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Waveguides induced by grey screening solitons*

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We investigate the properties of waveguides induced by one-dimensional grey screening solitons in biased photorefractive crystals. The results show that waveguides induced by grey screening solitons are always of single mode for all intensity ratios, i.e. the ratios between the peak intensity of the soliton and the dark irradiance. Our analysis indicates that the energy confined near the centre of the grey soliton and the propagation constant of the guided mode of the waveguide induced by the grey screening soliton increase monotonically with intensity ratio increasing. On the other hand, when the soliton greyness increases, the energy confined near the centre of the grey soliton and the propagation constant of the guided mode of the waveguide induced by the grey screening soliton decrease monotonically. Relevant examples are provided where photorefractive crystal is of the strontium barium niobate type.

Keywords: soliton waveguide, photorefractive effect, nonlinear optics
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1. Introduction

Photorefractive (PR) spatial solitons have been a topic of considerable research because of their robustness and simplicity of their creation at very low power levels.[1−16] Among the numerous types of photorefractive solitons, screening solitons are of the most commonly used type,[10] which occur in a steady state when an external bias voltage is appropriately applied to a photorefractive crystal. Thus far, bright, dark, and grey steady-state domains have been predicted. On the other hand, photorefractive spatial solitons induce waveguides that offer potential applications, some of which are truly unique[11,12] such as three-dimensional optical circuitry,[13] reconfigurable near-field interconnects,[12] nonlinear frequency conversion,[11] directional coupling,[14] and beam steering.[15] Recent work on screening-soliton-induced waveguides has shown theoretically and demonstrated experimentally waveguides induced by bright and dark solitons.[16] The number of guided modes in a waveguide induced by a bright screening soliton increases monotonically with the increase of intensity ratio, i.e. the ratio between the peak intensity of the soliton and the dark irradiance, whereas a waveguide induced by a dark screening soliton has a single guiding mode, of which the confined energy increases monotonically with intensity ratio increasing. Another interesting class of solitary waves is grey solitons, which are by nature more complicated. Evidently, it would be of interest to know whether such grey screening solitons induce waveguides.

In this paper, we analyse the properties of waveguides induced by one-dimensional grey screening solitons in biased photorefractive crystals. Our results show that the waveguide induced by a grey screening soliton has a single guiding mode, of which the confined energy increases monotonically with the increase of the intensity ratio i.e. the ratio between the peak intensity of the soliton and the dark irradiance, and the propagation constant of the guided mode of the waveguide induced by the grey screening soliton is monotonically increasing with intensity ratio. We find that when the soliton greyness increases, the energy confined near the centre of the grey soliton and the propagation constant of the guided mode of the

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waveguide induced by the grey screening soliton decrease monotonically. Relevant examples are provided where photorefractive crystal is assumed to be of the strontium barium niobate (SBN) type.

2. Theory and results

To investigate waveguides induced by grey screening solitons, we consider an optical beam that propagates in a photorefractive crystal along the z axis and is allowed to be diffracted only along the x direction. For the convenience of the analysis, the photorefractive crystal is taken here as strontium barium niobate with its optical c-axis oriented along the x direction. Moreover, we assume that the polarization of the optical beam and the external bias electric field are both parallel to the c-axis. Under these conditions, the perturbed refractive index n_b' (along the c-axis) is given by $(n_b')^2 = n_b^2 - n_b^4 r_{\rm eff} E_{\rm sc}$,[10] where n_b is the unperturbed index of refraction, $r_{\rm eff}$ is the electro-optic coefficient, and $\boldsymbol{E}_{\rm sc} = \boldsymbol{i}E_{\rm sc}$ is the space-charge field induced in this photorefractive crystal, $\boldsymbol{i}$ is the unit vector pointing to the x direction. Under steady-state conditions, the induced space-charge field in a typical photorefractive crystal can be directly obtained from the Vinetskii–Kukhtarev transport model and is given by[10]

$$E_{\rm sc} = E_0 \frac{I_\infty + I_{\rm d}}{I + I_{\rm d}}, \tag{1}$$

where $I = I(x, z)$ is the power density of the optical beam, $I_\infty = I(x \to \pm\infty, z)$, $I_{\rm d}$ is the so-called dark irradiance, and E_0 is the value of the space-charge electric field at $x \to \pm\infty$. If the spatial extent of the optical wave is much less than the x-width W of the PR crystal, then under a constant biased voltage V, E_0 is approximately given by $\pm V/W$. In Eq.(1), I is related to the slowly varying envelope φ through Poynting's vector, i.e. $I = (n_b/2\eta_0)|\varphi|^2$, where $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$ is the vacuum intrinsic impedance. By employing standard procedures, we readily obtain that the optical beam envelope φ obeys the following paraxial equation of diffraction:[10]

$$\mathrm{i}\varphi_z + \frac{1}{2k}\varphi_{xx} - \frac{k_0\left(n_b^3 r_{\rm eff} E_{\rm sc}\right)}{2}\varphi = 0, \tag{2}$$

where $k = k_0 n_b = (2\pi/\lambda_0) n_b$ is the wavevector, λ_0 is the free-space wavelength of the lightwave employed, and $\varphi_z = \partial\varphi/\partial z$, etc. For simplicity, we adopt the following dimensionless coordinates and variables, i.e. let $\xi = k_0 n_b^3 r_{\rm eff} |E_0| (z/2)$, $s = k_0 n_b^2 (r_{\rm eff}|E_0|/2)^{1/2} x$, and $U = [n_b/(2\eta_0 I_{\rm d})]^{1/2}\varphi$. In this case, substituting Eq.(1) into Eq.(2) leads to

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} + (1+\rho)\frac{U}{1+|U|^2} = 0, \tag{3}$$

where $\rho = I_\infty/I_{\rm d}$. To obtain the grey solitary wave solutions of Eq.(3), we express U in the following form:[10]

$$U = \rho^{1/2}\phi(s)\exp\left[\mathrm{i}\left(\mu\xi + \int_0^s \frac{\Gamma \mathrm{d}s'}{\phi^2(s')}\right)\right], \tag{4}$$

where μ is a nonlinear shift of the propagation constant, Γ is a real constant to be determined, and $\phi(s)$ is a normalized function bounded between 0 and 1, i.e. $0 \le \phi(s) \le 1$. Substituting Eq.(4) into Eq.(3) yields the following ordinary differential equation:

$$\phi'' - 2\mu\phi - \frac{\Gamma^2}{\phi^3} + (1+\rho)\frac{2\phi}{1+\rho\phi^2} = 0, \tag{5}$$

where $\phi'' = \mathrm{d}^2\phi/\mathrm{d}s^2$, etc. For grey solitons, one requires the boundary conditions: $\phi(0) = \sqrt{m}$ $(0 < m < 1)$, $\phi(s \to \pm\infty) = 1$, $\phi'(0) = 0$, and $\phi'(s \to \pm\infty) = \phi''(s \to \pm\infty) = 0$. The parameter m describes the soliton greyness, i.e. the intensity in the middle of the wave, defined as $I(0) = mI_\infty$. Using the boundary conditions $\phi(s \to \pm\infty) = 1$ and $\phi''(s \to \pm\infty) = 0$, and substituting $s \to \infty$ into Eq.(5) lead to

$$\Gamma = [2(1-\mu)]^{1/2}. \tag{6}$$

By integrating Eq.(5) once and by employing the boundary conditions $\phi(0) = \sqrt{m}$ and $\phi'(0) = 0$ we find that

$$\mu = \frac{1}{(m-1)^2}\left[\frac{m(1+\rho)}{\rho}\ln\left(\frac{1+\rho m}{1+\rho}\right) + (1-m)\right], \tag{7}$$

$$\begin{aligned}(\phi')^2 =& 2\mu\left(\phi^2 - 1\right) + \frac{2(1+\rho)}{\rho}\ln\left(\frac{1+\rho\phi^2}{1+\rho}\right) \\ &+ 2\mu\left(\frac{1-\phi^2}{\phi^2}\right),\end{aligned} \tag{8}$$

from which the grey envelope $\phi(s)$ can be obtained by numerical integration. After the grey soliton has formed, the space-charge field is present in the photorefractive crystal, which gives rise to a change in the refractive index. We consider a probe beam that propagates in this soliton-induced waveguide. The electric-field component $\bar{E}_{\rm opt}$ of the probe beam satisfies the Helmholtz equation

$$\nabla^2\bar{E}_{\rm opt} + \left(\bar{k}_0\bar{n}\right)^2\bar{E}_{\rm opt} = 0, \tag{9}$$

where $\bar{k}_0 = 2\pi/\bar{\lambda}$ is the wavevector in vacuum and $\bar{\lambda}$ is the common free-space wavelength used. The perturbed refractive index $\bar{n}$ (along the x axis) is given by[16]

$$\bar{n}(x) = \bar{n}_b + \Delta\bar{n}(x) = \bar{n}_b - \frac{1}{2}\bar{n}_b^3 \bar{r}_{\rm eff}\hat{E}(x). \qquad (10)$$

In the above equation, the values of unperturbed refractive index $\bar{n}_b$ and the electro-optic coefficient $\bar{r}_{\rm eff}$ are now subject to the new wavelength $\bar{\lambda}$. By defining $\bar{\boldsymbol{E}}_{\rm opt}(x,z,t) = \boldsymbol{i}\psi(x)\exp\left(\mathrm{i}\bar{\beta}z - \mathrm{i}\bar{\varpi}t\right) + \text{c.c.}$ ($\bar{\varpi}$ is the frequency of the probe beam, and $\bar{\beta}$ is the propagation constant), we find that Eq.(9) leads to the following wave equation with a slowly varying amplitude $\psi(x)$:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \left[\bar{k}_0^2\bar{n}^2(x) - \bar{\beta}^2\right]\psi = 0. \qquad (11)$$

Under paraxial approximation conditions $\bar{n}^2$ can be determined from Eq.(10) and is given by $\bar{n}^2 = \bar{n}_b^2 - \bar{n}_b^4\bar{r}_{\rm eff}E_{\rm sc}$. Substituting the expression $\bar{n}^2$ that we have just found into Eq.(11) leads to

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\bar{s}^2} = \left[\bar{\tau} - \frac{1+\rho}{1+\rho\phi^2\left(\bar{s}/\sqrt{2}\sigma_1\sigma_2\right)}\right]\psi, \qquad (12)$$

where the normalized propagation constant $\bar{\tau}$, the new normalized length $\bar{s}$, and normalization factors σ_1 and σ_2 are respectively defined as

$$\bar{\tau} = \frac{\bar{\beta}^2 - \sigma_1^2k^2}{n_b^2 r_{\rm eff}|E_0|k^2\sigma_1^2\sigma_2^2}, \qquad (13)$$

$$\bar{s} = \sqrt{2}\sigma_1\sigma_2 s, \qquad (14)$$

$$\sigma_1 = \frac{\bar{n}_b}{n_b}\frac{\lambda}{\bar{\lambda}}, \qquad (15)$$

$$\sigma_2 = \frac{\bar{n}_b}{n_b}\sqrt{\frac{\bar{r}_{\rm eff}}{r_{\rm eff}}}. \qquad (16)$$

In what follows, we will find the modes of the waveguide, $\psi(\bar{s})$, in a waveguide induced by a grey soliton. First, we solve Eq.(8) numerically for various values of ρ and find the soliton wave forms $\phi(s)$. We then solve Eq.(12) for $\psi(\bar{s})$ by the shooting method. In Eqs.(12) and (13), the eigenvalue, $\bar{\tau}$, must be in the range between unity and $\dfrac{1+\rho}{1+m\rho}$ to fulfill the condition that the propagation constant of a guided mode, $\bar{\beta}$, lies between the maximum and the minimum values of the product of refractive index $\bar{n}_b(x)$ by the wave vector $\bar{k}_0$. Figure 1 shows the normalized envelope of the grey soliton and the normalized envelope of the single guided mode $\psi(\bar{s})$ for three different values of ρ when $m = 0.3$ and $\alpha_1\alpha_2 = 0.697$, which corresponds to a grey soliton at $\lambda = 0.5\mu$m in a SBN crystal with $r_{\rm eff} = 224\times10^{-12}$m/V and a probe beam, guiding within it, at $\bar{\lambda} = 0.7\mu$m wavelength with $\bar{r}_{\rm eff} = 213\times10^{-12}$m/V, on the assumption of negligible difference between n_b and $\bar{n}_b$. This figure demonstrates that the bigger the value ρ, the more the optical energy is confined near the centre of the grey soliton. Figure 2 depicts the normalized envelope of the grey soliton and the normalized envelope of the single guided mode $\psi(\bar{s})$ for three different values of m when $\rho = 10$ and $\alpha_1\alpha_2 = 0.697$. Evidently, the energy confined near the centre of the grey soliton decreases as m increases. Figure 3 plots the propagation constant as a function of ρ for $\alpha_1\alpha_2 = 0.697$ with $m = 0.2$, 0.4, and 0.6. The number of guided modes for each ρ is given by the number of intersections of the dash-dot vertical lines with the curve of the propagation constant in Fig.3. The two dash-dot lines correspond to $\rho = 1$ and 10 (the examples given in Fig.1) separately.

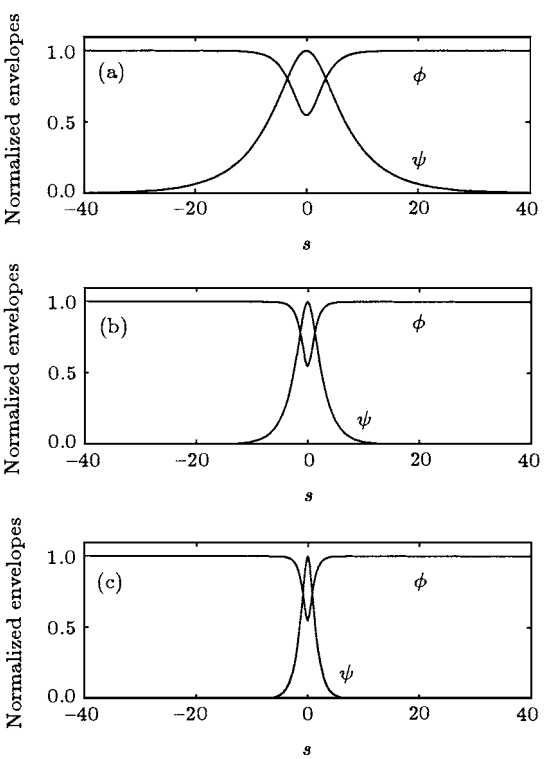


Fig.1. Normalized field profiles of the grey screening soliton and the single guided mode of the soliton-induced waveguide at (a) $\rho = 0.1$, (b) $\rho = 1$ and (c) $\rho = 10$ when $\sigma_1\sigma_2 = 0.697$ and $m = 0.3$. $\psi(s)$ is normalized to the maximum of $\psi(s)$.

It reveals that waveguides induced by grey screening solitons are always of single mode for all intensity rations. From Fig.3 it is also apparent that the propagation constant increases monotonically with ρ increasing. On the other hand, Fig.3 shows that when the soliton greyness increases, the propagation constant decreases monotonically.

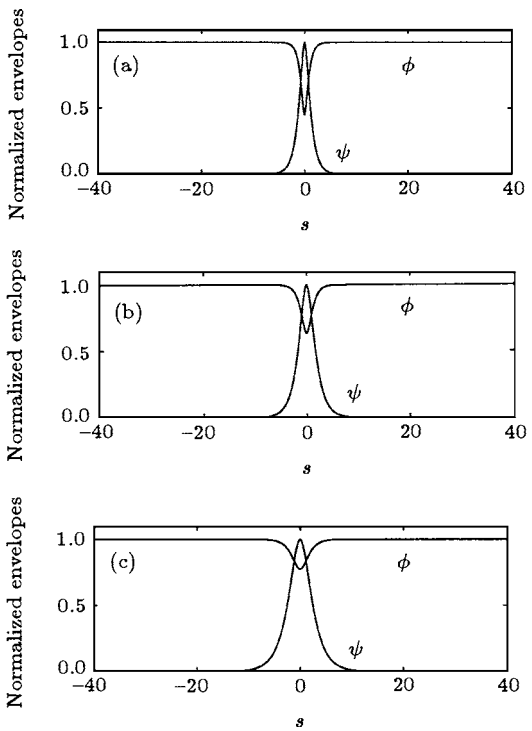


Fig.2. Normalized field profiles of the grey screening soliton and the single guided mode of the soliton-induced waveguide at (a) $m = 0.2$, (b) $m = 0.4$, and (c) $m = 0.6$ when $\sigma_1\sigma_2 = 0.697$ and $\rho = 10$. $\psi(s)$ is normalized to the maximum of $\psi(s)$.

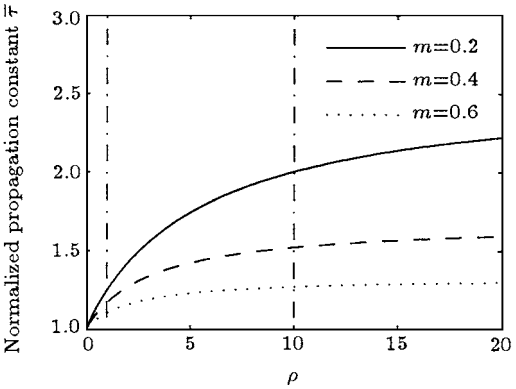


Fig.3. Propagation constants of the guided modes of the waveguide induced by the grey screening soliton at $m = 0.2$, $m = 0.4$, and $m = 0.6$ when $\sigma_1\sigma_2 = 0.697$.

3. Conclusion

We have investigated the properties of waveguides induced by one-dimensional grey screening solitons in biased photorefractive crystals. We have shown that waveguides induced by grey screening solitons are always of single mode for all intensity rations. Our analysis indicates that the energy confined near the centre of the grey soliton and the propagation constant of the guided mode of the waveguide induced by the grey screening soliton increase monotonically with the increase of intensity ratio. We have found that when the soliton greyness increases, the energy confined near the centre of the grey soliton and the propagation constant of the guided mode of the waveguide induced by the grey screening soliton decrease monotonically.

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