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Entanglement diversion and quantum teleportation of entangled coherent states*

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The proposals on entanglement diversion and quantum teleportation of entangled coherent states are presented. In these proposals, the entanglement between two coherent states, $|\alpha\rangle$ and $|-\alpha\rangle$, with the same amplitude but a phase difference of π is utilized as a quantum channel. The processes of the entanglement diversion and the teleportation are achieved by using the 50/50 symmetric beam splitters, the phase shifters and the photodetectors with the help of classical information.

Keywords: entangled coherent state, entanglement diversion, quantum teleportation, detection of photon

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1. Introduction

Quantum entanglement is one of the essential features of quantum mechanics. For decades, entanglement has been the focus of much work in the foundations of quantum mechanics, being associated particularly with quantum nonseparability and the violation of Bell's inequalities. In recent years, however, entanglement has aroused much interest in the quantum information processing, such as quantum teleportation,[1] superdense coding,[2] quantum key distribution,[3] telecoloning[4] and quantum positioning and clock synchronization.[5,6] Hence, quantum entanglement has been viewed as an essential resource for quantum information processing. The original study for quantum information processing focused on the systems with a finite-dimensional (discrete variable) state space, such as the polarizations of a photon or the discrete levels of an atom. Owing to the experimental demonstration of quantum teleportation of coherent states,[7] much interest have arisen in continuous variable quantum information processing.[8–11] Thereupon, the entangled states with continuous variable receive much attention in the study of quantum entanglement.[12–17]

The coherent state can be used to encode quantum information on continuous variables.[18] In quantum information processing, the entangled coherent state is normally categorized into a two-mode-continuous-variable state. However, there was a suggestion of implementing a logical qubit encoding through treating a coherent superposition state, which can make a single-mode-continuous-variable state as a qubit in two-dimensional Hilbert space.[19] Thereupon, many schemes have been proposed for realizing quantum information processing by using the coherent state.[20–23]

Recently van Enk and Hirota[20] considered the teleportation of the arbitrary coherent superposition state of two coherent states $|\alpha\rangle$ and $-|\alpha\rangle$ by using the entangled coherent state

$$|\Psi_\alpha\rangle_{1,2} = \frac{1}{\sqrt{N_\alpha}}(|\alpha\rangle_1|\alpha\rangle_2 + \mathrm{e}^{\mathrm{i}\theta}|-\alpha\rangle_1|-\alpha\rangle_2), \quad (1)$$

as quantum channel, where

$$|\alpha\rangle = \exp(-\mid\alpha\mid^2/2)\exp(\alpha\hat{a}^\dagger)|0\rangle \quad (2)$$

is a coherent state.

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Wang[22] has proposed a scheme for the teleportation of an entangled coherent state. In his scheme, the following tripartite maximally entangled state

$$|\Psi'_\alpha\rangle_{1,2,3} = \frac{1}{\sqrt{N'_\alpha}}(|\sqrt{2}\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 - |-\sqrt{2}\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3) \quad (3)$$

was used as a quantum channel.

Inspired by their idea, we propose some schemes for diverting the entanglement of entangled coherent state and teleporting an unknown entangled coherent state.

2. Entanglement diversion

We first briefly review the function of a beam splitter. The lossless symmetric 50/50 beam splitter is described by

$$\hat{B}_{12} = \exp\left[\mathrm{i}\frac{\pi}{4}(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1)\right], \quad (4)$$

where $\hat{a}_i$ and $\hat{a}_i^\dagger$ (i denotes 1 or 2) are the bosonic annihilation and creation operators of the system i, respectively. It transforms the input state from a direct product of two coherent states $|\alpha\rangle_1|\beta\rangle_2$ into an entangled state

$$|\alpha\rangle_1|\beta\rangle_2 \longrightarrow |(\alpha+\mathrm{i}\beta)/\sqrt{2}\rangle_1|(\beta+\mathrm{i}\alpha)/\sqrt{2}\rangle_2, \quad (5)$$

where the normalization factor is omitted on the right-hand side. Further equipping the beam splitter by a pair of $-\pi/2$ phase shifters described by the unitary operator $P_2 = \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}\hat{a}_2^\dagger\hat{a}_2}$, we can have the operator $B'_{12} = P_2B_{12}P_2$ that transforms the state $|\alpha\rangle_1|\beta\rangle_2$ into

$$|\alpha\rangle_1|\beta\rangle_2 \longrightarrow |(\alpha+\beta)/\sqrt{2}\rangle_1|(\alpha-\beta)/\sqrt{2}\rangle_2. \quad (6)$$

This transform plays an important role in realizing the quantum information processing that involves coherent states.

Suppose that the input state of the beam splitter $B'S_1$ is a direct product of a coherent superposition state

$$|\Psi\rangle = \frac{1}{\sqrt{N_\alpha}}(|\sqrt{2}\alpha\rangle_1 - |-\sqrt{2}\alpha\rangle_1) \quad (7)$$

and a vacuum state $|0\rangle_2$, where N_α is a normalization factor. And it is possible to generate a superposition of two coherent states $|\sqrt{2}\alpha\rangle$ and $|-\sqrt{2}\alpha\rangle$ from a coherent state $|\sqrt{2}\alpha\rangle$ propagating through a nonlinear medium.[24] After interaction with the beam-splitter, the output state emerging from the beam splitter is given by

$$|\Phi\rangle = \frac{1}{\sqrt{N'_\alpha}}(|\alpha\rangle_1|\alpha\rangle_2 - |-\alpha\rangle_1|-\alpha\rangle_2), \quad (8)$$

which is a maximally entangled state, and its amount of entanglement is exactly one ebit.

Assume there are three remote partners (Alice, Bob and Clara) who want to establish a connection. First, Alice shared the quantum channel $|\Phi\rangle_{13}$ with Bob and quantum channel $|\Phi\rangle_{24}$ with Clara, where

$$|\Phi\rangle_{13} = \frac{1}{\sqrt{N'_\alpha}}(|\alpha\rangle_1|\alpha\rangle_3 - |-\alpha\rangle_1|-\alpha\rangle_3) \quad (9)$$

and

$$|\Phi\rangle_{24} = \frac{1}{\sqrt{N'_\alpha}}(|\alpha\rangle_2|\alpha\rangle_4 - |-\alpha\rangle_2|-\alpha\rangle_4). \quad (10)$$

Note that at this time the modes 1 and 2 are located at the Alice's location, the mode 3 is at Bob's location and the mode 4 is at Clara's location.

Now Bob wants to establish a direct connection with Clara for realizing quantum communication between her and Clara. Then Bob sends a request through a classical channel to Alice. Alice, here as an operator, accepts this request and lets her modes 1 and 2 enter the input ports of the beam splitter $B'S_2$. After interacting with the beam-splitter, we see from Eq.(6) that the output state of the whole system becomes

$$|\Psi\rangle_{1234} = \frac{1}{N'_\alpha}(|\sqrt{2}\alpha\rangle_1|0\rangle_2|\alpha\rangle_3|\alpha\rangle_4 - |0\rangle_1|\sqrt{2}\alpha\rangle_2|\alpha\rangle_3|-\alpha\rangle_4 - |0\rangle_1|-\sqrt{2}\alpha\rangle_2|-\alpha\rangle_3|\alpha\rangle_4 + |-\sqrt{2}\alpha\rangle_1|0\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4). \quad (11)$$

Subsequently Alice makes the two-mode photon number measurement on the modes 1 and 2 at her side and sends the classical information to Bob and Clara to finish the entanglement diversion process. When zero photon is detected at the mode 1 and n (nonzero) photons are detected at the mode 2 simultaneously, the Bob's and Clara's state collapses into

$$|\Psi'\rangle_{34} \longrightarrow |\alpha\rangle_3|-\alpha\rangle_4 + (-1)^n|-\alpha\rangle_3|\alpha\rangle_4. \quad (12)$$

If n (nonzero) photons are detected at the mode 1 and zero photon is detected at the mode 2 simultaneously, the Bob's and Clara's state collapses into

$$|\Phi'\rangle_{34} \longrightarrow |\alpha\rangle_3|\alpha\rangle_4 - (-1)^n|-\alpha\rangle_3|-\alpha\rangle_4. \quad (13)$$

For n being even or odd, Bob and Clara obtain one of the four quasi-Bell states of entangled coherent states

that are defined by the following four entangled coherent states

$$|\Psi'\rangle_{\pm} = \frac{1}{\sqrt{N_{\pm}}}(|\alpha\rangle_3|-\alpha\rangle_4 \pm |-\alpha\rangle_3|\alpha\rangle_4), \quad (14)$$

$$|\Phi'\rangle_{\pm} = \frac{1}{\sqrt{N_{\pm}}}(|\alpha\rangle_3|\alpha\rangle_4 \pm |-\alpha\rangle_3|-\alpha\rangle_4). \quad (15)$$

Then the quantum channel between Bob and Clara has been established.

3. Quantum teleportation

Quantum teleportation is a process in which an unknown quantum state is transmitted from a system at the sender's (Alice's) location to another system at the receiver's (Bob's) location through a quantum channel with the help of some classical information.

Now we begin with a description of our teleportation scheme. We assume that Alice wants to teleport a completely unknown entanglement coherent state

$$|\Phi\rangle_{12} = \epsilon_+|\alpha\rangle_1|\alpha\rangle_2 - \epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2 \quad (16)$$

to Bob, where the amplitudes ϵ_+ and ϵ_- are arbitrary complex numbers. Normalization condition leads to

$$|\epsilon_+|^2 + |\epsilon_-|^2 = 1. \quad (17)$$

The two other entangled coherent states that they share and take as quantum channels are

$$|\Phi\rangle_{35} = \frac{1}{\sqrt{N'_{\alpha}}}(|\alpha\rangle_3|\alpha\rangle_5 - |-\alpha\rangle_3|-\alpha\rangle_5), \quad (18)$$

and

$$|\Phi\rangle_{46} = \frac{1}{\sqrt{N'_{\alpha}}}(|\alpha\rangle_4|\alpha\rangle_6 - |-\alpha\rangle_4|-\alpha\rangle_6). \quad (19)$$

After sharing the quantum channel given by Eq.(18) and Eq.(19) with Bob, first Alice mixes her parts of the entangled states (the modes 3 and 4) with the state to be teleported. Then the initial state of the whole system consisting of the modes 1, 2, 3, 4, 5, 6 is given by

$$\begin{aligned}|\Psi'\rangle =& |\Phi\rangle_{12} \otimes |\Phi\rangle_{35} \otimes |\Phi_{46}\\ =& \frac{1}{\sqrt{N'^3_{\alpha}}}(\epsilon_+|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5|\alpha\rangle_6\\ &- \epsilon_+|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|-\alpha\rangle_4|\alpha\rangle_5|-\alpha\rangle_6\\ &- \epsilon_+|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|\alpha\rangle_4|-\alpha\rangle_5|\alpha\rangle_6\\ &+ \epsilon_+|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4|-\alpha\rangle_5|-\alpha\rangle_6\\ &- \epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5|\alpha\rangle_6\\ &+ \epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|-\alpha\rangle_4|\alpha\rangle_5|-\alpha\rangle_6\\ &+ \epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|\alpha\rangle_4|-\alpha\rangle_5|\alpha\rangle_6\\ &- \epsilon_-|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4|-\alpha\rangle_5|-\alpha\rangle_6). \end{aligned} \quad (20)$$

Note that at this time the modes 1, 2, 3 and 4 are located at the Alice's side and the modes 5 and 6 are at the Bob's side.

Secondly, Alice lets the modes 1 and 3 enter the input ports of the beamsplitter $B'S_3$, and lets the modes 2 and 4 enter the input ports of the beamsplitter $B'S_4$. After interacting with the beamsplitters, the state of the whole system becomes

$$\begin{aligned}|\Psi''\rangle =& \frac{1}{\sqrt{N'^3_{\alpha}}}(\epsilon_+|\sqrt{2}\alpha\rangle_1|\sqrt{2}\alpha\rangle_2|0\rangle_3|0\rangle_4|\alpha\rangle_5|\alpha\rangle_6\\ &- \epsilon_+|\sqrt{2}\alpha\rangle_1|0\rangle_2|0\rangle_3|\sqrt{2}\alpha\rangle_4|\alpha\rangle_5|-\alpha\rangle_6\\ &- \epsilon_+|0\rangle_1|\sqrt{2}\alpha\rangle_2|\sqrt{2}\alpha\rangle_3|0\rangle_4|-\alpha\rangle_5|\alpha\rangle_6\\ &+ \epsilon_+|0\rangle_1|0\rangle_2|\sqrt{2}\alpha\rangle_3|\sqrt{2}\alpha\rangle_4|-\alpha\rangle_5|-\alpha\rangle_6\\ &- \epsilon_-|0\rangle_1|0\rangle_2|-\sqrt{2}\alpha\rangle_3|-\sqrt{2}\alpha\rangle_4|\alpha\rangle_5|\alpha\rangle_6\\ &+ \epsilon_-|0\rangle_1|-\sqrt{2}\alpha\rangle_2|-\sqrt{2}\alpha\rangle_3|0\rangle_4|\alpha\rangle_5|-\alpha\rangle_6\\ &+ \epsilon_-|-\sqrt{2}\alpha\rangle_1|0\rangle_2|0\rangle_3|-\sqrt{2}\alpha\rangle_4|-\alpha\rangle_5|\alpha\rangle_6\\ &- \epsilon_-|-\sqrt{2}\alpha\rangle_1|-\sqrt{2}\alpha\rangle_2|0\rangle_3|0\rangle_4|-\alpha\rangle_5|-\alpha\rangle_6). \end{aligned} \quad (21)$$

Thirdly, Alice makes the four-modes photon number measurement on the modes 1, 2, 3 and 4 at her side and transmits a classical information on the results of the determination through a classical channel to Bob for finishing the teleportation process. When zero photon is detected at the modes 3 and 4 and nonzero photon is detected at the modes 1 and 2 simultaneously, the Bob's state collapses into

$$|\Phi'\rangle_{56} = \epsilon_+|\alpha\rangle_5|\alpha\rangle_6 - (-1)^{n_1+n_2}\epsilon_-|-\alpha\rangle_5|-\alpha\rangle_6, \quad (22)$$

where n_i (i=1, 2, 3 and 4) is the photon numbers to be detected in the mode i. For $n_1 + n_2$ being even, Bob's state becomes

$$|\Phi'\rangle_{56} = \epsilon_+|\alpha\rangle_5|\alpha\rangle_6 - \epsilon_-|-\alpha\rangle_5|-\alpha\rangle_6. \quad (23)$$

Then Bob does nothing in his state and the teleportation scheme works perfectly.

After Alice measures nonzero photon in the modes 1 and 4 and zero photon in the modes 2 and 3 simultaneously, Bob's state becomes

$$|\Phi'\rangle_{56} = \epsilon_+|\alpha\rangle_5|-\alpha\rangle_6 - (-1)^{n_1+n_4}\epsilon_-|-\alpha\rangle_5|\alpha\rangle_6. \quad (24)$$

For $n_1 + n_4$ being even, Bob's state becomes

$$|\Phi'\rangle_{56} = \epsilon_+|\alpha\rangle_5|-\alpha\rangle_6 - \epsilon_-|-\alpha\rangle_5|\alpha\rangle_6. \quad (25)$$

Now, Alice transmits a classical information on the results of the determination through a classical channel to Bob, and Bob makes a local transform $R(\varphi) = \mathrm{e}^{-\mathrm{i}\pi\hat{a}_6^\dagger\hat{a}_6}$ in his state $|\Phi'\rangle_{56}$. This local transform is a phase shifter that makes phase shift by $-\pi$, which performs $|\alpha\rangle \longleftrightarrow |-\alpha\rangle$ on the mode 6. After using the phase shifter, Bob's state becomes

$$|\Phi''\rangle_{56} = \epsilon_+|\alpha\rangle_5|\alpha\rangle_6 - \epsilon_-|-\alpha\rangle_5|-\alpha\rangle_6. \qquad (26)$$

Then the teleportation process is finished.

If nonzero photon is detected in the modes 2 and 3 and zero photon in the modes 1 and 4 simultaneously and $n_2 + n_3$ is even, Bob's state becomes

$$|\Phi'\rangle_{56} = \epsilon_+|-\alpha\rangle_5|\alpha\rangle_6 - \epsilon_-|\alpha\rangle_5|-\alpha\rangle_6. \qquad (27)$$

Following the same steps as before, Bob makes a local transform $R(\varphi) = \mathrm{e}^{-\mathrm{i}\pi\hat{a}_5^\dagger\hat{a}_5}$ in his state $|\Phi'\rangle_{56}$. Then Bob's state becomes

$$|\Phi''\rangle_{56} = \epsilon_+|\alpha\rangle_5|\alpha\rangle_6 - \epsilon_-|-\alpha\rangle_5|-\alpha\rangle_6. \qquad (28)$$

When nonzero photon is detected in the modes 3 and 4 and zero photon in the modes 1 and 2 simultaneously and $n_3 + n_4$ is even, Bob's state becomes

$$|\Phi'\rangle_{56} = \epsilon_+|-\alpha\rangle_5|-\alpha\rangle_6 - \epsilon_-|\alpha\rangle_5|\alpha\rangle_6. \qquad (29)$$

Here Bob makes a local transform

$$R(\varphi) = \mathrm{e}^{-\mathrm{i}\pi(\hat{a}_5^\dagger\hat{a}_5 + \hat{a}_6^\dagger\hat{a}_6)}$$

in his state $|\Phi'\rangle_{56}$. Then Bob's state becomes

$$|\Phi''\rangle_{56} = \epsilon_+|\alpha\rangle_5|\alpha\rangle_6 - \epsilon_-|-\alpha\rangle_5|-\alpha\rangle_6. \qquad (30)$$

4. Summary

In summary, we have presented the simple schemes to divert the entanglement among Alice, Bob and Clara and to teleport a bipartite entangled coherent state. In this scheme, we exploited the important features of the beam splitter transforming entangled coherent state. Comparing our teleportation scheme with that given in Ref.[22], the unknown entangled coherent state to be transmitted does not need to be converted to the direct product of a coherent superposition state $(|\sqrt{2}\alpha\rangle_2 - |-\sqrt{2}\alpha\rangle_2)$ and a vacuum state $|0\rangle_1$ before it is to be teleported. The key factor in our teleportation scheme is photon number measurement that must be sensitive enough to measuring the number of photons and determining whether the number is even or odd. In practice this is difficult, especially in the case of large numbers of photons, but in principle this can be done. In Ref.[22] Wang described a method of distinguishing between even and odd n by coupling the field to a two-level atom through dispersive interaction, which can enhance the possibility to realize this scheme.

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