Vol 15 No 3, March 2006
1009-1963/2006/15(03)/0475-03
Chinese Physics
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A hyperbolic function approach to constructing exact solitary wave solutions of the Hybrid lattice and discrete mKdV lattice*

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(Received 12 October 2005; revised manuscript received 26 October 2005)

Some new exact solitary wave solutions of the Hybrid lattice and discrete mKdV lattice are obtained by using a hyperbolic function approach. This approach can also be applied to other nonlinear differential-difference equations.

Keywords: hyperbolic function approach, nonlinear differential-difference equation, exact solitary wave solution

PACC: 0340K, 0290

1. Introduction

The soliton theory is an important branch of nonlinear science. Finding explicit and exact solutions, in particular, solitary wave solutions of nonlinear partial differential equations (PDEs) in mathematical physics plays an important role in soliton theory. In recent years, several powerful direct algebraic methods have been employed to obtain many types of exact solutions of PDEs with the help of computer symbolic systems, such as, the hyperbolic tangent function expansion method,[1] the homogeneous balance method,[2] Jacobi elliptic function method,[3−5] F-expansion method,[6−8] the auxiliary equation method,[9] the hyperbolic function method,[10−12] and so on. But a little work has been done on the exact solutions for the nonlinear differential-difference equations (DDEs). Recently, some exact solutions of hybrid lattice are obtained by use of the improved tanh-method in Refs.[1, 13]. In Ref.[5], the Jacobian elliptic function method was applied to obtain the periodic wave solutions for the discrete mKdV lattice. In this paper, we shall obtain some new exact solitary wave solutions of the Hybrid lattice and discrete mKdV lattice by using a hyperbolic function approach.

2. Exact solutions of Hybrid lattice

The Hybrid lattice reads[1,13]

$$\dot{u}_n(t) = (1 + \alpha u_n + \beta u_n^2)(u_{n-1} - u_{n+1}), \qquad (1)$$

where α and β are constants.

By use of $u_n(x,t,n) = u_n(\xi), \xi = dn + c_1 t + c_2 x + c_0$, Eq.(1) becomes

$$c_1 \frac{\mathrm{d}u_n(\xi)}{\mathrm{d}\xi} = (1+\alpha u_n(\xi)+\beta u_n^2(\xi))(u_{n-1}(\xi)-u_{n+1}(\xi)). \qquad (2)$$

According to the homogeneous balance principle, we can choose

$$u_n(\xi) = A_0 + A_1 \sinh(\xi) + B_1 \cosh(\xi), \qquad (3)$$

$$\frac{\mathrm{d}u_n(\xi)}{\mathrm{d}\xi} = A_1 \cosh(\xi) + B_1 \sinh(\xi), \qquad (4)$$

*Project supported by the National Natural Science Foundation of China (Grant No 10461006), the Natural Science Foundation (Grant No 200408020103), the High Education Science Research Program (Grant No NJ02035) of Inner Mongolia, China and the Youth Foundation (Grant No QN004024) of Inner Mongolia Normal University, China.

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$$u_{n-1}(\xi) = A_0 + \frac{A_1(\sinh(\xi)\cosh(d) + \cosh(\xi)\sinh(d) + d)}{\cosh(\xi)\sinh(d) + a\,\cosh(\xi)\cosh(d) + b\,\sinh(\xi)\sinh(d) + c\,\sinh(\xi)\cosh(d)} + \frac{B_1(\sinh(\xi)\cosh(d) - \cosh(\xi)\sinh(d) + d)}{\cosh(\xi)\sinh(d) + a\,\cosh(\xi)\cosh(d) + b\,\sinh(\xi)\sinh(d) + c\,\sinh(\xi)\cosh(d)}, \tag{5}$$

$$u_{n+1}(\xi) = A_0 + \frac{A_1(\sinh(\xi)\cosh(d) - \cosh(\xi)\sinh(d) + (d+1))}{\cosh(\xi)\sinh(d) + a\,\cosh(\xi)\cosh(d) + b\,\sinh(\xi)\sinh(d) + c\,\sinh(\xi)\cosh(d)} + \frac{B_1(\sinh(\xi)\cosh(d) + \cosh(\xi)\sinh(d) + (d+1))}{\cosh(\xi)\sinh(d) + a\,\cosh(\xi)\cosh(d) + b\,\sinh(\xi)\sinh(d) + c\,\sinh(\xi)\cosh(d)}, \tag{6}$$

where A_0, A_1, B_1, a, b, c are constants to be determined later, d is an arbitrary constant. Substituting Eqs.(3)–(6) into Eq.(2), using $\cosh^2(\xi) = 1 + \sinh^2(\xi)$ and setting the coefficients of power terms like $\cosh^j(\xi)\sinh^i(\xi)$, (i=0, 1, 2, and 3; j=0 and 1) to zero yield a system of algebraic equations as follows:

$$\begin{aligned}
&A_1 + B_1 + A_0A_1\alpha + A_0B_1\alpha + A_0^2A_1\beta + A_0^2B_1\beta + A_1B_1^2\beta + B_1^3\beta + aA_1c_1\cosh(d) + A_1c_1 \\
&\times \sinh(d) - 2A_1B_1\alpha\sinh(d) + 2B_1^2\alpha\sinh(d) - 4A_0A_1B_1\beta\sinh(d) + 4A_0B_1^2\beta\sinh(d) = 0, \\
&A_1B_1\alpha + B_1^2\alpha + 2A_0A_1B_1\beta + 2A_0B_1^2\beta - 2A_1\sinh(d) + 2B_1\sinh(d) - 2A_0A_1\alpha\sinh(d) + 2A_0 \\
&\times B_1\alpha\sinh(d) - 2A_0^2A_1\beta\sinh(d) + 2A_0^2B_1\beta\sinh(d) - 2A_1B_1^2\beta\sinh(d) + 2B_1^3\beta\sinh(d) = 0, \\
&A_1^2\alpha + A_1B_1\alpha + 2A_0A_1^2\beta + 2A_0A_1B_1\beta - 4A_1^2B_1\beta\sinh(d) + 4A_1B_1^2\beta\sinh(d) = 0, \\
&2A_1^2B_1\beta + 2A_1B_1^2\beta + aB_1c_1\cosh(d) + A_1cc_1\cosh(d) + A_1bc_1\sinh(d) + B_1c_1\sinh(d) \\
&- 2A_1^2\alpha\sinh(d) + 2A_1B_1\alpha\sinh(d) - 4A_0A_1^2\beta\sinh(d) + 4A_0A_1B_1\beta\sinh(d) = 0, \\
&A_1^3\beta + A_1^2B_1\beta + A_1B_1^2\beta + B_1^3\beta + aA_1c_1\cosh(d) + B_1cc_1\cosh(d) + A_1c_1\sinh(d) + bB_1c_1 \\
&\times \sinh(d) - 2A_1B_1\alpha\sinh(d) + 2B_1^2\alpha\sinh(d) - 4A_0A_1B_1\beta\sinh(d) + 4A_0B_1^2\beta\sinh(d) = 0, \\
&- 2A_1^3\beta\sinh(d) + 2A_1^2B_1\beta\sinh(d) - 2A_1B_1^2\beta\sinh(d) + 2B_1^3\beta\sinh(d) = 0, \\
&- 4A_1^2B_1\beta\sinh(d) + 4A_1B_1^2\beta\sinh(d) = 0.
\end{aligned}$$

From the above, a type of solution is obtained with the help of Mathematica:

$$A_0 = -\frac{\alpha}{2\beta}, A_1 = B_1 = \mp\frac{\sqrt{(\alpha^2 - 4\beta)((a+c)\cosh(d) + (1+b)\sinh(d))}}{2\beta\sqrt{(-a+c)\cosh(d) + (-1+b)\sinh(d)}},$$
$$c_1 = \frac{\alpha^2 - 4\beta}{\beta((a-c)\cosh(d) - (-1+b)\sinh(d))}.$$

Substituting Eq.(7) into Eq.(3), we obtain the following new exact solitary wave solution of Eq.(1):

$$\begin{aligned}
u_n(t) = &-\frac{\alpha}{2\beta} \mp \frac{\sqrt{(\alpha^2 - 4\beta)((a+c)\cosh(d) + (1+b)\sinh(d))}}{2\beta\sqrt{(-a+c)\cosh(d) + (-1+b)\sinh(d)}} \\
&\times \sinh\left(dn + \frac{(\alpha^2 - 4\beta)t}{\beta((a-c)\cosh(d) - (-1+b)\sinh(d))} + c_2x + c_0\right) \\
&\mp \frac{\sqrt{(\alpha^2 - 4\beta)((a+c)\cosh(d) + (1+b)\sinh(d))}}{2\beta\sqrt{(-a+c)\cosh(d) + (-1+b)\sinh(d)}} \\
&\times \cosh\left(dn + \frac{(\alpha^2 - 4\beta)t}{\beta((a-c)\cosh(d) - (-1+b)\sinh(d))} + c_2x + c_0\right),
\end{aligned}$$

where a, b, c, α and β are constants, d, c_0 and c_2 are arbitrary constants.

When $a = 3$, $b = 2$, and $c = 1$ we obtain the following exact solitary wave solution of Eq.(1):

$$\begin{aligned}
u_n(t) = &-\frac{\alpha}{2\beta} \mp \frac{\sqrt{(-\alpha^2 + 4\beta)(4\cosh(d) + 3\sinh(d))}}{2\beta\sqrt{2\cosh(d) - \sinh(d)}}\sinh\left(dn + \frac{\alpha^2 - 4\beta}{\beta(2\cosh(d) - \sinh(d))}t + c_2x + c_0\right) \\
&\mp \frac{\sqrt{(-\alpha^2 + 4\beta)(4\cosh(d) + 3\sinh(d))}}{2\beta\sqrt{2\cosh(d) - \sinh(d)}}\cosh\left(dn + \frac{\alpha^2 - 4\beta}{\beta(2\cosh(d) - \sinh(d))}t + c_2x + c_0\right),
\end{aligned}$$

where c_0, c_2, and d are arbitrary constants. α and β are constants and $\beta \neq 0$.

3. Exact solutions of the discrete mKdV lattice

The discrete mKdV lattice reads[5]

$$\dot{u}_n(t) = (\alpha - u_n^2)(u_{n+1} - u_{n-1}), \qquad (8)$$

where α is a constant.

By using the hyperbolic function approach introduced above, we obtain the following new exact solitary wave solution of Eq.(8):

$$u_n(t) = \mp \frac{\sqrt{\alpha}\sqrt{(a+c)\cosh(d)+(1+b)\sinh(d)}}{\sqrt{(-a+c)\cosh(d)+(-1+b)\sinh(d)}}\sinh\left(dn - \frac{4\alpha t}{(a-c)\cosh(d)-(-1+b)\sinh(d)} + c_2 x + c_0\right)$$
$$\mp \frac{\sqrt{\alpha}\sqrt{(a+c)\cosh(d)+(1+b)\sinh(d)}}{\sqrt{(-a+c)\cosh(d)+(-1+b)\sinh(d)}}\cosh\left(dn - \frac{4\alpha t}{(a-c)\cosh(d)-(-1+b)\sinh(d)} + c_2 x + c_0\right),$$

where a, b, c, d, c_0 and c_2 are all arbitrary constants.

When $a = 2$, $b = 4$ and $c = 4$, we obtain the following exact solitary wave solution of Eq.(8):

$$u_n(t) = \mp \frac{\sqrt{\alpha}\sqrt{6\cosh(d)+5\sinh(d)}}{\sqrt{2\cosh(d)+3\sinh(d)}}\sinh\left(dn + \frac{4\alpha t}{2\cosh(d)+3\sinh(d)} + c_2 x + c_0\right)$$
$$\mp \frac{\sqrt{\alpha}\sqrt{6\cosh(d)+5\sinh(d)}}{\sqrt{2\cosh(d)+3\sinh(d)}}\cosh\left(dn + \frac{4\alpha t}{2\cosh(d)+3\sinh(d)} + c_2 x + c_0\right),$$

with c_0, c_2 and d being arbitrary constants.

4. Summary

In this paper, we have successfully utilized a hyperbolic function approach for obtaining new exact solitary wave solutions for the Hybrid lattice and discrete mKdV lattice. The hyperbolic function approach is a more effective approach to finding the exact solutions for nonlinear discrete system, and it is worthy of further studies.

References

[1] Zhu J M, Ma Z Y and Zheng C L 2005 *Acta Phys. Sin.* **54** 483 (in Chinese)

[2] Wang M L 1995 *Phys. Lett.* A **199** 169

[3] Shi Y R, Guo P, Lü K P and Duan W S 2004 *Acta Phys. Sin.* **53** 3265 (in Chinese)

[4] Liu S K, Fu Z T, Liu S D and Zhao Q 2001 *Acta Phys. Sin.* **50** 2068 (in Chinese)

[5] Zhu J M and Ma Z Y 2005 *Chin. Phys.* **14** 17

[6] Wang M L, Wang Y M and Zhang J L 2003 *Chin. Phys.* **12** 1341

[7] Zhou Y B, Wang M L and Wang Y M 2003 *Phys. Lett.* A **308** 31

[8] Zhang J L, Ren D M, Wang M L, Wang Y M and Fang Z D 2003 *Chin. Phys.* **12** 825

[9] Sirendaoreji and Sun J 2003 *Phys. Lett.* A **309** 387

[10] Zhang G X, Li Z B and Duan Y S 2000 *Science in China* (Series A) **30** 1103 (in Chinese)

[11] Taogetusang and Sirendaoreji 2004 *Acta Phys. Sin.* **53** 4052 (in Chinese)

[12] Zhao X Q, Tang D B, Wang L M and Zhang Y M 2003 *Acta Phys. Sin.* **52** 1827 (in Chinese)

[13] Yu Y X, Wang Q, Zhao X Q, Zhi H Y and Zhang H Q 2005 *Acta Phys. Sin.* **54** 3992 (in Chinese)