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Trichromatic phase manipulation of the response of a two-level medium to an arbitrarily intense probe field*

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The phase dependence and independence of the response of a trichromatically driven two-level medium to an arbitrarily intense probe field have been studied. The sum of the relative phases of the sideband components of the trichromatic field compared to the central component plays a crucial role in the response of the medium. For a weak probe field, as the sum of the relative phases changes from 0 to π, multiple switching can be achieved, in which switching from normal to anomalous dispersion occurs in multiple separate frequency regimes. The remarkable dependence on the sum phase is also shown for a strong probe field. On the other hand, when the sum of the two relative phases is fixed, the changes in the respective phases have no influence on the response of the medium.

Keywords: absorption and dispersion, phase dependence and independence, multiple switching from normal to anomalous dispersion

PACC: 4250, 4265

1. Introduction

The properties of absorption and refraction of a medium are significantly modified by employing coherent fields to drive the medium. Among those examples are coherent population trapping,[1,2] electromagnetically induced transparency,[3−14] lasing without inversion,[15−18] enhancement of index of refraction,[19] subluminal and superluminal light propagation without absorption,[20−22] and efficient four-wave mixing in an ultraslow propagation regime.[23−25] The atomic coherence has been prepared in a variety of systems such as two-level systems, multilevel systems,[26,27] and quantum well.[28] Approaches for investigation include the monochromatic pump and the polychromatic excitation.[29,30] Recently, the absorption and dispersion properties of two-level atoms under the trichromatic excitation were investigated by Yoon *et al*[29] and Ficek *et al.*[30] Yoon *et al*[29] focused on the response of the trichromatically driven two-level atoms to an arbitrarily intense probe field and presented a calculation method for such a case. Ficek *et al*[30] noted the phase dependence of the absorption and dispersion properties. The opposite features were obtained by changing the relative phases of the sideband components with respect to that of the central component. It seems that the response of the medium depends on the respective relative phases.

However, this is not the case, as will be shown in this paper. It is the sum of the relative phases that leads to remarkably different effects. For a weak probe field, simply by varying the sum phase, switching from normal to anomalous dispersion can be achieved in multiple different frequency regimes. Likewise, the response of the medium to a strong probe field is governed by the sum phase. On the other hand, when the sum phase is fixed, the response of the medium is no longer changed no matter how we vary the respective phases.

The switching from normal to anomalous dispersion, which corresponds to the switching from subluminal to superluminal light propagation, has recently been paid considerable attention. It has been

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achieved by two kinds of ways. One is to employ additional coupling in electromagnetically induced transparency systems. The examples include microwave coupling[31] or a single coherent field coupling[32] in a three-level Λ system, two beams of lasers and one beam of microwave field in a double Λ system,[33] and the additional coupling in a four-level N system.[34,35] The other kind is to use polychromatic excitation, which was recently considered by Ficek *et al.*[30] It should be noted that the above studies have been confined to the dispersion switching in a single frequency regime. Via the trichromatic phase manipulation of electromagnetically induced transparency, one obtains double-switching, in which switching from normal to anomalous dispersion occurs in two different frequency regimes.[36] Here we will show that via trichromatic phase manipulation in a two-level medium, it is possible to obtain the dispersion switching in three pairs of separate frequency regimes.

2. Absorption, dispersion, and group velocity

The model system under consideration is an ensemble of two-level atoms, which have a ground state $|1\rangle$ and an excited state $|2\rangle$. The two-level atom is driven by a strong trichromatic field $\frac{1}{2}\left(\boldsymbol{E}_0+\boldsymbol{E}_1\mathrm{e}^{\mathrm{i}\delta t}+\boldsymbol{E}_2\mathrm{e}^{-\mathrm{i}\delta t}\right)\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{c}}t}$+c.c., and probed by a monochromatic field $\frac{1}{2}\,\boldsymbol{E}_{\mathrm{p}}\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{p}}t}$+c.c., where $\boldsymbol{E}_j$ (j=0, 1, 2, p) are the complex electric amplitudes, ω_{c} and $\omega_{\mathrm{c}}\pm\delta$ are the frequency components of the trichromatic driving field, δ is the frequency difference, and ω_{p} is the frequency of the probe field. The equations for the reduced density-matrix elements are derived as

$$\dot{\rho}_{21}=-\left(\frac{\gamma}{2}+\mathrm{i}\Delta_0\right)\rho_{21}-\frac{\mathrm{i}}{2}\left(\Omega_0+\Omega_1\mathrm{e}^{\mathrm{i}\delta t}+\Omega_2\mathrm{e}^{-\mathrm{i}\delta t}+\Omega_{\mathrm{p}}\mathrm{e}^{-\mathrm{i}\Delta_{\mathrm{p}}t}\right)(\rho_{22}-\rho_{11}), \tag{1}$$

$$\dot{\rho}_{12}=-\left(\frac{\gamma}{2}-\mathrm{i}\Delta_0\right)\rho_{12}+\frac{\mathrm{i}}{2}\left(\Omega_0^*+\Omega_1^*\mathrm{e}^{-\mathrm{i}\delta t}+\Omega_2^*\mathrm{e}^{\mathrm{i}\delta t}+\Omega_{\mathrm{p}}^*\mathrm{e}^{\mathrm{i}\Delta_{\mathrm{p}}t}\right)(\rho_{22}-\rho_{11}), \tag{2}$$

$$\begin{aligned}\dot{\rho}_{22}-\dot{\rho}_{11}=&-\gamma(\rho_{22}-\rho_{11})+\mathrm{i}\left(\Omega_0+\Omega_1\mathrm{e}^{\mathrm{i}\delta t}+\Omega_2\mathrm{e}^{-\mathrm{i}\delta t}+\Omega_{\mathrm{p}}\mathrm{e}^{-\mathrm{i}\Delta_{\mathrm{p}}t}\right)\rho_{12}\\&-\mathrm{i}\left(\Omega_0^*+\Omega_1^*\mathrm{e}^{-\mathrm{i}\delta t}+\Omega_2^*\mathrm{e}^{\mathrm{i}\delta t}+\Omega_{\mathrm{p}}^*\mathrm{e}^{\mathrm{i}\Delta_{\mathrm{p}}t}\right)\rho_{21}-\gamma.\end{aligned} \tag{3}$$

Here, $\Omega_j=\boldsymbol{d}\cdot\boldsymbol{E}_j/\hbar$ (j=0, 1, 2, p) are the complex Rabi frequencies associated with the respective fields, and $\boldsymbol{d}$ is the atomic transition electric dipole moment. $\Delta_0=\omega_{\mathrm{a}}-\omega_{\mathrm{c}}$ is the detuning between the atomic resonance transition frequency ω_{a} and the frequency of the central driving component ω_{c}, and $\Delta_{\mathrm{p}}=\omega_{\mathrm{p}}-\omega_{\mathrm{c}}$ is the detuning between the probe field frequency ω_{p} and the frequency ω_{c}. γ is the atomic decay rate from the excited state $|2\rangle$ to the ground state $|1\rangle$.

In order to show the strong probe field effects, Yoon *et al*[29] solved these equations by using the double harmonic expansion and inversion matrix, but they did not take the phase dependence into account. For a weak probe field, Ficek *et al*[30] solved the set of equations by employing quantum regression theorem and a harmonic expansion of the two-time expectation values. They observed the effects of the respective relative phases but they did not find that such effects should be governed by the sum of the respective phases. Here, we follow the same technique as that in Ref.[29] to solve the set of equations (1–3) in three steps. (i) Make double harmonic expansion

$$\rho_{jk}(t)=\sum_{n,m=-\infty}^{+\infty}\rho_{jk}^{(n,m)}(t)\,\mathrm{e}^{\mathrm{i}n\delta t+\mathrm{i}m\Delta_{\mathrm{p}}t},\quad(j,k=1,2), \tag{4}$$

where $\rho_{jk}^{(n,m)}(t)$ represent the slowly varying amplitudes, which are needed to determine the absorption and dispersion properties. (ii) Substitute Eq.(4) into Eqs.(1–3), equate the coefficients of harmonics of δ and Δ_{p}, and obtain a set of equations for the slowly varying amplitudes in the steady state. (iii) Expressing $\rho_{12}^{(n,m)}$ and $\rho_{21}^{(n,m)}$ in terms of $\rho_{22}^{(n,m)}-\rho_{11}^{(n,m)}$ leads to a recurrence relation for $\rho_{22}^{(n,m)}-\rho_{11}^{(n,m)}$, which are arranged into an appropriate column vector and solved by the matrix inversion. Numerical calculations are performed by truncating the column vector at large values (N,N) necessary to achieve the required accuracy. The complex polarization of the medium is

written as

$$P(t) = n_{\rm a} d^* \rho_{21} = n_{\rm a} d^* \sum_{n,m} \rho_{21}^{(n,m)} {\rm e}^{-{\rm i}\omega_{\rm p} t + {\rm i}n\delta t + {\rm i}(m+1)\Delta_{\rm p} t}, \quad (5)$$

where $n_{\rm a}$ is the atom number density. The susceptibility $\chi(\omega_{\rm p})$, which measures the response of the medium to the probe field, is given by a complex polarization component $p(\omega_{\rm p})$ that oscillates at a frequency $\omega_{\rm p}$. As in Ref.[29], we focus only on the continuous part of the susceptibility. Thus the complex polarization in the present rotating frame is rewritten as $p(\omega_{\rm p}) = n_{\rm a} d^* \rho_{21}^{(0,-1)}$. In terms of the definition of $p(\omega_{\rm p}) = \varepsilon_0 \chi(\omega_{\rm p}) E_{\rm p}$, the susceptibility is expressed as $\chi = \chi' + {\rm i}\chi''$, with its real and imaginary parts given by

$$\chi'(\omega_{\rm p}) = \frac{3 n_{\rm a} \lambda_{\rm p}^3}{8\pi^2} {\rm Re}\left(\frac{\rho_{21}^{(0,-1)}}{\Omega_{\rm p}/\gamma}\right), \quad (6)$$

$$\chi''(\omega_{\rm p}) = \frac{3 n_{\rm a} \lambda_{\rm p}^3}{8\pi^2} {\rm Im}\left(\frac{\rho_{21}^{(0,-1)}}{\Omega_{\rm p}/\gamma}\right), \quad (7)$$

by which the dispersion and absorption properties are determined respectively. In Eqs. (6,7) we have used the expression $\frac{|d|^2}{\hbar\varepsilon_0} = \frac{3\lambda_{\rm p}^3 \gamma}{8\pi^2}$ for decay rate. The group velocity $v_{\rm g}$ of the probe pulse is given by

$$v_{\rm g} = \frac{c}{n_{\rm R} + \omega_{\rm p} \partial n_{\rm R}/\partial\omega_{\rm p}}, \quad (8)$$

where $n_{\rm R} = 1 + 2\pi\chi'(\omega_{\rm p})$ is the real part of the refractive index. The group velocity can be lower than the phase velocity ($v_{\rm g} < c/n_{\rm R}$, subluminal light) when $\partial\chi'/\partial\omega_{\rm p} > 0$ (normal dispersion) in the region of $\chi' \approx 0$, and can be higher than the phase velocity or becomes negative ($v_{\rm g} > c/n_{\rm R}$ or $v_{\rm g} < 0$, superluminal light) when $\partial\chi'/\partial\omega_{\rm p} < 0$ (anomalous dispersion) in the same region.

3. Results and conclusion

For numerical estimation, we set the probe wavelength $\lambda_{\rm p}$, the transition rate γ and the atom density $n_{\rm a}$ as 500nm, 10^7Hz, and $10^{12}{\rm cm}^{-3}$ respectively. Frequencies, detunings and Rabi frequencies are scaled in units of γ. The susceptibility is in unit of $\frac{3 n_{\rm a} \lambda_{\rm p}^3}{8\pi^2}$ as shown in Eqs.(6, 7). Without loss of generality, we assume that the Rabi frequencies Ω_0 and $\Omega_{\rm p}$ and the dipole moment d are real. At the same time we use $\Omega_j = |\Omega_j| {\rm e}^{-{\rm i}\phi_j}$ $(j = 1, 2)$, where $\phi_{1,2}$ are the relative phases of the sideband components $\boldsymbol{E}_{1,2}$ with respect to that of the central component $\boldsymbol{E}_0$, respectively; we also define the sum of the relative phases of the sideband components with respect to that of the central component as $\Phi = \phi_1 + \phi_2$.

Firstly, we focus on the linear susceptibility, which is obtained simply by assuming that the probe field is weak, i.e. $|\Omega_{\rm p}| \ll \gamma, |\Omega_{\rm i}|, (i = 0, 1, 2)$. The real part (solid line) and the imaginary part (dashed line) of the susceptibility are plotted in Fig.1 as functions of the probe detuning $\Delta_{\rm p}$ for $\Omega_0 = |\Omega_1| = |\Omega_2| = \delta = 10$. The other parameters are $\Delta_0 = 0$, $\Omega_{\rm p} = 0.01$, and $\Phi = 0$ (a), $\Phi = \pi$ (b). The sum phase Φ plays a crucial role in the susceptibility. As the sum phase is varied from $\Phi = 0$ to $\Phi = \pi$, the remarkable change in the absorption spectrum is the switching from absorption to gain (negative absorption) at the centre. Switching from normal to anomalous dispersion occurs in three pairs of different frequency regimes, which are indicated by letters A_i, B_i and C_i $(i = 1, 2)$. This results in switching from subluminal to superluminal light propagation.

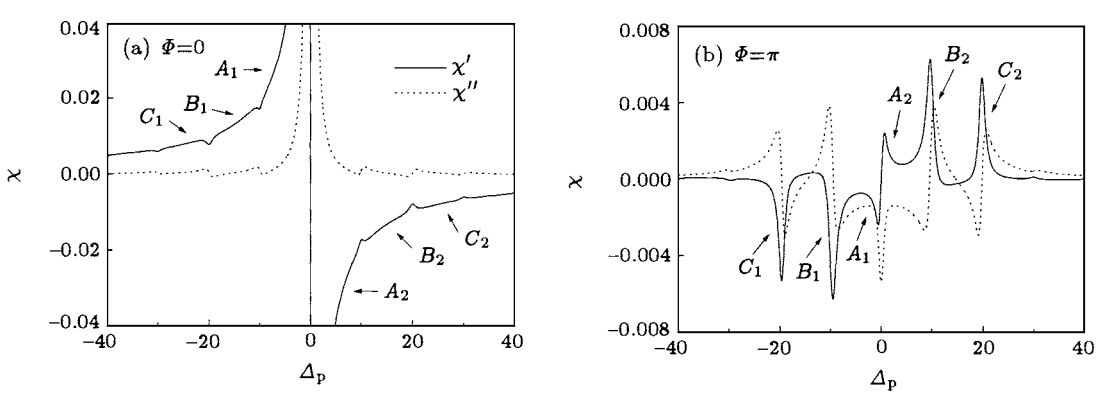


Fig.1. The imaginary (dashed lines) and real (solid lines) parts of the susceptibility as functions of the probe detuning for a weak probe field. (a) $\Phi = 0$ and (b) $\Phi = \pi$.

The group velocity v_g is plotted in Fig.2 for the same parameters as in Fig.1. For clearness, we only show the velocities in the three pairs of regimes, A_1A_2 (top), B_1B_2 (middle), and C_1C_2 (bottom), where slow group velocities (solid lines) occur at $\Phi = 0$ and negative group velocities (dashed lines) appear at $\Phi = \pi$. Note that the slow group velocities are indicated by the left coordinate axes, while the negative group velocities are measured by the right coordinate axes. The overlapped regions are $\Delta_p \approx (\pm 0.60 - \pm 4.09)$, $\Delta_p \approx (\pm 10.72 - \pm 13.14)$, and $\Delta_p \approx (\pm 21.25 - \pm 27.62)$. The group velocity spectra are symmetrical with respect to the atomic resonant transition frequency ω_a ($\Delta_0 = \Delta_p = 0$) because $\omega_p |\partial n_R/\partial \omega_p| = \omega_p |\partial n_R/\partial \Delta_p| \gg n_R$ except those near the frequency at which $|\chi'|$'s take their maximum values. For the atomic density given above, the positive group velocities are reduced to $v_g \sim \left(10^{-6} - 10^{-5}\right) c$ (A_1) and $v_g \sim \left(10^{-4} - 10^{-3}\right) c$ (B_1, C_1), while the negative group velocities are $v_g \sim 10^{-3} c$ (A_2, B_2, C_2).

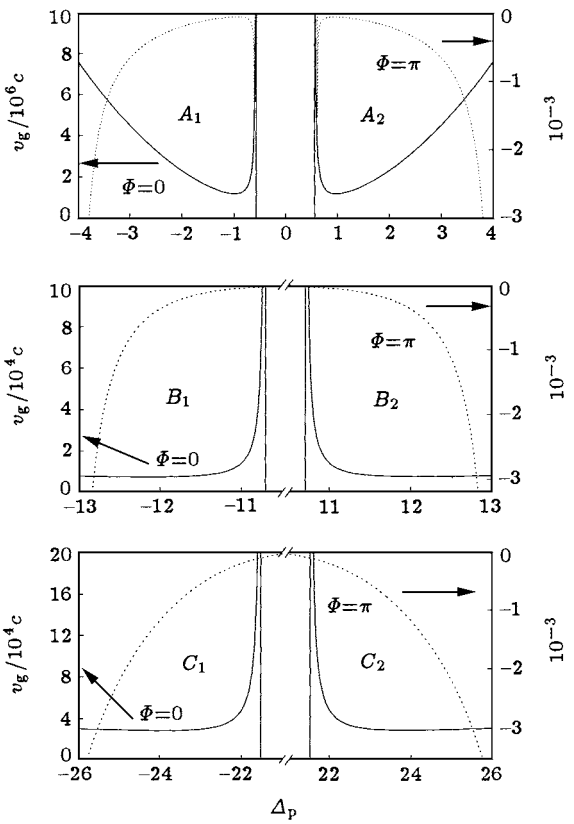


Fig.2. The group velocities versus the probe detuning, with the same parameters as in Fig.1. The group velocities for $\Phi = 0$ are expressed by solid lines and the left coordinate axes, while the group velocities for $\Phi = \pi$ are denoted by dashed lines and the right coordinate axes.

Next, in the same way, the response of the medium to a strong probe field depends strongly on the sum phase. For $\Omega_p = 10$ we plot the imaginary (a) and real (b) parts of the susceptibility in Fig.3, in which solid lines correspond to $\Phi = 0$ and dashed lines to $\Phi = \pi$. The other parameters are the same as in Fig.1. When the sum phase is varied from $\Phi = 0$ to $\Phi = \pi$, absorption is transformed into gain for many frequency regimes, which include the central and sideband ones. At the same time, the real part of the susceptibility changes from positive to negative values for the left sideband regime while it is reversed for the right sideband regime.

Thirdly, when the sum of the two relative phases is fixed, i.e., $\phi_1 + \phi_2 = C$ (constant), the response of the medium is no longer dependent on the respective relative phases. The change in the respective phases $\phi_1 = C - \phi$ and $\phi_2 = \phi$ (ϕ is any real number) does not exert any effect on the absorption and dispersion so long as C is kept unchanged. The remarkably different features for different values $C = 0$ and $C = \pi$ as shown in Fig.1–3 are kept unchanged regardless of the change in the respective values of ϕ_1 and ϕ_2.

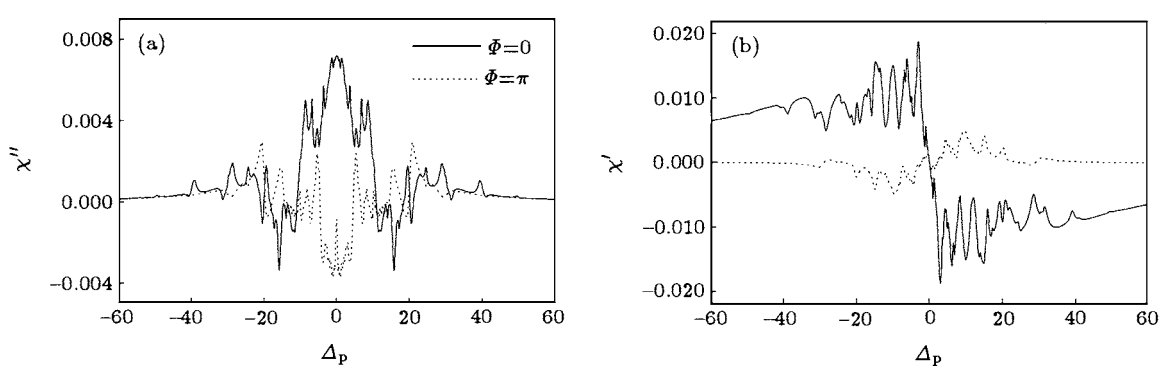


Fig.3. The imaginary (a) and real (b) parts of the susceptibility as functions of the probe detuning for a strong probe field. Solid lines correspond to $\Phi = 0$ and dashed lines to $\Phi = \pi$.

The above phase dependence and independence can be analysed by using the dressed states basis associated with the central driving component. The energy spectrum of the driven two-level system is composed of an infinite set of dressed states doublets.[37]

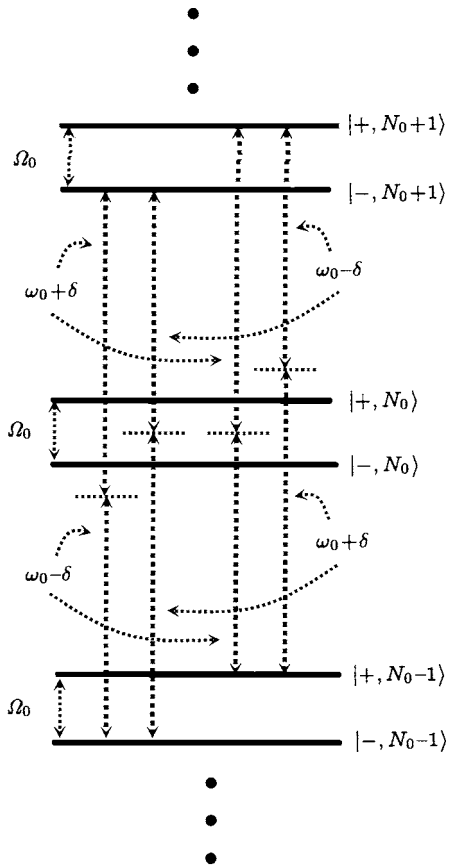


Fig.4. The dressed states associated with the central driving component.

We plot in Fig.4 the energy level diagram of the dressed states $|\pm, N_0\rangle$, $|\pm, N_0 \pm 1\rangle$, where N_0 is the number of photons of the central component. It is well known that phase dependence occurs in a closed loop system in which all transitions are in the loop configuration. For the bichromatic case ($\Omega_0 = 0$), the dressed states are absent, and we are confined to the bare atomic states basis, in which the closed loop is nonexistent and no phase dependence occurs. For the trichromatic case, however, the sideband driving components lead to the two-step transitions such as $|\pm, N_0 - 1\rangle \leftrightarrow |\pm, N_0 + 1\rangle$. Two closed loops appear in the dressed states representation.[30] For each two-step transition, the polarization is determined by the same sum phase $(\phi_1 + \phi_2)$ but not by the respective phases.

In summary, we have identified the phase dependence and independence in the response of the trichromatically driven two-level system to an arbitrarily intense probe field. On one hand, the absorption and dispersion of the medium depend crucially on the sum of the relative phases of the sideband components of the trichromatic field with respect to that of the central component. For the weak probe field, as the sum phase is varied from 0 to π, switching from normal to anomalous dispersion occurs in three pairs of different frequency regimes. For the strong probe field, the response of the medium is governed by the sum phase in the same way. On the other hand, so long as the sum phase is kept unchanged, the absorption and dispersion are independent of the respective phases.

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