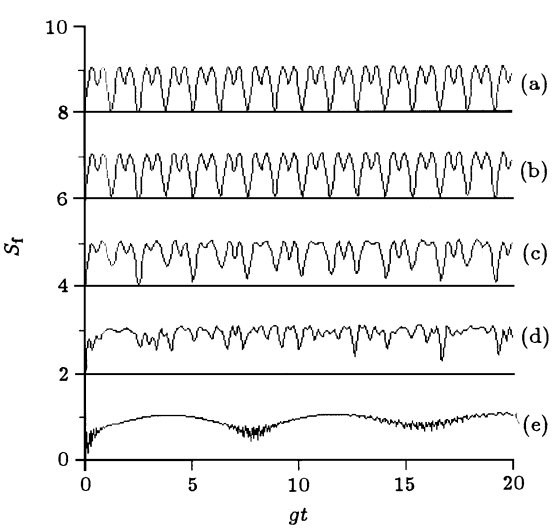


Fig.2. Time evolution of the field entropy for $\varphi = \pi$ (a) $|\alpha| = 0.1$, (b) $|\alpha| = 0.5$, (c) $|\alpha| = 1$, (d) $|\alpha| = 2$, (e) $|\alpha| = 5$.

In Fig.3 is shown the influence of the strength of light field on the time evolution of the field entropy $S_{\rm f}(t)$ in a system in the even coherent states of the light field interacting with entangled atoms. When $|\alpha|$ is small, the amplitude of vibration of the field entropy $S_{\rm f}(t)$ presents periodicity, and resembles Fig.1. With increasing $|\alpha|$, the field entropy $S_{\rm f}(t)$ increases gradually, also. When $|\alpha|$ is large enough, the atoms and the light field may be in the same entangled state as the Yurke–Stoler coherent state, and the even coherent state interacting with entangled atoms.

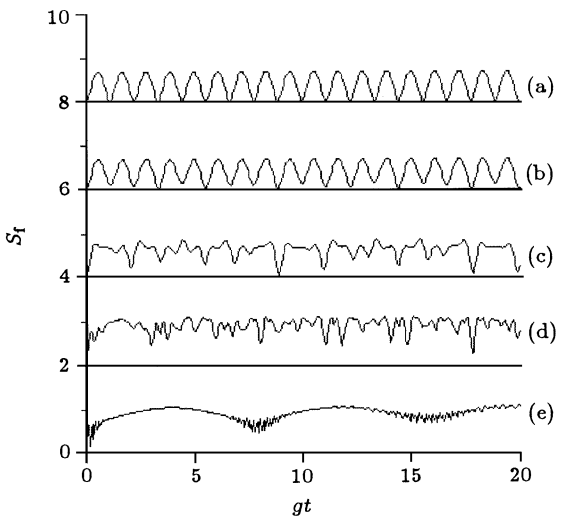


Fig.3. Time evolution of the field entropy with $\varphi = 0$ (a) $|\alpha| = 0.1$, (b) $|\alpha| = 0.5$, (c) $|\alpha| = 1$, (d) $|\alpha| = 2$, (e) $|\alpha| = 5$.

4. Conclusion

In conclusion, the time evolution of the field entropy has been studied on a system of two entangled atoms interacting with light field, where the two two-level atoms are initially in an entangled state interacting with the Schrödinger cat state. The influence of the phase angle of the two coherent states and the strength of the light field on the field entropy by using numerical calculations is discussed. The result shows that the field entropy is not zero, and the degrees of entanglement between the atoms and three different light fields are equal when the light field is large enough. However, when the light field is small, the degree of entanglement is maximum for a system of the two entangled atoms interacting with odd coherent state of the light field, and it is intermediate for the system of the two entangled atoms interacting with the Yurke–Stoler coherent state of the light field, and it is minimum for a system of two entangled atoms interacting with an even coherent state of the light field. This result is important for experimental realization of the preparation of entangled states.

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