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Formation mechanism of the low-frequency locally resonant band gap in the two-dimensional ternary phononic crystals*

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The low-frequency band gap and the corresponding vibration modes in two-dimensional ternary locally resonant phononic crystals are restudied successfully with the lumped-mass method. Compared with the work of C. Goffaux and J. Sánchez-Dehesa (*Phys. Rev.* B **67** 14 4301(2003)), it is shown that there exists an error of about 50% in their calculated results of the band structure, and one band is missing in their results. Moreover, the in-plane modes shown in their paper are improper, which results in the wrong conclusion on the mechanism of the ternary locally resonant phononic crystals. Based on the lumped-mass method and better description of the vibration modes according to the band gaps, the locally resonant mechanism in forming the subfrequency gaps is thoroughly analysed. The rule used to judge whether a resonant mode in the phononic crystals can result in a corresponding subfrequency gap is also verified in this ternary case.

Keywords: phononic crystals, locally resonant, band gap mechanism
PACC: 4320, 4335, 0260, 8160H

1. Introduction

In the last decade, a great deal of work has been devoted to the study of the propagation of elastic or acoustic wave in periodic structures made of materials with different elastic properties. These new structures have been named the phononic crystals (PCs)[1] by analogy with the photonic crystals[2,3] for electromagnetic waves. Because of the periodicity in such structures, there exist frequency ranges in which elastic waves are forbidden. This phenomenon can be of real interest because of the rich physics of elastic systems, where the wave can have mixed longitudinal and transverse modes, and a large contrast between the elastic parameters is allowed. A famous example is the locally resonant (LR) phononic crystals (PCs) consisting of very soft rubber[4−10] (with an elastic constant of five orders lower than common solids) and other components. This new material is most likely to depart from the conventional Bragg scattering mechanism[11,12] and obtain low-frequency gaps with structures of small dimensions, which can lead to promising applications such as low-frequency vibration/noise insulation.

Several theoretical methods have been developed in order to study the elastic response of PCs, such as the traditional[13] and improved[14] plane-wave expansion (PWE) method, the multiple scattering (MS) theory[15−17], the finite-difference algorithms,[18−20] the variational method (VM),[8] and the lumped-mass (LM) method.[9,21,22] The LM method[9,21,22] is proposed recently as a new procedure to compute the band structure of PCs. Finite particles were employed as the simplification model of a lattice of PCs in the LM method. It has been concluded[9,10,22] that the LM method converges faster than the traditional methods and its convergence is insensitive to the sharp variation of elastic constants on the interfaces inside PCs. This advantage was unique in comparison with

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other works[8,14] on the improvement of traditional methods. As there exists large contrasts of elastic parameters in the LR PCs, this advantage is especially useful in the calculation of their band structures. The MS method can also be used successfully in the study of strongly heterogeneous elastic composites. However, due to its complicated deductions, the MS theory can only be used to handle specific arrays of spheres[16,17] (three-dimensional case) or cylinders[15] (two-dimensional case) now. This limitation is the main shortcoming of the MS technique. However, It can be easily overcome with the LM method.

In this paper, the LM method is used to calculate the band structure and the corresponding vibration modes of the two-dimensional (2D) ternary LR PCs. It is shown that there exists an error about 50% in the band structure of the 2D ternary LR PCs calculated by Goffaux *et al*,[8] and one band is missing in their results.[8] Therefore the capability of their VM[8] in the calculation of band structures in LR PCs has to be doubted. Moreover, the in-plane vibration modes shown in their paper are improper, which results in wrong conclusions on the mechanism of the ternary locally resonant phononic crystals. With the accurate calculation results properly shown in this paper, we study successfully the mechanism of the LR band gaps and validate the rule used to judge whether a resonant mode in LR PCs can result in a corresponding subfrequency gap. The rule has recently been concluded in the 2D binary case.[10]

2. Band structure calculations

Figure 1 illustrates the band structures of the ternary LR PCs with square lattice (same as in Ref.[8]) calculated with the LM method. Compared with the corresponding previous result (Fig.6 in Ref.[8]) of Goffaux *et al*, there are two mismatches. Firstly, the flat branch L_4 in Fig.1(a) which is real and convergent with the LM technique is missing in the previous result. Secondly, the frequencies of the higher modes in the previous result are much higher (about 50%) than that calculated with the LM method. For example, the eigenfrequency of mode $L_{5,6}$ calculated by Goffaux *et al* is about 0.033, while the corresponding value calculated with the LM method is 0.0196, as shown in Fig.1.

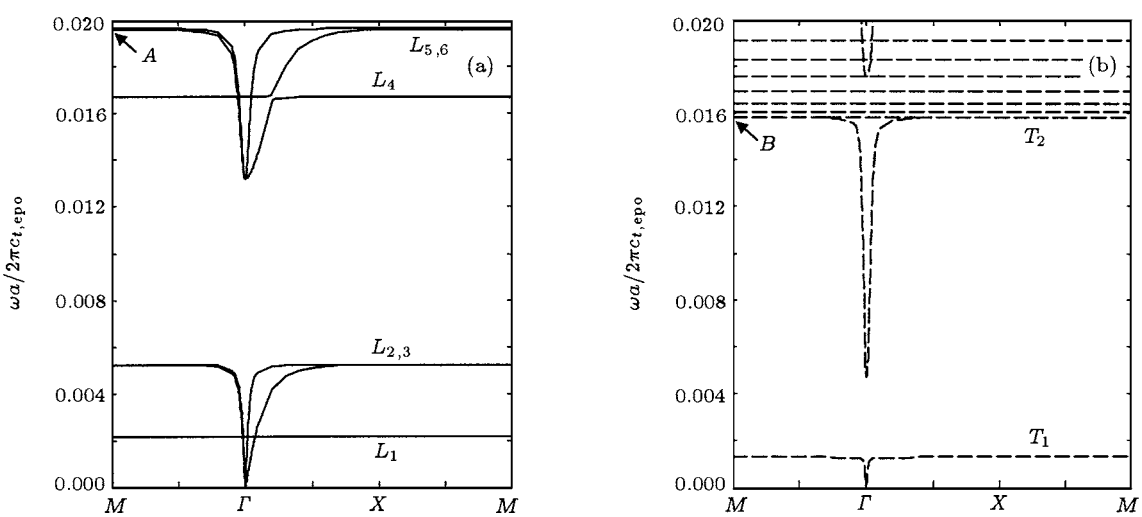


Fig.1. (a) Band structure of the in-plane modes existing in a square lattice of the same ternary LR PCs as in Ref.[8]. (b) Band structure of the corresponding out-of-plane modes. The band structures are calculated with the LM method by employing 428 lumped masses.

The convergence of the LM method has also been analysed in Fig.2 in which backs the results in Fig.1. It is shown that the LM method has very good convergence even at higher frequencies. Only a set of 428 lumped masses is needed to guarantee a convergence better then 0.5%. The dimension of the calculation in the LM methods with 428 lumped masses equals to that of the PWE methods with 428 plane waves or that of the VM with 16 nodes. Thus, from Fig.2 in this paper and Fig.7 in Ref.[8], we can conclude that the convergence of LM method is much better than that of the PWE method and the variation method (VM). In fact, the convergence of the LM method is independent of the sharp variation of elastic constants

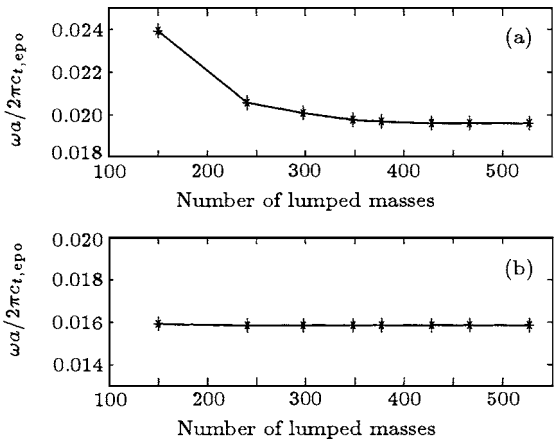


Fig.2. (a) Convergence of the frequency of the 6th in-plane mode for the lumped-mass method at point A in Fig.1. (b) Convergence of the frequency of the 2nd out-of-plane mode for the lumped-mass method at point B in Fig.1. The horizontal axis represents the number of lumped masses, i.e. the size of matrix, employed in the lumped-mass calculations.

on the interfaces inside the PCs.[22] Thus we can conclude that the improvement of the VM compared with the traditional PWE method is small.

3. Band gap mechanism

In order to study the origin of the LR band gaps properly, lattice vibrations of several selected eigenmodes are calculated with the LM method and illustrated in Figs.3 and 4. Figure 3 shows the in-plane modes marked with L_1, $L_{2,3}$, L_4 and $L_{5,6}$ in Fig.1(a), and Fig.4 illustrates the out-of-plane modes marked with T_1 and T_2 in Fig.1(b), respectively.

With Figs.3(a) and 3(b), i.e. the modes at the first and second flat bands, respectively, we can see obviously that the vibrations are localized in both the polymer coating layer and the gold core. Due to the improper description of its in-plane modes by Goffaux *et al* (see Fig.8(b) in Ref.[8]), their conclusion that

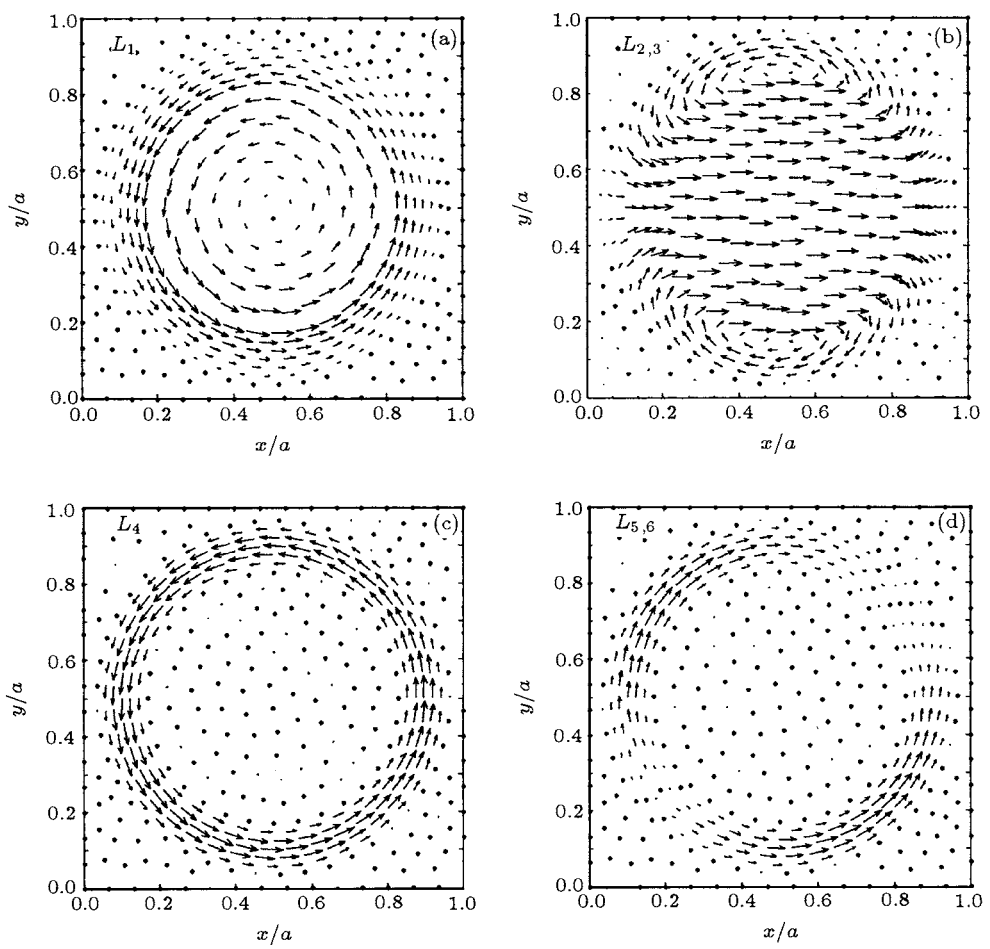


Fig.3. Lattice displacement vectors of the in-plane modes marked with (a) L_1, (b) $L_{2,3}$, (c) L_4, and (d) $L_{5,6}$ in Fig.1(a). The direction and length of each arrow represent the direction and amplitude of the displacement vector at the starting point of the arrow.

the vibrations are only localized in the polymer coating is erroneous. Actually, with the proper arrow diagram in Fig.3(a), we can see clearly that the lattice vibration of the first flat band is localized in the clockwork-like torsion mass-spring structure, i.e. the core acts as the mass and the polymer is the torsion spring. Consequently, this mode has no relation with the formation of the first band gap (see the following for detailed discussions). The vibration mode at the second flat band illustrated in Fig.3(b) shows that the gold core vibrates holistically and the polymer acts as a spring linking the core and the host media. Actually, this mode has brought about the formation of the first band gap.

As for the third and fourth modes at L_4 and $L_{5,6}$ in Fig.1(a), lattice vibrations are all localized in the coating layer. We believe that the second flat band calculated by Goffaux *et al* (see Fig.8(b) in Ref.[8]) belongs to the latter.

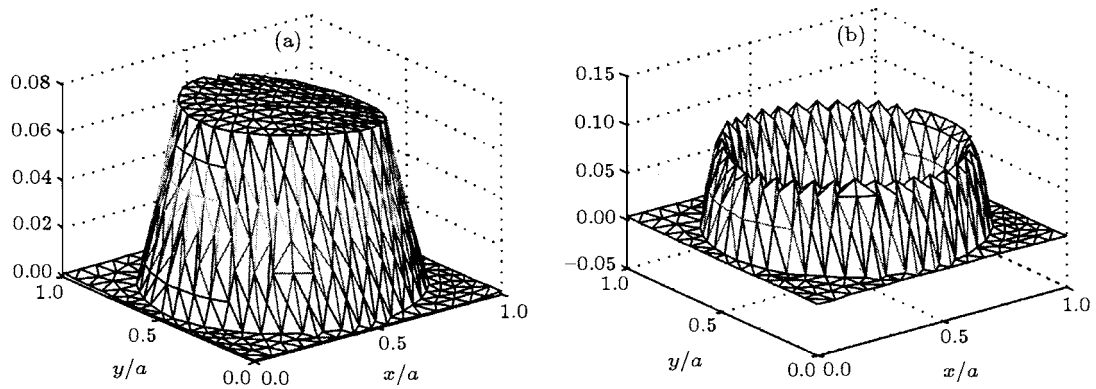


Fig.4. Lattice displacement of the out-of-plane modes marked with (a) T_1 and (b) T_2 in Fig.1(b).

As for the out-of-plane modes illustrated in Figs.4, the lattice vibration mode of the first flat band (see Fig.4(a)) is similar to that of the second in-plane flat branch except that the displacements are along the axis of cylinders, i.e. the out-of-plane direction. The vibration mode of the second flat band is localized only in the coating layer (see Fig.4(b)).

A remarkable feature of the in-plane band structure illustrated in Fig.1(a) is that some of the localized resonant modes (marked with $L_{2,3}$ and $L_{5,6}$ in Fig.1(a)) result in the corresponding LR band gaps, while the others (marked with L_1 and L_4 in Fig.1(a)) just go through the other dispersion curves without any gap being generated. This feature can be explained with the rule recently concluded in the binary case[10] and used to judge whether a resonant mode can result in a corresponding subfrequency gap. To be specific, for the first flat band (L_1 in Fig.1(a)), because the composition of the forces to the host structure contributed by the polymer-gold torsion oscillator is zero, the oscillator vibrates solely and no gap is generated. As for the 2nd and 3rd LR flat branches ($L_{2,3}$ in Fig.1 (a)) where the oscillator vibrates holistically (see Fig.3(b)), the composition of the forces from the oscillator to the host structure is nonzero and time-harmonic. Therefore, the dispersion curves are cut off at its eigenfrequency and a corresponding subfrequency gap is generated. For the other modes illustrated in Figs.3(c) and 3(d), the same rule can be easily concluded.

4. Conclusions

In summary, there exists a large error in the band structure calculated by Goffaux *et al* of the 2D ternary LR PCs, and the advantages of the VM need to be doubted. Moreover, their improper description of the vibration modes has hindered the deduction of correct conclusions concerning the origin of the LR band gaps. We have restudied the 2D ternary LR PCs successfully with the LM method, and shown that the first and second in-plane locally resonant modes are localized in both the coating layer and the core. Furthermore, It is shown that the rule recently concluded and used to judge whether a resonant mode in the 2D binary LR PCs can result in a corresponding subfrequency gap is also valid in this ternary case.

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