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Teleportation of a two-atom entangled state using a single EPR pair in cavity QED*

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We propose a scheme for teleporting a two-atom entangled state in cavity quantum electrodynamics (QED). In the scheme, we choose a single Einstein–Podolsky–Rosen (EPR) pair as the quantum channel which is shared by the sender and the receiver. By using the atom–cavity-field interaction and introducing an additional atom, we can teleport the two-atom entangled state successfully with a probability of 1.0. Moreover, we show that the scheme is insensitive to cavity decay and thermal field.

Keywords: quantum teleportation, two-atom entangled state, cavity QED
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1. Introduction

In quantum teleportation,[1] an unknown quantum state will be sent from the sender to the receiver via a quantum channel with the help of classical communication. In the process, the original particle, the carrier of the unknown quantum state, is not teleported. In order to realize teleportation, the sender Alice can make the original particle interact with another system, or "ancilla", initially in a known state. After the interaction, the original particle is left in a standard state and the "ancilla" is in an unknown state containing the information about the original quantum state. If Alice sends Bob the "ancilla", Bob can prepare a replica of the original state. Thus, the unknown quantum state is teleported to Bob.

The first quantum teleporting protocol was suggested by Bennett *et al*[1] in 1993. Since then, many teleporting schemes have been proposed by many authors.[2−20] Recent years, several theoretical schemes for teleporting a two-particle entangled state have been proposed.[21−24] Lu and Guo[21] realized the teleportation of the two-particle entangled state by using two Einstein–Podolsky–Rosen (EPR) pairs as the quantum channel. Shi *et al*[22] and Ye *et al*[23] realized the teleportation by using a non-maximally entangled Greninger–Horne–Zeilinger (GHZ) state and a maximally entangled GHZ state, respectively. Cao and Song[24] also realized the teleportation by using a non-maximally entangled W state. In these schemes, however, the quantum channels are complicated to some extent. Moreover, it is well known that the entanglement in three qubits is more intricate than that in two qubits. In order to reduce the requirements for the quantum channel, Cola and Paris[25] proposed a scheme for teleporting an unknown bipartite entangled state using a single EPR pair with the help of an additional qubit. But, the joint Bell measurement is needed for teleportation in these schemes.[21−25] Nevertheless, for atomic states, the joint Bell measurement is still a formidable problem for quantum teleportation. In 2004, Ye and Guo[26] suggested a scheme for teleporting a two-atom entangled state without the joint Bell measurement. In their scheme, the GHZ state is chosen as the quantum channel.

In this paper, we use a single EPR pair as the quantum channel to teleport a two-atom entangled state in cavity quantum electrodynamics (QED). In our scheme, we not only reduce the requirements for the quantum channel by using a single EPR pair as the

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quantum channel, but also avoid the joint Bell measurement by using the cavity QED techniques. Using this scheme, we can realize the teleportation with probability of 1.0.

Suppose that Alice has a two-atom entangled state in the following form:

$$|\psi\rangle_{12} = (\alpha|ee\rangle + \beta|gg\rangle)_{12}, \tag{1}$$

and she wants to teleport this state to Bob. We assume that Alice and Bob only share a single EPR pair

$$|\psi\rangle_{34} = \frac{1}{\sqrt{2}}(|ee\rangle + |gg\rangle)_{34}, \tag{2}$$

where atom 3 belongs to Alice while atom 4 belongs to Bob. Here, atoms 1, 2, 3, and 4 are identical two-level atoms. The initial state of the whole system is given by

$$|\psi\rangle_{1234} = (\alpha|ee\rangle + \beta|gg\rangle)_{12} \otimes \frac{1}{\sqrt{2}}(|ee\rangle + |gg\rangle)_{34}. \tag{3}$$

Then, in order to realize quantum teleportation, Alice sends atoms 1 and 3, driven by a classical field, through a single-mode cavity. In the rotating-wave approximation, the interaction Hamiltonian between atoms and the single-mode cavity can be described as follows:[27]

$$\begin{aligned} H =& \omega_0 \sum_{j=1}^{2} S_{z,j} + \omega_a a^\dagger a \\ &+ \sum_{j=1}^{2} [g(a^\dagger S_j^- + a S_j^\dagger) \\ &+ \Omega(S_j^\dagger \mathrm{e}^{-\mathrm{i}\omega t} + S_j^- \mathrm{e}^{\mathrm{i}\omega t})], \end{aligned} \tag{4}$$

where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^\dagger = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, and $|e_j\rangle$, $|g_j\rangle$ are the excited and ground states of the jth atom, respectively. $a^\dagger$, a are the creation and annihilation operators for the cavity mode, g is the atom-cavity coupling strength, Ω is the Rabi frequency, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Supposing ω_0=ω, in the interaction picture, the interaction Hamiltonian is[27]

$$\begin{aligned} H_i =& \Omega \sum_{j=1,2} (S_j^\dagger + S_j^-) \\ &+ g \sum_{j=1}^{2} (\mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^- + \mathrm{e}^{\mathrm{i}\delta t} a S_j^\dagger), \end{aligned} \tag{5}$$

where δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_a. If $\Omega \gg \delta, g$ and $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. So the effect of cavity decay is eliminated. Then in the interaction picture, the effective interaction Hamiltonian is[27]

$$\begin{aligned} H_e =& \frac{\lambda}{2}\Bigg[\sum_{j=1}^{2}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) \\ &+ \sum_{i,j=1,i\neq j}^{2} (S_i^\dagger S_j^- + S_i^\dagger S_j^\dagger + \mathrm{H.c})\Bigg], \end{aligned} \tag{6}$$

where $\lambda = g^2/2\delta$. We note that the effective Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state. So the effect of the thermal field is also eliminated.

Then the evolution operator of the system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}H_0 t}\mathrm{e}^{-\mathrm{i}H_e t}, \tag{7}$$

where $H_0 = \Omega \sum_{j=1,2} (S_j^\dagger + S_j^-)$.

After an interaction time t, the state of the whole system will undergo the following evolution:

$$\begin{aligned} |\psi\rangle_{1234} \longrightarrow & \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\lambda t}\{\alpha[\cos\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1)(\cos\Omega t|e\rangle_3 - \mathrm{i}\sin|g\rangle_3) \\ & - \mathrm{i}\sin\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1)(\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3)]|ee\rangle_{24} \\ & + \alpha[\cos\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1)(\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3) \\ & - \mathrm{i}\sin\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1)(\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3)]|eg\rangle_{24} \\ & + \beta[\cos\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1)(\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3) \\ & - \mathrm{i}\sin\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1)(\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3)]|ge\rangle_{24} \\ & + \beta[\cos\lambda t(\cos\Omega t|g\rangle_1 - \mathrm{i}\sin\Omega t|e\rangle_1)(\cos\Omega t|g\rangle_3 - \mathrm{i}\sin\Omega t|e\rangle_3) \\ & - \mathrm{i}\sin\lambda t(\cos\Omega t|e\rangle_1 - \mathrm{i}\sin\Omega t|g\rangle_1)(\cos\Omega t|e\rangle_3 - \mathrm{i}\sin\Omega t|g\rangle_3)]|gg\rangle_{24}\}. \end{aligned} \tag{8}$$

If we choose the interaction time t and Rabi frequency Ω appropriately so that $\lambda t = \pi/4$, $\Omega t = \pi$, then Eq.(8) becomes

$$\begin{aligned}|\psi\rangle_{1234} \longrightarrow & \frac{1}{2}[|ee\rangle_{13}(\alpha|ee\rangle - \mathrm{i}\beta|gg\rangle)_{24} \\ & + |eg\rangle_{13}(\alpha|eg\rangle - \mathrm{i}\beta|ge\rangle)_{24} \\ & + |ge\rangle_{13}(-\mathrm{i}\alpha|eg\rangle + \beta|ge\rangle)_{24} \\ & + |gg\rangle_{13}(-\mathrm{i}\alpha|ee\rangle + \beta|gg\rangle)_{24}], \end{aligned} \tag{9}$$

where the global phase factor is discarded. Now, Alice detects her atoms 1 and 3. If the result is $|ee\rangle_{13}$, the state of atoms 2 and 4 will be

$$|\psi\rangle_{24} \longrightarrow \frac{1}{2}(\alpha|ee\rangle - \mathrm{i}\beta|gg\rangle)_{24}. \tag{10}$$

For Bob to restructure the initial state, firstly, Alice introduces an additional identical two-level atom 5, initially prepared in the ground state $|g\rangle_5$, and sends atoms 5 and 2 through another identical cavity simultaneously. The evolution operator is the same as Eq.(7). After an interaction time t', the state of atoms 2, 4, and 5 will be

$$\begin{aligned}|\psi\rangle_{245} \longrightarrow & \frac{1}{2}\{\alpha[\cos\lambda t'(\cos\Omega t'|e\rangle_2 - \mathrm{i}\sin\Omega t'|g\rangle_2)(\cos\Omega t'|g\rangle_5 - \mathrm{i}\sin\Omega t'|e\rangle_5) \\ & - \mathrm{i}\sin\lambda t'(\cos\Omega t'|g\rangle_2 - \mathrm{i}\sin\Omega t'|e\rangle_2)(\cos\Omega t'|e\rangle_5 - \mathrm{i}\sin\Omega t'|g\rangle_5)]|e\rangle_4 \\ & - \mathrm{i}\beta[\cos\lambda t'(\cos\Omega t'|g\rangle_2 - \mathrm{i}\sin\Omega t'|e\rangle_2)(\cos\Omega t'|g\rangle_5 - \mathrm{i}\sin\Omega t'|e\rangle_5) \\ & - \mathrm{i}\sin\lambda t'(\cos\Omega t'|e\rangle_2 - \mathrm{i}\sin\Omega t'|g\rangle_2)(\cos\Omega t'|e\rangle_5 - \mathrm{i}\sin\Omega t'|g\rangle_5)]|g\rangle_4\}. \end{aligned} \tag{11}$$

If $\lambda t' = \pi/4$ and $\Omega t' = \pi$ are satisfied, Eq.(11) becomes

$$\begin{aligned}|\psi\rangle_{245} \longrightarrow & \frac{1}{2\sqrt{2}}[|e\rangle_2(\alpha|eg\rangle - \beta|ge\rangle)_{45} \\ & + |g\rangle_2(-\mathrm{i}\alpha|ee\rangle - \mathrm{i}\beta|gg\rangle)_{45}]. \end{aligned} \tag{12}$$

Then, Alice sends atom 5 to Bob and detects atom 2. With the two atoms (atom 4 and 5) in his possession, Bob can restructure the initial state after he knows Alice's detection result by classical communication. No matter what result ($|e\rangle_2$ or $|g\rangle_2$) it is , Bob only needs to make standard rotations on his atoms to restructure the initial state of Eq.(1) with a probability of $\frac{1}{4}$. It can be easily proven that when Alice's first detection result is $|eg\rangle_{13}$, or $|ge\rangle_{13}$, or $|gg\rangle_{13}$, the initial state will be restructured successfully with the similar procedure as the above. So, the total success probability is 1.0.

In conclusion, we have proposed a scheme for teleporting a two-atom entangled state in cavity QED. In our scheme, we reduce not only the requirements for the quantum channel by using a single EPR pair as the quantum channel instead of two EPR pairs in Ref.[21], a non-maximally entangled GHZ-state in Ref.[22], a maximally entangled GHZ state in Ref.[23], and a non-maximally entangled W-state in Ref.[24], but also avoid the joint Bell measurement by using cavity QED techniques. By introducing an additional atom and using the atom–cavity-field interaction, we have realized the teleportation of the two-atom entangled state successfully with a probability of 1.0. The scheme is insensitive to the cavity decay and thermal field.

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