Vol 15 No 3, March 2006
1009-1963/2006/15(03)/0492-04
Chinese Physics
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Quantum teleportation by entanglement swapping with trapped ions*

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(Received 3 November 2005; revised manuscript received 17 November 2005)

An effective teleportation scheme for an unknown ionic internal state via trapped ions is proposed without joint Bell-state measurement (BSM). In the constructed quantum channel process, we make use of entanglement swapping to avoid decrease in entanglement during the distributing of particles. Thus our scheme provides new prospects for quantum teleportation in a longer distance. The distinct advantage of our scheme is insensitive to the heating of vibrational mode. Furthermore, our scheme has no any individual optical access, and the successful probability also can reach 1.

Keywords: entanglement, entanglement swapping, quantum teleportation, trapped ions
PACC: 0365, 4250

Quantum entanglement is of fundamental interest in testing quantum mechanics against local hidden theories,[1−4] and has many practical applications in quantum information processing, for example, quantum cryptography,[5] quantum dense coding,[6] and quantum teleportation.[7−10]

Quantum teleportation was proposed first by Bennett *et al*[11] and experimentally realized by Bouwmeester *et al*[12] and Boschi *et al*.[13] It is a process to transmit unknown states to a remote location via a quantum channel aided by some classical communication. In original contributions, the joint Bell state measurement (BSM) is a key step in quantum teleportation. Hitherto, we still cannot discriminate the Bell states entirely and conclusively.[14] In addition, the joint BSM is very complex for realization experimentally,[15] thus it is a bottle-neck. To overcome the difficulty, Zheng[8] proposed a more feasible teleportation scheme using a resonant optical cavity without BSM and with a successful teleportation probability of 0.25. But the scheme is subjected to effects of both the cavity decay and the thermal field. Then, Ye and Guo[9] proposed another scheme, also without BSM, to teleport atomic states, where the effect of cavity decay has been eliminated and the successful probability of teleportation is 0.5. But the effect of thermal field is still present. The scheme presented by Yang and Cao[16] have eliminated the affections of thermal field and cavity decay, and successful probability of teleportation is 1.

Although schemes given by Yang, Ye and Zheng are very successful, the generation of quantum channel is confined in a cavity QED, in addition after maximally entangled states are generated, entangled pair of atoms are directly distributed to Alice(the sender) and Bob(the acceptor), who should be separated by large distances. During the distribution process, the entanglement of the atoms will inevitably decrease. The longer the distance, the more the decrease. Thus schemes offered by Yang, Ye and Zheng are probably applied only in shorter distances. In the paper, in order to realize teleportation in a longer distance, we present a feasible scheme of quantum teleportation by entanglement swapping with trapping ions. The distinct advantage of the scheme is that we adopt the trapped-ions system in the constructed quantum channel, which is insensitive to the heating of vibrational mode. In addition, our scheme has no indi-

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vidual optical access, or even could succeed without BSM, and the successful probability can reach 1.

We now consider a collection of two identical two-level ions in a linear trap driven by a different laser field separately. The choice of a linear trap eliminates the so-called micromotion of ions on the axis. The interaction between the ions and the lasers can be described as follows:

$$H = \nu a^+ a + \omega_0 \sum_i S_i^z + \sum_i \frac{\Omega_i}{2}(S_i^+ \mathrm{e}^{\mathrm{i}[\eta_i(a+a^+)-\omega_i t]} + \mathrm{H.c.}), \quad (1)$$

where $a^+(a)$ denotes the creation (annihilation) operator for the centre-of-mass mode of the collective motion of the two ions (frequency ν), ω_0 is the atomic transition frequency, $S_i^z = \frac{1}{2}(|e\rangle_i\langle e| - |g\rangle_i\langle g|)$, $S_i^+ = |e\rangle_i\langle g|$, and $S_i^- = |g\rangle_i\langle e|$ are ionic operators and $|e\rangle_i$, and $|g\rangle_i$ are the excited and ground states of ith ion, respectively. Ω_i is the Rabi frequency of the laser field addressing the ith atom, and η_i is Lamb–Dicke parameter.

For the laser addressing the first ion, we choose a detuning close to the upper sideband, i.e. $\omega_1 = \omega_0 + \delta$, and for the laser addressing the second ion, we choose the detuning to be the negative of the detuning of the first laser, i.e. $\omega_2 = \omega_0 - \delta$. In addition, we choose the detuning from the sideband to be so large that the internal state transition is insensitive to the vibrational quantum number n.

Rabi frequency Ω can be determined in the second-order perturbation theory.[17,18]

$$\widetilde{\Omega} = -\frac{(\Omega\eta)^2}{2(\nu-\delta)}, (\nu > \delta). \quad (2)$$

Then the evolution of a two-ion system for different initial states can be given by

$$\begin{aligned}
|gg\rangle &\longrightarrow \cos\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle + \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle,\\
|ee\rangle &\longrightarrow \cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle + \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle,\\
|ge\rangle &\longrightarrow \cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle - \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle,\\
|eg\rangle &\longrightarrow \cos\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle - \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle. \quad (3)
\end{aligned}$$

Suppose that three users Alice, Bob and Tom are spatially separated. Tom is in the middle between Alice and Bob. Alice possesses the ion pair (1, 2), and Bob possesses the ion 5. The ion pair (3, 4) belongs to Tom. The ion 1 to be teleported is initially in the state

$$|\phi\rangle_1 = c_g|g\rangle_1 + c_e|e\rangle_1. \quad (4)$$

The ion pair (2, 3), and the ion pair (4, 5) have been previously prepared in the following entangled states

$$|\phi\rangle_{23} = a|ge\rangle_{23} + b|eg\rangle_{23}, \quad (5)$$

$$|\phi\rangle_{45} = c|ge\rangle_{45} + d|eg\rangle_{45}, \quad (6)$$

where $|g\rangle_i$ and $|e\rangle_i$ are the ground and excited states of the ith ion, and c_g, c_e, a, b, c and d are coefficients (c_g and c_e are unknown). To realize the entanglement swapping, Tom confines the ion pair (3, 4) in a linear trap, he addresses each ion with a single different laser. Before entanglement swapping,[19,20] the state of the ion pair (2, 5) and ion pair (3, 4) is

$$\begin{aligned}
|\phi\rangle_{2345} =& ac|ge\rangle_{25}|eg\rangle_{34} + ad|gg\rangle_{25}|ee\rangle_{34}\\
&+ bc|ee\rangle_{25}|gg\rangle_{34} + bd|eg\rangle_{25}|ge\rangle_{34}. \quad (7)
\end{aligned}$$

After an interaction time $t_1 = \dfrac{\pi}{2|\widetilde{\Omega}|}$, the whole state evolves according to Eq.(3) to

$$\begin{aligned}
|\phi\rangle'_{2345} =& \frac{1}{\sqrt{2}}|eg\rangle_{34}(ac|ge\rangle_{25} - \mathrm{i}bd|eg\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|ge\rangle_{34}(bd|eg\rangle_{25} - \mathrm{i}ac|ge\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|ee\rangle_{34}(ad|gg\rangle_{25} + \mathrm{i}bc|ee\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|gg\rangle_{34}(bc|ee\rangle_{25} + \mathrm{i}ad|gg\rangle_{25}). \quad (8)
\end{aligned}$$

If the initial states of the ion pair(2,3) and the ion pair(4,5) satisfy the condition: $a = c$ and $b = d$, i.e. the ion pair(2,3) and the ion pair(4,5) are initially in the same state, the state in Eq.(8) will become

$$\begin{aligned}
|\phi\rangle'_{2345} =& \frac{1}{\sqrt{2}}|eg\rangle_{34}(a^2|ge\rangle_{25} - \mathrm{i}b^2|eg\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|ge\rangle_{34}(b^2|eg\rangle_{25} - \mathrm{i}a^2|ge\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|ee\rangle_{34}ab(|gg\rangle_{25} + \mathrm{i}|ee\rangle_{25})\\
&+ \frac{1}{\sqrt{2}}|gg\rangle_{34}ab(|ee\rangle_{25} + \mathrm{i}|gg\rangle_{25}). \quad (9)
\end{aligned}$$

Next, Tom measures the ion pair(3,4) in the state $|ee\rangle_{34}$, or $|gg\rangle_{34}$, the ion pair(2,5) will be in maximally entangled state:

$$|\phi\rangle_{25} = \frac{1}{\sqrt{2}}ab(|gg\rangle_{25} + \mathrm{i}|ee\rangle_{25}), \quad (10)$$

or

$$|\phi\rangle_{25} = \frac{1}{\sqrt{2}}ab(|ee\rangle_{25} + \mathrm{i}|gg\rangle_{25}). \quad (11)$$

The probability for obtaining each state is $P = a^2(1-a^2)$. If the measurement result is $|ge\rangle_{34}$, or $|eg\rangle_{34}$, the ion pair(2,5) will be left the nonmaximally entangled. Through analysis, the successful probability for obtaining maximally entangled states is $P = 2a^2(1-a^2)$. For the condition $a = c$, $b = d$, if there is a small error $c = a + ka$ and we choose $k = 0.1$, and $a = 0.7$, through calculation, the fidelity of the obtained maximally entangled states of the ion pair(2,5) can reach 0.989.[19] Therefore, a small error of the coefficients only slightly affects the fidelity of the result state.

We can assume that, without loss of generality, the quantum state of the ion pair(2,5) is

$$|\phi\rangle_{25} = \frac{1}{\sqrt{2}}(|gg\rangle_{25} + \mathrm{i}|ee\rangle_{25}). \quad (12)$$

Then

$$|\phi\rangle_{125} = \frac{1}{\sqrt{2}}(|gg\rangle_{25} + \mathrm{i}|ee\rangle_{25})(c_g|g\rangle_1 + c_e|e\rangle_1). \quad (13)$$

Now Alice confines the ion pair(1,2) in a linear trap, she addresses each ion with a single different laser. The state of the ion pair(1,2) evolves according to Eq.(3). After the interaction time $t_2 = \frac{\pi}{2|\widetilde{\Omega}|}$, the total state of the ion 1, ion 2 and ion 5 becomes the following

$$\begin{aligned}|\phi\rangle'_{125} =& \frac{1}{2}[|gg\rangle_{12}(c_g|g\rangle_5 - c_e|e\rangle_5) \\ &+ \mathrm{i}|ee\rangle_{12}(c_g|g\rangle_5 + c_e|e\rangle_5) \\ &- \mathrm{i}|ge\rangle_{12}(c_e|g\rangle_5 - c_g|e\rangle_5) \\ &+ |eg\rangle_{12}(c_e|g\rangle_5 + c_g|e\rangle_5)]. \end{aligned} \quad (14)$$

Next Alice will detect the ion pair(1,2). If the result is $|ee\rangle_{12}$, the state of the ion 5 is $|\phi\rangle_5 = c_g|g\rangle_5 + c_e|e\rangle_5$, so the teleportation succeeds with a probability of $\frac{1}{4}$. If the measurement result is $|gg\rangle_{12}$, or $|ge\rangle_{12}$ or $|eg\rangle_{12}$, Bob can reconstruct the original state by corresponding unitary transformation; the teleportation also can succeed with a probability of $\frac{1}{4}$. Thus the total probability of successful teleportation reaches 1.

Here we discuss the experimental feasibility of the proposed scheme. Because during the interaction between trapped ions and bichromatic light, the detuning from the sideband is very large, and the thermal field is eliminated. Thus we only need to consider the radiative time of ions. For the Rydberg ions with principal numbers 50 and 51, the radiative time is $T_a = 3 \times 10^{-2}$ s. For the interaction between trapped ions and the laser field, no particular assumption is required for the experimental parameters. With a vibrational frequency $\frac{\nu}{2\pi}$=200kHz, for example, we choose $\delta = 0.90\nu, \eta = 0.10$, and $\Omega = 0.10\nu$, then the interaction time is 5×10^{-3} s, which is much shorter than the atomic radiative time T_a, so our scheme is realizable.

Although in the processes of teleporting bipartite entangled state, Xue *et al*[10] also adopt entanglement swapping, but their goal is the same as that of Bennett *et al*,[11] i.e. the atom pairs or the atoms which belong to Alice are required to be in Bell states to be measured. In addition, to achieve the goal of the joint BSM, Alice must send two atom-pairs to the cavity separately, thus the total interaction time between the cavity and the atoms increases. Specially, after the quantum channels are generated, entangled pair of atoms are directly distributed. Thus Xue's scheme can only be applied in quantum teleportation in a shorter distance.

There exists still a disadvantage in our scheme. On the one hand, our scheme must use two classical channels: one belongs to Alice and Tom, and the other belongs to Alice and Bob. On the other hand, if Alice detects the ion pair(1,2) in the state $|ge\rangle_{12}$ or $|eg\rangle_{12}$, she must distinguish between the ion 1 and the ion 2. Fortunately, Riebe *et al*[21] have solved this problem, and they have developed a technique to address any specified target ion using tightly focused laser beams and to 'hide' the remaining ions from the target ion's fluorescence by changing their internal states such that they are insensitive to the fluorescent light. So the scheme is feasible in the current technology.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Greenberger D M, Horne M A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131

[3] Bell J S 1965 *Physics* (New York: Long Island City) **1** 195

[4] Zheng S B 2001 *Phys. Rev. Lett.* **87** 230404

[5] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[6] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[7] Zheng Y Z, Gu Y J, Wu G C and Guo G C 2003 *Chin. Phys.* **12** 1070
Cai X H 2003 *Chin. Phys.* **12** 1066

[8] Zheng S B 2004 *Phys. Rev.* A **69** 0643024

[9] Ye L and Guo G C 2004 *Phys. Rev.* A **70** 054303

[10] Xue Z Y, Yang M, Yi Y M and Cao Z L 2005 quant-ph/0508040
[11] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[12] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[13] Boschi D, Branca S, De Martini F, Hardy L and Popescu S 1998 *Phys. Rev. Lett.* **80** 1121
[14] Zheng S B and Cuo G C 2001 *Phys. Rev.* A **63** 044302
[15] Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737
[16] Yang M and Cao Z L 2004 quant-ph/0411195
[17] Mølmer K and Sørensen A 1999 *Phys. Rev. Lett.* **82** 1835
[18] Sørensen A and Mølmer K 1999 *Phys. Rev. Lett.* **82** 1971
[19] Yang M, Song W and Cao Z L 2005 *Phys. Rev.* A **71** 034312
[20] Yang M, Zhao Y, Song W and Cao Z L 2005 *Phys. Rev.* A **71** 044302
[21] Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Körber T W, Becher C, Schmidt-Kaler F, James D F V and Blatt R 2004 *Nature* (London) **429** 734