Vol 15 No 3, March 2006
1009-1963/2006/15(03)/0482-06

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Quantum circuits for realizing deterministic and exact teleportation via two partially entangled pairs of particles*

Li Wen-Dong(李文东), Zhang Jian-Li(张建立), and Gu Yong-Jian(顾永建)†

Department of Physics, Ocean University of China, Qingdao 266071, China

(Received 10 September 2005; revised manuscript received 18 October 2005)

Deterministic and exact teleportation can be achieved via two partially entangled pairs of particles [Gu Y J 2006 *Opt. Comm.* **259** 385]. The key point of the protocol is a generalized measurement described by a positive operator-valued measure, which can be realized by performing a unitary operation in the extended space and a conventional Von Neumann orthogonal measurement. By decomposing the evolution process from the initial state to the final state, we construct the quantum circuits for realizing the unitary operation with quantum Toffoli gates, and thus provide a physical means to realize the teleportation. Our method for constructing quantum circuits differs from the usual methods based on decomposition of unitary matrices, and is convenient for a large class of quantum processes involving generalized measurements.

Keywords: quantum circuit, teleportation, partially entangled state, generalized measurement
PACC: 0365, 4250

1. Introduction

Quantum teleportation,[1] a key ingredient in quantum information,[2,3] is a process in which an unknown quantum state of a particle is transferred from a sender (Alice) to a receiver (Bob) via local operations and classical communication (LOCC) and previously shared entangled pairs of particles. It is known that a single pair of two-state particles in a maximally entangled state is sufficient as a quantum channel for deterministic and exact teleportation (i.e., with unit success probability and unit fidelity) of a two-state particle (qubit). However, deterministic and exact teleportation of a qubit cannot be achieved if only a single partially entangled pair of qubits is available. In this case, we can only achieve approximate teleportation deterministically (with imperfect fidelity but unit success probability) or exact teleportation probabilistically (with unit fidelity but certain probability of failure). A lot of efforts have been made to find protocols maximizing the average fidelity of approximate teleportation [4−6] or optimizing the success probability of exact teleportation.[7−10]

On the other hand, efforts are being made to overcome the difficulty due to non-maximal entanglement to make teleportation deterministic and exact. Recently, a protocol was proposed[11] for deterministic and exact teleportation of a d-state particle (qudit) via a single partially entangled pair of n-state particles (with $n > d$). One of the authors of the present paper presented a protocol[12] for deterministic and exact teleportation of a qubit via two partially entangled pairs of qubits, where Alice's generalized measurement described by a positive operator-valued measure (POVM) lies at the heart. As the most general quantum measurement, the POVM measurement plays a key role in quantum information processing. Recently, a scheme[13] was proposed for teleporting an unknown two-particle entangled state via W class states, in which probabilistic teleportation is achieved by means of optimal discrimination between two non-orthogonal quantum states via a POVM measurement.

Clearly, it is important to obtain a physical means

*Project supported by the National Key Basic Research Special Foundation of China (Grant No 2001CB309305), the National Natural Science Foundation of China (Grant No 10204020) and the Scientific Research Foundation for Introduced Talent of Ocean University of China.

†Corresponding author. E-mail: yjgu@ustc.edu

to carry out this deterministic and exact teleportation. In this paper, we provide a method to realize the teleportation presented by Gu in Ref.[12]. To do this, we only need to show how to realize the required POVM measurement. It is known that a POVM measurement performed on a quantum system A can always be realized by first entangling system A with an ancillary system B via unitary evolution, and then performing a conventional Von Neumann orthogonal measurement on system B.[2,3] As any unitary evolution can be implemented via quantum circuits,[14,15] i.e., quantum logic networks composed of universal quantum logic gates, the key to realizing our teleportation is to construct the quantum circuits for realizing the required unitary evolution. The usual methods for constructing quantum circuits are based on decomposition of unitary matrices. Here we obtain our quantum circuits by using a different method: i.e., by decomposing the evolution process from initial states to final states. One will see that our method is convenient for a large class of quantum processes involving generalized measurements.

The paper is organized as follows. In Section 2, we briefly review the protocol for deterministic and exact teleportation via two partially entangled pairs of particles presented in Ref.[12]. Then, in Section 3, after explaining our basic ideas for constructing quantum circuits, we construct the quantum circuits for realizing the required unitary operation. Finally, we summarize the paper in Section 4.

2. Protocol for deterministic and exact teleportation

First, let us briefly review the protocol we presented in Ref.[12] for deterministic and exact teleportation via two partially entangled pairs of particles. Suppose that Alice and Bob share two partially entangled pairs of two-state particles, the states of which can be written in Schmidt decomposition as

$$|\psi\rangle_{AB} = \sqrt{a}\,|00\rangle_{AB} + \sqrt{1-a}\,|11\rangle_{AB}\,,$$
$$|\varphi\rangle_{A'B'} = \sqrt{b}\,|00\rangle_{A'B'} + \sqrt{1-b}\,|11\rangle_{A'B'}\,, \quad (1)$$

where the Schmidt coefficients a and b are real and positive. The particles A and A' belong to Alice, and particles B and B' to Bob. Without loss of generality, we can set $1/2 < a \leq b < 1$, and we require that a and b satisfy the condition $ab \leq 1/2$. Further, suppose that Alice is to teleport to Bob a single qubit X in the state

$$|\chi\rangle_X = x\,|0\rangle_X + y\,|1\rangle_X\,, \quad |x|^2 + |y|^2 = 1. \quad (2)$$

To start teleportation, Alice performs a generalized measurement on her particles X, A, A', which can be represented by the measurement operators M^{ij} $(i,j = 0,1,2,3)$ satisfying the completeness relation $\sum_{ij} M^{ij\dagger}M^{ij} = I$ (One can refer to Ref.[12] for their specific forms). All these operators $M^{ij\dagger}M^{ij}$ constitute a POVM. Corresponding to different measurement results ij, the initial state collapses into one of the following 16 states:

$$\begin{aligned}
&|000\rangle_{XAA'}\,(x\,|0\rangle_B + y\,|1\rangle_B)\,|0\rangle_{B'}\,, \quad i=0, \quad j=0,\\
&|010\rangle_{XAA'}\,(x\,|1\rangle_B + y\,|0\rangle_B)\,|0\rangle_{B'}\,, \quad i=1, \quad j=0,\\
&|100\rangle_{XAA'}\,(x\,|0\rangle_B - y\,|1\rangle_B)\,|0\rangle_{B'}\,, \quad i=2, \quad j=0,\\
&|110\rangle_{XAA'}\,(x\,|1\rangle_B - y\,|0\rangle_B)\,|0\rangle_{B'}\,, \quad i=3, \quad j=0,\\
&|000\rangle_{XAA'}\,|0\rangle_B\,(x\,|0\rangle_{B'} + y\,|1\rangle_{B'})\,, \quad i=0, \quad j=1,\\
&|010\rangle_{XAA'}\,|0\rangle_B\,(x\,|1\rangle_{B'} + y\,|0\rangle_{B'})\,, \quad i=1, \quad j=1,\\
&|100\rangle_{XAA'}\,|0\rangle_B\,(x\,|0\rangle_{B'} - y\,|1\rangle_{B'})\,, \quad i=2, \quad j=1,\\
&|110\rangle_{XAA'}\,|0\rangle_B\,(x\,|1\rangle_{B'} - y\,|0\rangle_{B'})\,, \quad i=3, \quad j=1,\\
&|000\rangle_{XAA'}\,(x\,|00\rangle_{BB'} + y\,|11\rangle_{BB'})\,, \quad i=0, \quad j=2,\\
&|010\rangle_{XAA'}\,(x\,|11\rangle_{BB'} + y\,|00\rangle_{BB'})\,, \quad i=1, \quad j=2,\\
&|100\rangle_{XAA'}\,(x\,|00\rangle_{BB'} - y\,|11\rangle_{BB'})\,, \quad i=2, \quad j=2,\\
&|110\rangle_{XAA'}\,(x\,|11\rangle_{BB'} - y\,|00\rangle_{BB'})\,, \quad i=3, \quad j=2,\\
&|000\rangle_{XAA'}\,(x\,|1\rangle_B + y\,|0\rangle_B)\,|1\rangle_{B'}\,, \quad i=0, \quad j=3,\\
&|010\rangle_{XAA'}\,(x\,|0\rangle_B + y\,|1\rangle_B)\,|1\rangle_{B'}\,, \quad i=1, \quad j=3,\\
&|100\rangle_{XAA'}\,(x\,|1\rangle_B - y\,|0\rangle_B)\,|1\rangle_{B'}\,, \quad i=2, \quad j=3,\\
&|110\rangle_{XAA'}\,(x\,|0\rangle_B - y\,|1\rangle_B)\,|1\rangle_{B'}\,, \quad i=3, \quad j=3, \quad (3)
\end{aligned}$$

with respective probabilities (see also Ref.[12]). Having received Alice's measurement results ij, Bob tries to reconstruct the state of particle X on particle B or B'. First, he chooses one from particles B and B'. If $j = 0,3$, he chooses particle B and discards particle B'; if $j = 1,2$, he chooses particle B' (when $j = 2$, he must first perform an additional Control-NOT operation on particles B and B', with B' as the control qubit and B as the target qubit, which will change the related states into exactly the same states as when $j = 1$, and then he chooses particle B'). Then he applies Pauli operators to the chosen particle according to ij. For example, when $j = 0$, he applies I, σ_x, σ_z, σ_y respectively to particle B corresponding to $i = 0,1,2,3$. With that the state of particle X is reconstructed on particle B or B' and the teleportation process is completed.

We note that the measurement operators M^{ij} can

be decomposed into

$$M^{ij} = \left(B^i_{XA} \otimes I_{A'}\right)\left(I_X \otimes A^j_{AA'}\right), \qquad (4)$$

where B^i_{XA} are Bell measurement operators acting on particles X and A, and $A^j_{AA'}$ are operators acting on particles A and A'. In the basis $\{|00\rangle_{AA'}, |01\rangle_{AA'}, |10\rangle_{AA'}, |11\rangle_{AA'}\}$, $A^j_{AA'}$ read

$$A^0_{AA'} = \mathrm{diag}\left(\sqrt{\frac{1-a}{a}}, 0, 1, 0\right),$$

$$A^1_{AA'} = \begin{pmatrix} \sqrt{\frac{2a-1}{2ab}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{2a-1}{2a(1-b)}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A^2_{AA'} = \begin{pmatrix} \sqrt{\frac{(2a-1)(2b-1)}{2ab}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{(2a-1)(2b-1)}{2(1-a)(1-b)}} \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A^3_{AA'} = \begin{pmatrix} 0 & 0 & 0 & \sqrt{\frac{1-2ab}{2(1-a)(1-b)}} \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{1-2ab}{2a(1-b)}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (5)$$

It is easy to check that $\sum_j A^{j\dagger}_{AA'} A^j_{AA'} = I_{AA'}$, so $A^j_{AA'}$ are measurement operators of a generalized measurement on particles A and A'. Equation (4) means that the generalized measurement on particles X, A, A' described by M^{ij} is equivalent to a generalized measurement on particles A, A' described by $A^j_{AA'}$ followed by a Bell measurement on particles X, A.

3. Quantum circuits for deterministic and exact teleportation

It is easy to see from the above section that the key to the deterministic and exact teleportation is to implement the POVM measurement described by the operators $A^j_{AA'}$. To do this, we extend the Hilbert space, i.e., introduce two ancillary qubits C and C' having an orthonormal basis $\{|j\rangle_{CC'},\ j = 0, 1, 2, 3\}$ in one-to-one correspondence with the possible results of the measurement we wish to implement, where j in kets represents a binary string, e.g. $|3\rangle_{CC'} = |11\rangle_{CC'}$. Let C and C' be prepared in the initial state $|11\rangle_{CC'}$ and define an operator U acting on particles A, A', C, C' by

$$U|\psi\rangle_{AB} \otimes |\varphi\rangle_{A'B'} |11\rangle_{CC'} = \sum_j A^j_{AA'} |\psi\rangle_{AB} \otimes |\varphi\rangle_{A'B'} |j\rangle_{CC'}. \qquad (6)$$

Using the completeness relation $\sum_j A^{j\dagger}_{AA'} A^j_{AA'} = I_{AA'}$, we can see that U preserves inner products, therefore can be extended to a unitary operator. We first carry out the unitary operation, and then perform a Von Neumann orthogonal measurement on the ancillary qubits. From Eq.(6), it follows that the unitary evolution together with the conventional orthogonal measurement is equivalent to the required generalized measurement. As is well known, an arbitrary unitary operation can be implemented with a quantum circuit composed of universal quantum logic gates, so our main task is to construct the quantum circuits for implementing the unitary evolution in Eq.(6).

The usual methods for constructing quantum circuits are by first obtaining the explicit unitary representation, then decomposing the unitary representa-

tion into universal quantum logic operations. Here we obtain the required quantum circuits by using a different method, i.e., by decomposing the evolution process from initial states to final states. The idea is as follows. Deutsch[14] showed that a simple generalization of the Toffoli gate is sufficient as a universal gate for quantum computation. A quantum Toffoli gate is an $(m+1)$-bit gate that applies a unitary operation to the $(m+1)$th bit (target bit) if and only if the other m bits (control bits) are all in the state $|1\rangle$. Barenco *et al*[15] further demonstrated that an arbitrary quantum Toffoli gate can be executed by the combination of one-bit quantum gates and two-bit Controlled-NOT gates. The key point of our method is to decompose the evolution process from initial states to final states using generalized quantum Toffoli gates, namely quantum gates that apply unitary operations to some bits (target bits) under the control of the other bits (control bits). This decomposition is repeated until each state change can be implemented by a quantum Toffoli gate. Specifically, suppose that the initial state $|\psi_0\rangle$ evolves unitarily into the final state $|\psi_e\rangle$. In the first cycle, we decompose this process into multiple steps such as $|\psi_0\rangle \Rightarrow |\psi_1\rangle \Rightarrow ... \Rightarrow |\psi_i\rangle \Rightarrow ... \Rightarrow |\psi_e\rangle$, by the analysis of available expressions such as Eq.(6). The state change in each step can be realized by applying a unitary operation to some bits under the control of the other bits. Then in the second cycle, we take $|\psi_{i-1}\rangle$ and $|\psi_i\rangle$ as the new initial state and final state respectively, and repeat the above decomposition. Evidently, in this cycle, fewer bits (namely only the target bits in the last cycle) are involved, and they are further divided into target bits and control bits. The decomposition is repeated until only one target bit is left in each step, i.e., each state change can be implemented by a quantum Toffoli gate. In the following we illustrate the method by constructing the quantum circuits for realizing the unitary evolution defined by Eq.(6).

For the unitary evolution in Eq.(6), the initial state reads

$$\begin{aligned}|\psi_0\rangle =&\sqrt{ab}\,|110000\rangle \\ &+\sqrt{a(1-b)}\,|110101\rangle \\ &+\sqrt{(1-a)b}\,|111010\rangle \\ &+\sqrt{(1-a)(1-b)}\,|111111\rangle\,, \end{aligned} \tag{7}$$

where and below, the particles are arranged in the order $CC'AA'BB'$, and we omit the subscripts indicating particles. According to Eq.(6), the components of $|\psi_0\rangle$ evolve in the following manner:

$$\begin{aligned}\sqrt{ab}\,|110000\rangle \Rightarrow&\sqrt{(1-a)b}\,|000000\rangle \\ &+\sqrt{(2a-1)/2}\,|010000\rangle \\ &+\sqrt{(2a-1)(2b-1)/2}\,|100000\rangle\,, \end{aligned} \tag{8}$$

$$\sqrt{(1-a)b}\,|111010\rangle \Rightarrow \sqrt{(1-a)b}\,|001010\rangle\,, \tag{9}$$

$$\begin{aligned}\sqrt{a(1-b)}\,|110101\rangle \Rightarrow&\sqrt{(2a-1)/2}\,|011001\rangle \\ &+\sqrt{(1-2ab)/2}\,|111001\rangle\,, \end{aligned} \tag{10}$$

$$\begin{aligned}&\sqrt{(1-a)(1-b)}\,|111111\rangle \\ \Rightarrow&\sqrt{(2a-1)(2b-1)/2}\,|101011\rangle \\ &+\sqrt{(1-2ab)/2}\,|110011\rangle\,. \end{aligned} \tag{11}$$

As a result, $|\psi_0\rangle$ changes into

$$\begin{aligned}|\psi_e\rangle =&\sqrt{(1-a)b}\,|000000\rangle \\ &+\sqrt{(1-a)b}\,|001010\rangle \\ &+\sqrt{(2a-1)/2}\,|010000\rangle \\ &+\sqrt{(2a-1)/2}\,|011001\rangle \\ &+\sqrt{(2a-1)(2b-1)/2}\,|100000\rangle \\ &+\sqrt{(2a-1)(2b-1)/2}\,|101011\rangle \\ &+\sqrt{(1-2ab)/2}\,|110011\rangle \\ &+\sqrt{(1-2ab)/2}\,|111001\rangle\,. \end{aligned} \tag{12}$$

Correspondingly, we decompose the evolution process from $|\psi_0\rangle$ to $|\psi_e\rangle$ into four steps: $|\psi_0\rangle \Rightarrow |\psi_1\rangle \Rightarrow |\psi_2\rangle \Rightarrow |\psi_3\rangle \Rightarrow |\psi_e\rangle$, with each step implementing one of the evolutions (8)–(11) of the four components while not affecting the evolutions of the other components (For simplicity, here we do not explicitly write out $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$). This can be realized by applying unitary operations to CC' under the control of AA'. The evolutions (8)–(11) correspond respectively to the control conditions that AA' is in the state $|00\rangle$, $|10\rangle$, $|01\rangle$ and $|11\rangle$. In the first step, the evolution (8) is realized by applying to CC' a unitary operation U_1 satisfying

$$\begin{aligned}U_1\sqrt{ab}\,|11\rangle =&\sqrt{(1-a)b}\,|00\rangle+\sqrt{(2a-1)/2}\,|01\rangle \\ &+\sqrt{(2a-1)(2b-1)/2}\,|10\rangle\,, \end{aligned} \tag{13}$$

if and only if AA' is in $|00\rangle$ (namely under the control of AA'). Similarly, in the second step the evolution (9) is realized by applying a NOT operation to C and C' separately, if and only if AA' is in $|10\rangle$. For the evolution (10), in the third step we first apply to C a unitary operation U_2 satisfying

$$\begin{aligned}U_2\sqrt{a(1-b)}\,|1\rangle =&\sqrt{(2a-1)/2}\,|0\rangle \\ &+\sqrt{(1-2ab)/2}\,|1\rangle\,, \end{aligned} \tag{14}$$

under the condition that AA' is in $|01\rangle$. This operation will transform

$$\sqrt{a(1-b)}\,|110101\rangle$$

into

$$\sqrt{(2a-1)/2}\,|010101\rangle+\sqrt{(1-2ab)/2}\,|110101\rangle\,.$$

Then we apply a swap operation U_3 to AA' under the condition that CC' is in $|01\rangle$. This operation will transform $\sqrt{(2a-1)/2}\,|010101\rangle$ into $\sqrt{(2a-1)/2}\,|011001\rangle$ while not affecting the evolutions of the other components. We leave the transformation

$$\sqrt{(1-2ab)/2}\,|110101\rangle\Rightarrow\sqrt{(1-2ab)/2}\,|111001\rangle$$

to the next step. In the basis $|0\rangle,|1\rangle$, the matrix of U_2 can be written as

$$U_2=\begin{pmatrix}\sqrt{\dfrac{1-2ab}{2a(1-b)}} & \sqrt{\dfrac{2a-1}{2a(1-b)}}\\ -\sqrt{\dfrac{2a-1}{2a(1-b)}} & \sqrt{\dfrac{1-2ab}{2a(1-b)}}\end{pmatrix}. \qquad (15)$$

As for the evolution (11), in the fourth step we first apply to C' a unitary operation U_4 satisfying

$$\begin{aligned}&U_4\sqrt{(1-a)(1-b)}\,|1\rangle\\ =&\sqrt{(2a-1)(2b-1)/2}\,|0\rangle+\sqrt{(1-2ab)/2}\,|1\rangle\,,\end{aligned} \qquad (16)$$

under the condition that AA' is in $|11\rangle$. This operation will transform

$$\sqrt{(1-a)(1-b)}\,|111111\rangle$$

into

$$\begin{aligned}&\sqrt{(2a-1)(2b-1)/2}\,|101111\rangle\\ &+\sqrt{(1-2ab)/2}\,|111111\rangle\,.\end{aligned}$$

Then we consecutively apply a NOT operation to A' under the condition that $CC'A$ is in $|101\rangle$, and a NOT operation to A and A' separately under the condition that CC' is in $|11\rangle$. These operations will transform $\sqrt{(2a-1)(2b-1)/2}\,|101111\rangle$, $\sqrt{(1-2ab)/2}\,|111111\rangle$ and $\sqrt{(1-2ab)/2}\,|110101\rangle$ into

$$\sqrt{(2a-1)(2b-1)/2}\,|101011\rangle\,,$$

$$\sqrt{(1-2ab)/2}\,|110011\rangle$$

and $\sqrt{(1-2ab)/2}\,|111001\rangle$ respectively, while not affecting the evolutions of the other components. In the basis $|0\rangle,|1\rangle$, the matrix of U_4 can be written as

$$U_4=\begin{pmatrix}\sqrt{\dfrac{1-2ab}{2(1-a)(1-b)}} & \sqrt{\dfrac{(2a-1)(2b-1)}{2(1-a)(1-b)}}\\ -\sqrt{\dfrac{(2a-1)(2b-1)}{2(1-a)(1-b)}} & \sqrt{\dfrac{1-2ab}{2(1-a)(1-b)}}\end{pmatrix}. \qquad (17)$$

With that we complete the first cycle. The quantum circuit constructed in the first cycle is shown in Fig.1. The meaning of the symbols is also illustrated in Fig.1, in which X represents a quantum NOT gate.

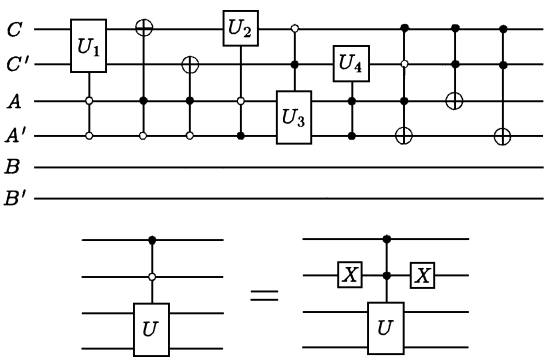


Fig.1. The quantum circuit constructed in the first cycle for realizing the unitary operation in Eq.(6).

In the second cycle, we use the same procedure to U_1 in Fig.1. It is not difficult to obtain that U_1 can be decomposed as in Fig.2, where

$$U_{11}=\begin{pmatrix}\sqrt{\dfrac{2a-1}{2ab}} & \sqrt{\dfrac{2ab-2a+1}{2ab}}\\ -\sqrt{\dfrac{2ab-2a+1}{2ab}} & \sqrt{\dfrac{2a-1}{2ab}}\end{pmatrix},$$

$$U_{12}=\begin{pmatrix}\sqrt{\dfrac{(2a-1)(2b-1)}{2ab-2a+1}} & \sqrt{\dfrac{2(1-a)b}{2ab-2a+1}}\\ -\sqrt{\dfrac{2(1-a)b}{2ab-2a+1}} & \sqrt{\dfrac{(2a-1)(2b-1)}{2ab-2a+1}}\end{pmatrix}, \qquad (18)$$

and we here do not give the detailed analysis of the decomposition of U_1. As for U_3 in Fig.1, it is easy to know from the structure of a quantum swap operation[2] that it can be decomposed as in Fig.3.

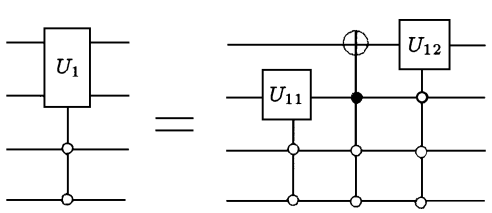


Fig.2. The decomposition of U_1 in Fig.1.

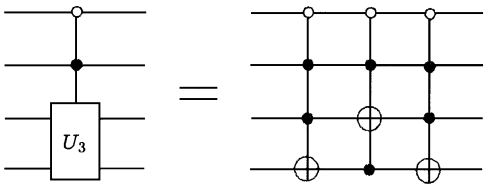


Fig.3. The decomposition of U_3 in Fig.1.

Until now the quantum circuit has been decomposed into an array of quantum Toffoli gates, therefore it can further be decomposed into the combination of one-bit quantum gates and two-bit Controlled-NOT gates. The final quantum circuit implementing the generalized measurement on AA' described by the operators $A^j_{AA'}$ in Eq.(5) is shown in Fig.4.

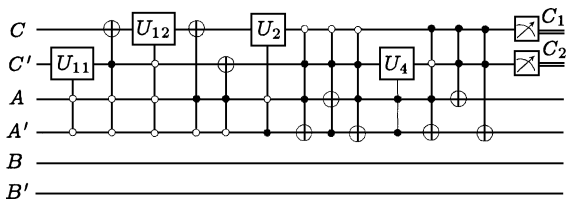


Fig.4. The final quantum circuit for realizing the generalized measurement described by the operators $A^j_{AA'}$ in Eq.(5).

4. Conclusions

Quantum teleportation has been the subject of intensive study because of the key role it plays in quantum information science. Since non-maximal entanglement corrupts the quality of teleportation, efforts have been made to overcome the difficulty. Although we have presented a protocol[12] for deterministic and exact teleportation via two partially entangle pairs of particles, it is more important to find ways to implement the protocol. The implementation of the generalized measurement in the protocol demands a small-scale quantum computer, namely quantum circuit composed of universal quantum gates, to implement the corresponding unitary operation. In this paper, we have constructed the quantum circuits. Alice can use the constructed quantum computer together with orthogonal measurements to implement the generalized measurement. Then she sends the measurement results to Bob, who recovers the exact state in a deterministic fashion according to the results he has received.

In Section 3, we construct the quantum circuits by decomposing the evolution process from initial states to final states. This method is different from the usual methods based on decomposition of unitary matrices. The idea of the method is to decompose the evolution process using generalized quantum Toffoli gates, namely quantum gates that apply unitary operations to some bits under the control of the other bits. We repeat this decomposition until all circuit components reduce to quantum Toffoli gates. It can be seen that this method is convenient, especially for quantum processes involving generalized measurements.

Finally, we point out that by multiplying all the unitary representations obtained in each step, we can finally obtain the whole explicit unitary representation mapping the initial state to the final state, while for the usual methods, we decompose the whole unitary representation into a multiplication of many unitary representations. From this we can see that these two kinds of method go in opposite directions.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[3] Preskill J 1998 *Quantum Information and Computation* (CIT)

[4] Vidal G, Jonathan D and Nielsen M A 2000 *Phys. Rev.* A **62** 012304

[5] Henderson L, Hardy L and Vedral V 2000 *Phys. Rev.* A **61** 062306

[6] Banaszek K 2000 *Phys. Rev.* A **62** 024301

[7] Li W L, Li C F and Guo G C 2000 *Phys. Rev.* A **61** 034301

[8] Mor T and Horodecki P 1999 quant-ph/9906039

[9] Son W, Lee J, Kim M S and Park Y J 2001 *Phys. Rev.* A **64** 064304

[10] Gu Y J, Zheng Y Z and Guo G C 2002 *Phys. Lett.* A **296** 157

[11] Gour G 2004 *Phys. Rev.* A **70** 042301

[12] Gu Y J 2006 *Opt. Comm.* **259** 385

[13] Cao Z L and Song W 2005 *Physica* A **347** 177

[14] Deutsch D 1985 *Proc. R. Soc. London* Ser. A **400** 97
Deutsch D 1989 *Proc. R. Soc. London* Ser. A **425** 73

[15] Barenco A, Bennett C H, Cleve R, Di Vincenzo D P, Margolus N, Shor P, Sleator T, Smolin J A and Weinfurter H 1995 *Phys. Rev.* A **52** 3457