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Three-body entanglement induced by spontaneous emission in a three two-level atoms system*

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We study three-body entanglement induced by spontaneous emission in a three two-level atoms system by using the entanglement tensor approach. The results show that the amount of entanglement is strongly dependent on the initial state of the system and the species of atoms. The three-body entanglement is the result of the coherent superposition of the two-body entanglements. The larger the two-body entanglement is, the stronger the three-body entanglement is. On the other hand, if there exists a great difference in three two-body entanglement measures, the three-body entanglement is very weak. We also find that the maximum of the two-body entanglement obtained with nonidentical atoms is greater than that obtained with identical atoms via adjusting the difference in atomic frequency.

Keywords: spontaneous emission, entanglement measures
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1. Introduction

The subject of generation of entangled states has aroused much interest of all since it became clear that entanglement lay at the heart of many new applications which are listed under the general heading of quantum information and quantum computation. Several methods of creating entanglement involving trapped and cooled ions or neutral atoms have been proposed.[1−8] Recently, the possibility of environment induced entanglement in an open quantum system has opened new intriguing perspectives.[9] For example, it has been shown that dissipation can be exploited to implement nearly decoherence-free quantum gates,[10] and transient entanglement between distant two-level atoms can be induced by atomic spontaneous decay or by cavity losses.[11] It has been also demonstrated that asymptotic (stationary) entangled states of two closely separated two-level atoms in free space can be created by spontaneous emission.[12] Therefore, the creation of the atomic entangled states induced by spontaneous emission is getting more and more interesting.

The properties, preparation, and applications of tripartite entangled states are now a hot topic in the field of quantum information. It is known that quantum correlations in the multipartite setting have a much richer structure than in the bipartite setting.[13] And, it has been shown that tripartite entangled states have an advantage over the two-particle entangled states in quantum teleportation,[14] dense coding[15] and quantum cloning.[16]

To assess how much entanglement is stored in a given quantum system, it is essential to have appropriate measures of entanglement. A number of measures have been proposed, which include entanglement of formation,[17] entanglement of distillation,[18] relative entropy of entanglement[19] and negativity.[20−23] The problem of describing quantitatively the entanglement in a two-particle system has been solved exactly. However, the quantification of the amount of entanglement in three and more subsystems is still under investigation due to the high complexity of entanglement in a multi-particle system. Recently, measures of entanglement in a three-qubit system in terms of

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the entanglement tensor approach[24] has been proposed in Ref.[25]. Entanglement measures of a three-qubit system involve both an inter-three-qubit entanglement measure and an inter-two-qubit entanglement measure. Moreover, according to the definition of entanglement measure, the quantum entanglement of the Tavis–Cummings model for two identical two-level atoms interacting with a single-mode field is studied in Ref.[26].

By using the entanglement tensor approach, we study three-body entanglement induced by spontaneous emission in a three two-level atoms system in this paper. It is shown that the amount of entanglement is strongly dependent on the initial state of the system and the species of atoms. The three-body entanglement is the result of the coherent superposition of the two-body entanglements. The larger the two-body entanglement is, the stronger the three-body entanglement is. On the other hand, if there exists a great difference in three two-body entanglement measures, the three-body entanglement is very weak. We also find that the maximum of the two-body entanglement obtained with nonidentical atoms is greater than that obtained with identical atoms via adjusting the difference in atomic frequency.

2. The three two-level atoms system

We consider a system of three non-overlapping two-level atoms connected by the dipole transition moments $\boldsymbol{\mu}_i$. The atoms are located at fixed positions $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$, and coupled to all modes of the electromagnetic field, which are assumed to be in the vacuum state. We consider spontaneous emission from identical as well as non-identical atoms prepared in different initial states. In the case of nonidentical atoms, we assume that atoms have equal dipole moments $\hat{\mu}_1 = \hat{\mu}_2 = \hat{\mu}_3 = \hat{\mu}$, but different transition frequencies ω_1, ω_2 and ω_3, such that $\omega_i - \omega_j \ll (\omega_i + \omega_j)/2$ $(i, j = 1, 2, 3)$, so that the rotating-wave approximation can be applied to calculate the dynamics of the system.

The time evolution of the system of atoms coupled through the vacuum field is given by the following master equation:[27–29]

$$\frac{\partial \rho}{\partial t} = -\frac{\mathrm{i}}{\hbar}[H_s, \rho] - \frac{1}{2}\sum_{i,j=1}^{3} \Gamma_{ij}(\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+), \quad (1)$$

where

$$H_s = \hbar \sum_{i=1}^{3} \omega_i S_i^z + \hbar \sum_{i \neq j}^{3} \Omega_{ij} S_i^+ S_j^-, \quad (2)$$

with, $S_i^+ (S_i^-)$ being the dipole raising(lowering) operators, S_i^z being the energy operator of the ith atom, and $\Gamma_{ij}(i = j)$ being the spontaneous emission rates of the atoms, equal to the Einstein coefficient for spontaneous emission. Γ_{ij} and $\Omega_{ij}(i \neq j)$ describe the interatomic coupling, and are the collective damping and the dipole–dipole interaction potential defined, respectively, by

$$\Gamma_{ij} = \Gamma_{ji} = \frac{3}{2}\Gamma\left\{[1 - (\hat{\mu}.\hat{r}_{ij})^2]\frac{\sin(k_0 r_{ij})}{k_0 r_{ij}} + [1 - 3(\hat{\mu}.\hat{r}_{ij})^2]\left[\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3}\right]\right\} \quad (3)$$

and

$$\Omega_{ij} = \frac{3}{4}\Gamma\left\{-[1 - (\hat{\mu}.\hat{r}_{ij})^2]\frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} + [1 - 3(\hat{\mu}.\hat{r}_{ij})^2]\left[\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3}\right]\right\}, \quad (4)$$

where $k_0 = \omega_0/c$, $r_{ij} = |\boldsymbol{r}_j - \boldsymbol{r}_i|$ is the distance between the atoms, $\hat{\mu}$ is the unit vector along the atomic transition dipole moments, which are assumed to be parallel to each other, and $\hat{r}_{ij}$ is the unit vector along the interatomic axis. In this paper, we consider the case where $\hat{\mu}$ is perpendicular to $\hat{r}_{ij}$.

Using the master equation (1), we can write down the equations of motion for the components of the density matrix of the three-atom system on the basis of the product states $|111\rangle$, $|110\rangle$, $|101\rangle$, $|100\rangle$, $|011\rangle$, $|010\rangle$, $|001\rangle$, $|000\rangle$ of the individual atoms (see appendix). From these equations, if three atoms are initially in the excited state ($|\phi_0\rangle = |111\rangle$, i.e. $\rho_{11}(0) = 1$), one can find that the nonzero components of the density matrix are

$$
\begin{aligned}
\dot{\rho}_{11} =& -(\Gamma_{11}+\Gamma_{22}+\Gamma_{33})\rho_{11},\\
\dot{\rho}_{22} =& -(\Gamma_{11}+\Gamma_{22})\rho_{22}+\Gamma_{33}\rho_{11}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{23}\\
&+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{25}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{32}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{52},\\
\dot{\rho}_{23} =& \left(-\mathrm{i}\omega_2+\mathrm{i}\omega_3-\Gamma_{11}-\frac{1}{2}\Gamma_{22}-\frac{1}{2}\Gamma_{33}\right)\rho_{23}+\Gamma_{23}\rho_{11}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{22}\\
&+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{25}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{33}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{53},\\
\dot{\rho}_{25} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_3-\Gamma_{22}-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{33}\right)\rho_{25}+\Gamma_{13}\rho_{11}+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{22}\\
&+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{23}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{55},\\
\dot{\rho}_{33} =& (-\Gamma_{11}-\Gamma_{33})\rho_{33}+\Gamma_{22}\rho_{11}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{32}\\
&+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{23}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{53},\\
\dot{\rho}_{35} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_2-\Gamma_{33}-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{22}\right)\rho_{35}+\Gamma_{12}\rho_{11}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{33}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{32}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{55}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{25},\\
\dot{\rho}_{44} =& -\Gamma_{11}\rho_{44}+\Gamma_{23}\rho_{32}+\Gamma_{32}\rho_{23}+\Gamma_{22}\rho_{22}+\Gamma_{33}\rho_{33}+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{46}\\
&+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{64}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{74},\\
\dot{\rho}_{46} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_2-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{22}\right)\rho_{46}+\Gamma_{12}\rho_{22}+\Gamma_{13}\rho_{32}+\Gamma_{32}\rho_{25}+\Gamma_{33}\rho_{35}\\
&+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{44}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{66}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{76},\\
\dot{\rho}_{47} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_3-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{33}\right)\rho_{47}+\Gamma_{12}\rho_{23}+\Gamma_{13}\rho_{33}+\Gamma_{23}\rho_{35}+\Gamma_{22}\rho_{25}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{44}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{46}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{77},\\
\dot{\rho}_{55} =& -(\Gamma_{22}+\Gamma_{33})\rho_{55}+\Gamma_{11}\rho_{11}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{53}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{52}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{25},\\
\dot{\rho}_{66} =& -\Gamma_{22}\rho_{66}+\Gamma_{11}\rho_{22}+\Gamma_{33}\rho_{55}+\Gamma_{13}\rho_{52}+\Gamma_{31}\rho_{25}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{64}\\
&+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{46}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{76},\\
\dot{\rho}_{67} =& \left(-\mathrm{i}\omega_2+\mathrm{i}\omega_3-\frac{1}{2}\Gamma_{22}-\frac{1}{2}\Gamma_{33}\right)\rho_{67}+\Gamma_{21}\rho_{25}+\Gamma_{13}\rho_{53}+\Gamma_{23}\rho_{55}+\Gamma_{11}\rho_{23}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{64}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{66}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{77},\\
\dot{\rho}_{77} =& -\Gamma_{33}\rho_{77}+\Gamma_{12}\rho_{53}+\Gamma_{21}\rho_{35}+\Gamma_{11}\rho_{33}+\Gamma_{22}\rho_{55}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{76}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{74}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{47}. \qquad (5)
\end{aligned}
$$

We note that all other components of the density matrix not only contain the terms oscillating at higher frequencies ω_i, $\omega_i+\omega_j$ $(i,j=1,2,3)$, but also are zero under this initial condition. Moreover, the above initial condition that three atoms are in the excited state $(|\phi_0\rangle = |111\rangle)$ is ideal and feasible experimentally. It is difficult to obtain the analytical solutions to Eqs.(5), so we solve these equations subject to the relevant initial condition by using the fourth-order Runge–Kutta method.

3. Entanglement measures of a three-particle system

We will study quantum entanglement in a three-particle system by making use of the entanglement tensor approach that was first proposed in Ref.[24] and further developed for a three-particle system in Ref.[25]. Entanglement measures of a three-qubit system involve both an inter-three-qubit entanglement measure denoted by E_3 and an inter-two-qubit entanglement measure labelled by E_2. Based on entanglement tensors $M(1,2,3)$ and $M(m,n)$ $(1\le m<n\le 3)$ introduced in Ref.[24], the entanglement measures E_3 and E_2 can be defined as follows:

$$E_3=\frac{1}{4}\sum_{i,j,k=1}^{3}M_{ijk}(1,2,3)M_{ijk}(1,2,3), \tag{6}$$

$$E_2(m,n)=\frac{1}{3}\sum_{i,j=1}^{3}M_{ij}(m,n)M_{ij}(m,n). \tag{7}$$

Here the entanglement tensors in Eqs.(6) and (7) are defined by

$$M_{ij}(m,n)=K_{ij}(m,n)-\lambda_i(m)\lambda_j(n), \tag{8}$$

$$\begin{aligned}M_{ijk}(1,2,3)=&K_{ijk}(1,2,3)-\lambda_i(1)M_{jk}(2,3)\\&-\lambda_j(2)M_{ik}(1,3)-\lambda_k(3)M_{ij}(1,2)\\&-\lambda_i(1)\lambda_j(2)\lambda_k(3),\end{aligned} \tag{9}$$

where $K_{ijk}(1,2,3)$ and $K_{ij}(m,n)$, and $\lambda_i(m)$ are the correlation tensors and coherence vectors defined, respectively, by

$$\begin{aligned}&K_{ij}(1,2)=\mathrm{Tr}(\rho\cdot\sigma_{\mathrm{i}}\otimes\sigma_j\otimes I),\\&K_{ik}(1,3)=\mathrm{Tr}(\rho\cdot\sigma_{\mathrm{i}}\otimes I\otimes\sigma_k),\\&K_{jk}(2,3)=\mathrm{Tr}(\rho\cdot I\otimes\sigma_j\otimes\sigma_k),\\&K_{ijk}(1,2,3)=\mathrm{Tr}(\rho\cdot\sigma_{\mathrm{i}}\otimes\sigma_j\otimes\sigma_k),\\&\lambda_i(1)=\mathrm{Tr}(\rho\cdot\sigma_{\mathrm{i}}\otimes I\otimes I),\\&\lambda_j(2)=\mathrm{Tr}(\rho\cdot I\otimes\sigma_j\otimes I),\\&\lambda_k(3)=\mathrm{Tr}(\rho\cdot I\otimes I\otimes\sigma_k),\end{aligned} \tag{10}$$

where σ_i, σ_j and σ_k are Pauli matrices, I is the unit matrix, and ρ is the density operator of the system. The values of entanglement measures can change between 0 and 1. For unentangled atoms, the values of entanglement measures are 0. For the maximally entangled states, the values of entanglement measures are 1, which is useful in quantum information processing. E_3 quantifies the three-qubit entanglement. The larger E_3 is, the stronger the three-qubit entanglement is. And $E_2(m,n)$ quantifies the entanglement between any two qubits m and n in the three-qubit system. The larger $E_2(m,n)$ is, the stronger the entanglement between two qubits m and n is. Physically, the practical application of entanglement requires a considerable amount of entanglements. Making use of Eqs.(10), we can obtain the nonzero components of the coherence vectors and correlation tensors

$$\begin{aligned}\lambda_3(1)=&2(\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44})-1,\\\lambda_3(2)=&2(\rho_{11}+\rho_{22}+\rho_{55}+\rho_{66})-1,\\\lambda_3(3)=&2(\rho_{11}+\rho_{33}+\rho_{55}+\rho_{77})-1,\\K_{11}(1,2)=&K_{22}(1,2)=\rho_{35}+\rho_{46}+\rho_{53}+\rho_{64},\\K_{12}(1,2)=&K_{21}^{*}(1,2)=-\mathrm{i}\rho_{35}-\mathrm{i}\rho_{46}+\mathrm{i}\rho_{53}+\mathrm{i}\rho_{64},\\K_{33}(1,2)=&-2(\rho_{33}+\rho_{44}+\rho_{55}+\rho_{66})+1,\\K_{11}(1,3)=&K_{22}(1,3)=\rho_{25}+\rho_{47}+\rho_{52}+\rho_{74},\\K_{12}(1,3)=&K_{21}^{*}(1,3)=-\mathrm{i}\rho_{25}-\mathrm{i}\rho_{47}+\mathrm{i}\rho_{52}+\mathrm{i}\rho_{74},\\K_{33}(1,3)=&-2(\rho_{22}+\rho_{44}+\rho_{55}+\rho_{77})+1,\\K_{11}(2,3)=&K_{22}(2,3)=\rho_{23}+\rho_{32}+\rho_{67}+\rho_{76},\\K_{12}(2,3)=&K_{21}^{*}(2,3)=-\mathrm{i}\rho_{23}-\mathrm{i}\rho_{67}+\mathrm{i}\rho_{32}+\mathrm{i}\rho_{76},\\K_{33}(2,3)=&-2(\rho_{22}+\rho_{33}+\rho_{66}+\rho_{77})+1,\\K_{113}(1,2,3)=&K_{223}(1,2,3)\\=&\rho_{35}-\rho_{46}+\rho_{53}-\rho_{64},\\K_{123}(1,2,3)=&K_{213}^{*}(1,2,3)\\=&-\mathrm{i}\rho_{35}+\mathrm{i}\rho_{46}+\mathrm{i}\rho_{53}-\mathrm{i}\rho_{64},\\K_{131}(1,2,3)=&K_{232}(1,2,3)\\=&\rho_{25}-\rho_{47}+\rho_{52}-\rho_{74},\\K_{132}(1,2,3)=&K_{231}^{*}(1,2,3)\\=&-\mathrm{i}\rho_{25}+\mathrm{i}\rho_{47}+\mathrm{i}\rho_{52}-\mathrm{i}\rho_{74},\\K_{311}(1,2,3)=&K_{322}(1,2,3)\\=&\rho_{23}-\rho_{67}+\rho_{32}-\rho_{76},\\K_{312}(1,2,3)=&K_{321}^{*}(1,2,3)\\=&-\mathrm{i}\rho_{23}+\mathrm{i}\rho_{67}+\mathrm{i}\rho_{32}-\mathrm{i}\rho_{76},\\K_{333}(1,2,3)=&2(\rho_{11}+\rho_{44}+\rho_{66}+\rho_{77})-1.\end{aligned} \tag{11}$$

From Eqs.(8) and (9), one can find the nonzero components of the entanglement tensor

$$
\begin{aligned}
M_{11}(1,2) =& M_{22}(1,2) = K_{11}(1,2),\\
M_{12}(1,2) =& M_{21}^{*}(1,2) = K_{12}(1,2),\\
M_{33}(1,2) =& K_{33}(1,2) - \lambda_3(1)\lambda_3(2),\\
M_{11}(1,3) =& M_{22}(1,3) = K_{11}(1,3),\\
M_{12}(1,3) =& M_{21}^{*}(1,3) = K_{12}(1,3),\\
M_{33}(1,3) =& K_{33}(1,3) - \lambda_3(1)\lambda_3(3),\\
M_{11}(2,3) =& M_{22}(2,3) = K_{11}(2,3),\\
M_{12}(2,3) =& M_{21}^{*}(2,3) = K_{12}(2,3),\\
M_{33}(2,3) =& K_{33}(2,3) - \lambda_3(2)\lambda_3(3),\\
M_{113}(1,2,3) =& M_{223}(1,2,3)\\
=& K_{113}(1,2,3) - \lambda_3(3)K_{11}(1,2),\\
M_{123}(1,2,3) =& M_{213}^{*}(1,2,3)\\
=& K_{123}(1,2,3) - \lambda_3(3)K_{12}(1,2),\\
M_{131}(1,2,3) =& M_{232}(1,2,3)\\
=& K_{131}(1,2,3) - \lambda_3(2)K_{11}(1,3),\\
M_{132}(1,2,3) =& M_{231}^{*}(1,2,3)\\
=& K_{132}(1,2,3) - \lambda_3(2)K_{12}(1,3),\\
M_{311}(1,2,3) =& M_{322}(1,2,3)\\
=& K_{311}(1,2,3) - \lambda_3(1)K_{11}(2,3),\\
M_{312}(1,2,3) =& M_{321}^{*}(1,2,3)\\
=& K_{312}(1,2,3) - \lambda_3(1)K_{12}(2,3),\\
M_{333}(1,2,3) =& K_{333}(1,2,3) - \lambda_3(1)K_{33}(2,3)\\
& - \lambda_3(2)K_{33}(1,3) - \lambda_3(3)K_{33}(1,2)\\
& - \lambda_3(1)\lambda_3(2)\lambda_3(3), \qquad (12)
\end{aligned}
$$

while all other components of the entanglement tensor vanish. In Section 4, we will study the three-body quantum entanglement induced by spontaneous emission in the three two-level atoms system by using Eqs.(6),(7),(11) and (12).

4. Entanglement in the three-atom system

Let us now study the three-body quantum entanglement induced by spontaneous emission from identical as well as non-identical atoms prepared in different initial states. We assume that three atoms are located in a line. The interatomic distance $r_{12} = r_{23} = \dfrac{\lambda}{12}$ that gives the values $\Gamma_{12} = \Gamma_{23} = 0.95\Gamma$ and $\Omega_{12} = \Omega_{23} = 9.30\Gamma$, and $r_{13} = \dfrac{\lambda}{6}$ that gives the values $\Gamma_{13} = 0.79\Gamma$ and $\Omega_{13} = 1.12\Gamma$.

4.1. Identical atoms

We begin with spontaneous emission from three identical atoms ($\Gamma_{11} = \Gamma_{22} = \Gamma_{33} = \Gamma$, $\omega_1 = \omega_2 = \omega_3$) and consider the evolutions of entanglement measures for three different initial states.

Case 1 Initial state with only the second atom excited: $|\phi_0\rangle = |010\rangle$. Here, we have $\rho_{66}(0) = 1$.

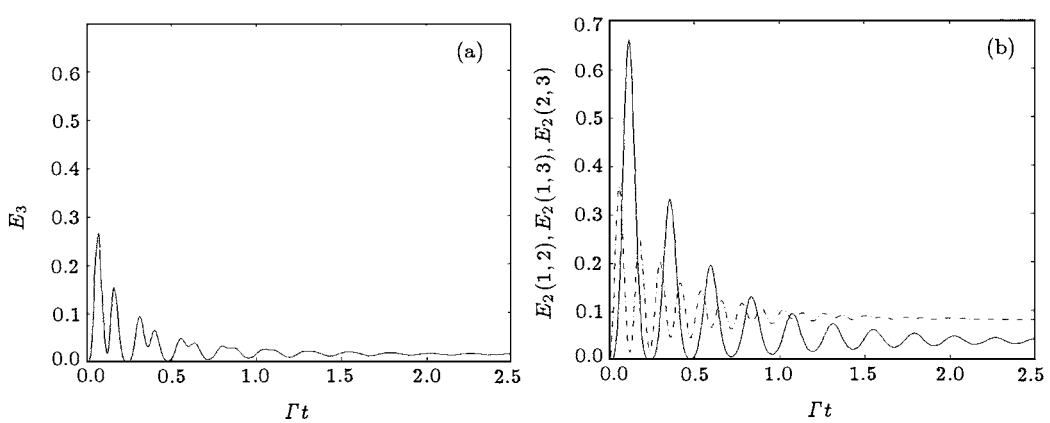


Fig.1. Time evolutions of entanglement measures for identical atoms with initially only the second atom excited ($|\phi_0\rangle = |010\rangle$). (a) the time evolution of E_3; (b) the time evolutions of $E_2(1,3)$ (solid line) and $E_2(1,2) = E_2(2,3)$ (dashed-dotted line).

In Fig.1, we plot the time evolutions of the entanglement measures E_3, $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ for initially only the second atom being excited ($|\phi_0\rangle = |010\rangle$). It can be seen from Figs.1(a) and (b) that the entanglement measures exhibit oscillatory behaviours because of the sufficiently large values of the dipole–dipole interaction, i.e. $\Omega_{12} = \Omega_{23} = 9.30\Gamma \gg \Gamma$. Moreover, due to the dissipation of energy in the sys-

tem, the oscillation and the peaks of the entanglement measures decrease as time goes on. Over a long time, the entanglement measures go to zero. And, the amount of $E_2(1,2)$ entanglement is always equal to the amount of $E_2(2,3)$ entanglement (see Fig.1(b)). This result can be explained as follows. From Eqs.(5), when $\rho_{66}(0)=1$, we have

$$\rho_{11}=\rho_{22}=\rho_{23}=\rho_{25}=\rho_{33}=\rho_{35}=\rho_{55}=0, \quad (13)$$

and

$$\begin{aligned}
\dot{\rho}_{44} =& -\Gamma_{11}\rho_{44}+\Big(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\Big)\rho_{46},\\
&+\Big(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\Big)\rho_{47}+\Big(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\Big)\rho_{64}+\Big(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{74},\\
\dot{\rho}_{46} =& \Big(-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{22}\Big)\rho_{46},\\
&+\Big(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12})\rho_{44}+\Big(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\Big)\rho_{47}+\Big(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\Big)\rho_{66}+\Big(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{76},\\
\dot{\rho}_{47} =& \Big(-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{33}\Big)\rho_{47},\\
&+\Big(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{44}+\Big(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\Big)\rho_{46}+\Big(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\Big)\rho_{67}+\Big(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{77},\\
\dot{\rho}_{67} =& \Big(-\frac{1}{2}\Gamma_{22}-\frac{1}{2}\Gamma_{33}\Big)\rho_{67},\\
&+\Big(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{64}+\Big(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\Big)\rho_{66}+\Big(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\Big)\rho_{47}+\Big(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\Big)\rho_{77},\\
\dot{\rho}_{77} =& -\Gamma_{33}\rho_{77}+\Big(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\Big)\rho_{76},\\
&+\Big(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\Big)\rho_{74}+\Big(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\Big)\rho_{67}+\Big(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\Big)\rho_{47}. \quad (14)
\end{aligned}$$

It is easy to obtain $\rho_{46}=\rho_{76}$, $\rho_{44}=\rho_{77}$ and $\rho_{47}=\rho_{74}$. From Eqs.(11), (12) and (7), we have $E_2(1,2)=E_2(2,3)\neq E_2(1,3)$. We also see that when the entanglement measure $E_2(1,3)$ intersects the entanglement measure $E_2(1,2)$, the peak of the entanglement measure E_3 occurs. And, the larger the value of intersecting point is, the higher the peak of E_3 is. This means that the three-body entanglement is the result of the coherent superposition of the two-body entanglements. The three-body entanglement will vary with two-body entanglements changing. The larger the two-body entanglement is, the stronger the three-body entanglement is. The above result sheds light on the idea that the dissipation can assist the generation of entanglement especially three-partite entanglement.

Case 2 Initial state with three atoms excited: $|\phi_0\rangle=|111\rangle$. Here, we have $\rho_{11}(0)=1$.

The time evolutions of the entanglement measures E_3, $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ are shown in Fig.2 for initially three atoms being excited ($|\phi_0\rangle=|111\rangle$). From this figure, we can see that the amount of $E_2(1,2)$ entanglement is equal to the amount of $E_2(2,3)$ entanglement. And, the amount of $E_2(1,3)$ entanglement is always less than that of $E_2(1,2)$ entanglement and also that of $E_2(2,3)$ entanglement. And, the maximal value of the entanglement measure $E_2(1,3)$ in the case of initially three atoms being excited is much less than that in the case of initially only the second atom being excited; the maximal value of $E_2(1,3)$ is 0.66 (see Fig.1(b)). We also see that the entanglement measure E_3 is very small because of the coherent superposition of the entanglement measures $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ in this case.

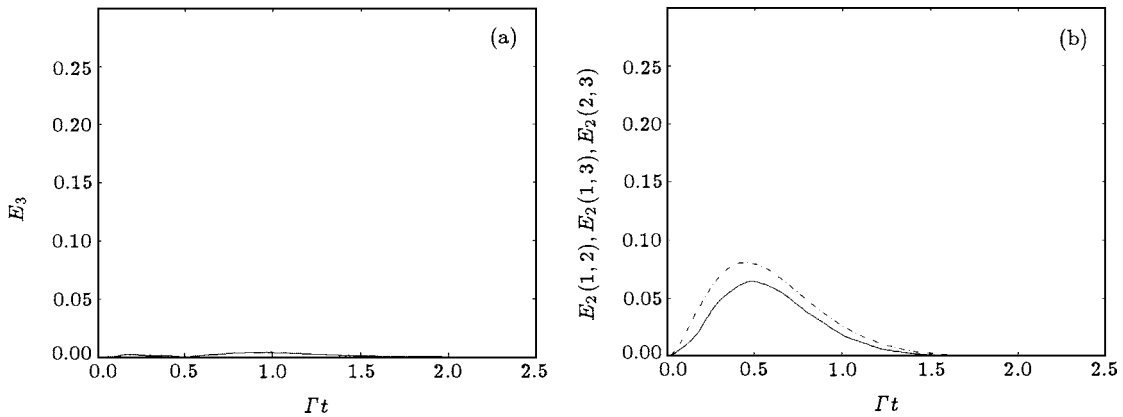


Fig.2. Time evolutions of entanglement measures for identical atoms with initially three atoms excited ($|\phi_0\rangle = |111\rangle$). (a) the time evolution of E_3; (b) the time evolutions of $E_2(1,3)$ (solid line) and $E_2(1,2) = E_2(2,3)$ (dashed-dotted line).

Case 3 Initial state with only the third atom excited: $|\phi_0\rangle = |001\rangle$. Here, we have $\rho_{77}(0) = 1$.

Figure 3 displays the time evolutions of the entanglement measures E_3, $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ for initially only the third atom being excited ($|\phi_0\rangle = |001\rangle$). Comparing Fig.3(a) with Fig.1(a), it can be seen that the maximum of the entanglement measure E_3 in the case of initially only the third atom being excited is less than that in the case of initially only the second atom being excited. We also see that the time evolution of the entanglement measure $E_2(2,3)$ is not identical with the time evolution of the entanglement measure $E_2(1,2)$ and that of $E_2(1,3)$ in Fig.3(b). The maximum of the entanglement measure $E_2(2,3)$ is 0.65, which is greater than that of the entanglement measure $E_2(1,2)$ and also that of $E_2(1,3)$.

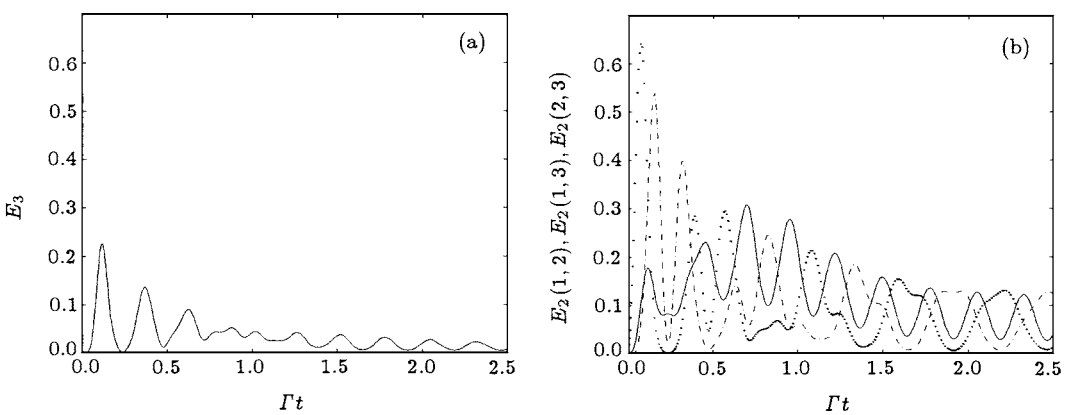


Fig.3. Time evolutions of entanglement measures for identical atoms with initially only the third atom excited ($|\phi_0\rangle = |001\rangle$). (a) the time evolution of E_3; (b) the time evolutions of $E_2(1,3)$ (solid line), $E_2(1,2)$ (dashed-dotted line) and $E_2(2,3)$ (dotted line).

4.2. Nonidentical atoms

For nonidentical atoms, we assume $\Gamma_{11} = \Gamma_{22} = \Gamma_{33} = \Gamma$, but $\omega_1 \neq \omega_2 \neq \omega_3$. And, we consider the evolutions of entanglement measures for different initial states.

In Fig.4, we present the time evolutions of the entanglement measures E_3, $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ for initially only the third atom being excited ($|\phi_0\rangle = |001\rangle$) with $\omega_1 - \omega_2 = 32\Gamma$, $\omega_2 - \omega_3 = 17\Gamma$ and $\omega_1 - \omega_3 = 49\Gamma$. It can be seen from Fig.4(b) that the entanglement measure $E_2(2,3)$ exhibits regular oscillatory behaviours because of the sufficiently large difference in the atomic frequency. Moreover, a comparison between Fig.4(b) and Fig.3(b) shows that the value of the entanglement measure $E_2(2,3)$ at the first maximum is 0.84 in the case of nonidentical atoms, which is greater than that in the case of identical atoms. However, it is obvious from Fig.4 that the amounts of $E_2(1,2)$, $E_2(1,3)$ and E_3 entanglements

are very small. This means that if there appears a great difference in three two-body entanglement measures , the three-body entanglement is very weak due to the coherent superposition of the two-body entanglements.

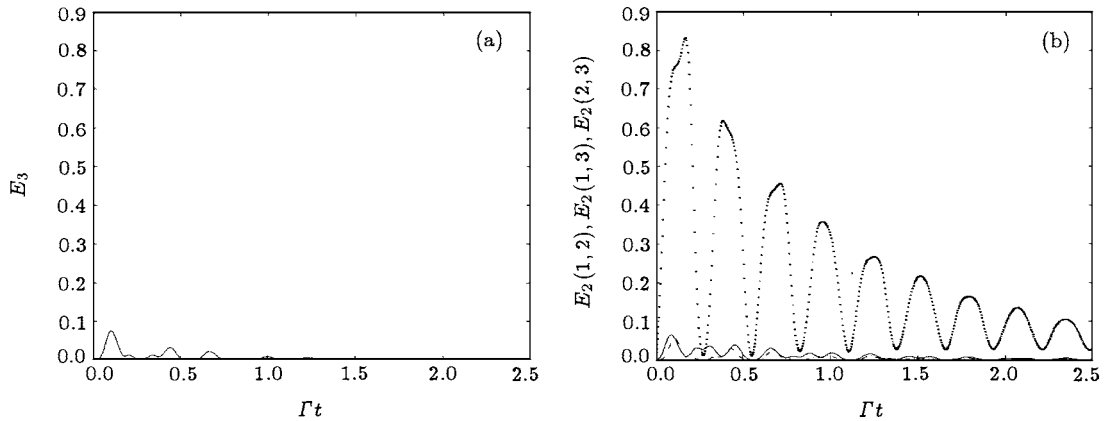


Fig.4. Time evolutions of entanglement measures for non-identical atoms with $\omega_1-\omega_2 = 32\Gamma$, $\omega_2-\omega_3 = 17\Gamma$ and $\omega_1-\omega_3 = 49\Gamma$ with initially only the third atom excited($|\phi_0\rangle = |001\rangle$). (a) the time evolution of E_3; (b) the time evolutions of $E_2(1,3)$ (solid line), $E_2(1,2)$ (dashed-dotted line) and $E_2(2,3)$ (dotted line).

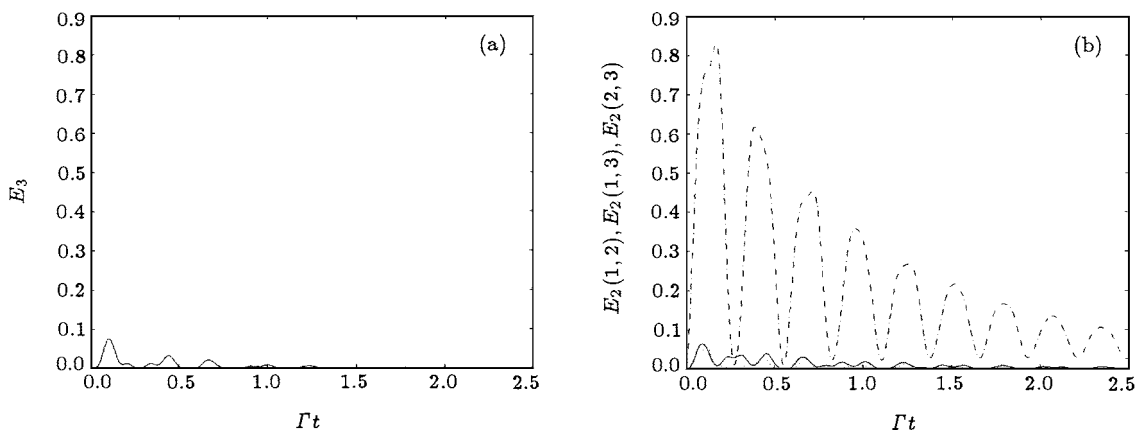


Fig.5. Time evolutions of entanglement measures for non-identical atoms with $\omega_1-\omega_2 = -17\Gamma$, $\omega_1-\omega_3 = -49\Gamma$ and $\omega_2-\omega_3 = -32\Gamma$ with initially only the first atom excited($|\phi_0\rangle = |100\rangle$). (a) the time evolution of E_3; (b) the time evolutions of $E_2(1,3)$ (solid line), $E_2(1,2)$ (dashed-dotted line) and $E_2(2,3)$ (dotted line).

In Fig.5, we plot the time evolutions of the entanglement measures E_3, $E_2(1,2)$, $E_2(2,3)$ and $E_2(1,3)$ for initially only the first atom being excited ($|\phi_0\rangle = |100\rangle$) with $\omega_1-\omega_2 = -17\Gamma$, $\omega_1-\omega_3 = -49\Gamma$ and $\omega_2-\omega_3 = -32\Gamma$. It is shown that the amount of $E_2(1,2)$ entanglement is very big and the value of the first maximum is 0.84, which is quite remarkable. While the amounts of $E_2(1,2)$, $E_2(2,3)$ and E_3 entanglements are very small. From Fig.4 and Fig.5, we can find that the amount of two-body entanglement can greatly increase via adjusting the difference in atomic frequency. It is useful for practical realization of quantum information processing due to the considerable amount of entanglement and the dissipation-assisted entanglement.

5. Conclusion

In this paper, we study the three-body entanglement induced by spontaneous emission in a three two-level atoms system by using the entanglement tensor approach. It is shown that the amount of entanglement is strongly dependent on the initial state of the system and the species of atoms. The three-body entanglement is the result of the coherent superposition of the two-body entanglements. The larger the two-

body entanglement is, the stronger the three-body entanglement is. On the other hand, if there exists a great difference in three two-body entanglement measures, the three-body entanglement is very weak. We also find that the maximum of the two-body entanglement obtained with nonidentical atoms is greater than that obtained with identical atoms via adjusting the difference in atomic frequency.

Appendix

In this appendix, we present the equations of motion for components of the density matrix of the three-atom system by using the master equation (1) as follows:

$$
\begin{aligned}
\dot{\rho}_{11} =& -(\Gamma_{11}+\Gamma_{22}+\Gamma_{33})\rho_{11},\\
\dot{\rho}_{22} =& -(\Gamma_{11}+\Gamma_{22})\rho_{22}+\Gamma_{33}\rho_{11}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{23}\\
&+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{25}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{32}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{52},\\
\dot{\rho}_{23} =& \left(-\mathrm{i}\omega_2+\mathrm{i}\omega_3-\Gamma_{11}-\frac{1}{2}\Gamma_{22}-\frac{1}{2}\Gamma_{33}\right)\rho_{23}+\Gamma_{23}\rho_{11}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{22}\\
&+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{25}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{33}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{53},\\
\dot{\rho}_{25} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_3-\Gamma_{22}-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{33}\right)\rho_{25}+\Gamma_{13}\rho_{11}+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{22}\\
&+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{23}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{55},\\
\dot{\rho}_{33} =& (-\Gamma_{11}-\Gamma_{33})\rho_{33}+\Gamma_{22}\rho_{11}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{32}\\
&+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{23}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{53},\\
\dot{\rho}_{35} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_2-\Gamma_{33}-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{22}\right)\rho_{35}+\Gamma_{12}\rho_{11}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{33}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{32}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{55}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{25},\\
\dot{\rho}_{44} =& -\Gamma_{11}\rho_{44}+\Gamma_{23}\rho_{32}+\Gamma_{32}\rho_{23}+\Gamma_{22}\rho_{22}+\Gamma_{33}\rho_{33}+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{46}\\
&+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{64}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{74},\\
\dot{\rho}_{46} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_2-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{22}\right)\rho_{46}+\Gamma_{12}\rho_{22}+\Gamma_{13}\rho_{32}+\Gamma_{32}\rho_{25}+\Gamma_{33}\rho_{35}\\
&+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{44}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{66}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{76},\\
\dot{\rho}_{47} =& \left(-\mathrm{i}\omega_1+\mathrm{i}\omega_3-\frac{1}{2}\Gamma_{11}-\frac{1}{2}\Gamma_{33}\right)\rho_{47}+\Gamma_{12}\rho_{23}+\Gamma_{13}\rho_{33}+\Gamma_{23}\rho_{35}+\Gamma_{22}\rho_{25}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{44}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{46}+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{77},\\
\dot{\rho}_{55} =& -(\Gamma_{22}+\Gamma_{33})\rho_{55}+\Gamma_{11}\rho_{11}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{53}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{52}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{35}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{25},\\
\dot{\rho}_{66} =& -\Gamma_{22}\rho_{66}+\Gamma_{11}\rho_{22}+\Gamma_{33}\rho_{55}+\Gamma_{13}\rho_{52}+\Gamma_{31}\rho_{25}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{64}\\
&+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{46}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{76},
\end{aligned}
$$

$$
\begin{aligned}
\dot{\rho}_{67} =& \left(-\mathrm{i}\omega_2+\mathrm{i}\omega_3-\frac{1}{2}\Gamma_{22}-\frac{1}{2}\Gamma_{33}\right)\rho_{67}+\Gamma_{21}\rho_{25}+\Gamma_{13}\rho_{53}+\Gamma_{23}\rho_{55}+\Gamma_{11}\rho_{23}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{64}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{66}+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{47}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{77},\\
\dot{\rho}_{77} =& -\Gamma_{33}\rho_{77}+\Gamma_{12}\rho_{53}+\Gamma_{21}\rho_{35}+\Gamma_{11}\rho_{33}+\Gamma_{22}\rho_{55}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{76}\\
&+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{74}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{67}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{47},\\
\dot{\rho}_{12} =& -\left(\mathrm{i}\omega_3+\Gamma_{11}+\Gamma_{22}+\frac{1}{2}\Gamma_{33}\right)\rho_{12}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{13}+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{15},\\
\dot{\rho}_{13} =& -\left(\mathrm{i}\omega_2+\Gamma_{11}+\Gamma_{33}+\frac{1}{2}\Gamma_{22}\right)\rho_{13}+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{15}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{12},\\
\dot{\rho}_{15} =& -\left(\mathrm{i}\omega_1+\Gamma_{22}+\Gamma_{33}+\frac{1}{2}\Gamma_{11}\right)\rho_{15}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{13}+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{12},\\
\dot{\rho}_{14} =& -\left(\mathrm{i}\omega_2+\mathrm{i}\omega_3+\Gamma_{11}+\frac{1}{2}\Gamma_{22}+\frac{1}{2}\Gamma_{33}\right)\rho_{14}+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{16}+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{17},\\
\dot{\rho}_{16} =& -\left(\mathrm{i}\omega_1+\mathrm{i}\omega_3+\frac{1}{2}\Gamma_{11}+\Gamma_{22}+\frac{1}{2}\Gamma_{33}\right)\rho_{16}+\left(\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{14}+\left(\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{17},\\
\dot{\rho}_{17} =& -\left(\mathrm{i}\omega_1+\mathrm{i}\omega_2+\frac{1}{2}\Gamma_{11}+\frac{1}{2}\Gamma_{22}+\Gamma_{33}\right)\rho_{17}+\left(\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{14}+\left(\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{16},\\
\dot{\rho}_{18} =& -\left(\mathrm{i}\omega_1+\mathrm{i}\omega_2+\mathrm{i}\omega_3+\frac{1}{2}\Gamma_{11}+\frac{1}{2}\Gamma_{22}+\frac{1}{2}\Gamma_{33}\right)\rho_{18},\\
\dot{\rho}_{28} =& -\left(\mathrm{i}\omega_1+\mathrm{i}\omega_2+\frac{1}{2}\Gamma_{11}+\frac{1}{2}\Gamma_{22}\right)\rho_{28}+\Gamma_{13}\rho_{14}+\Gamma_{23}\rho_{16}+\Gamma_{33}\rho_{17}\\
&+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{58}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{38},\\
\dot{\rho}_{38} =& -\left(\mathrm{i}\omega_1+\mathrm{i}\omega_3+\frac{1}{2}\Gamma_{11}+\frac{1}{2}\Gamma_{33}\right)\rho_{38}+\Gamma_{32}\rho_{17}+\Gamma_{22}\rho_{16}+\Gamma_{12}\rho_{14}\\
&+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{58}+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{28},\\
\dot{\rho}_{58} =& -\left(\mathrm{i}\omega_2+\mathrm{i}\omega_3+\frac{1}{2}\Gamma_{22}+\frac{1}{2}\Gamma_{33}\right)\rho_{58}+\Gamma_{21}\rho_{16}+\Gamma_{31}\rho_{17}+\Gamma_{11}\rho_{14}\\
&+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{38}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{28},\\
\dot{\rho}_{48} =& -\left(\mathrm{i}\omega_1+\frac{1}{2}\Gamma_{11}\right)\rho_{48}+\Gamma_{12}\rho_{24}+\Gamma_{13}\rho_{34}+\Gamma_{23}\rho_{36}+\Gamma_{32}\rho_{27}+\Gamma_{22}\rho_{26}+\Gamma_{33}\rho_{37}\\
&+\left(-\mathrm{i}\Omega_{12}-\frac{1}{2}\Gamma_{12}\right)\rho_{68}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{78},\\
\dot{\rho}_{68} =& -\left(\mathrm{i}\omega_2+\frac{1}{2}\Gamma_{22}\right)\rho_{68}+\Gamma_{21}\rho_{26}+\Gamma_{31}\rho_{27}+\Gamma_{11}\rho_{24}+\Gamma_{13}\rho_{54}+\Gamma_{23}\rho_{56}+\Gamma_{33}\rho_{57}\\
&+\left(-\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{48}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{78},\\
\dot{\rho}_{78} =& -\left(\mathrm{i}\omega_3+\frac{1}{2}\Gamma_{33}\right)\rho_{78}+\Gamma_{12}\rho_{54}+\Gamma_{21}\rho_{36}+\Gamma_{32}\rho_{57}+\Gamma_{31}\rho_{37}+\Gamma_{11}\rho_{34}+\Gamma_{22}\rho_{56}\\
&+\left(-\mathrm{i}\Omega_{32}-\frac{1}{2}\Gamma_{32}\right)\rho_{68}+\left(-\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{48},\\
\dot{\rho}_{24} =& \left(-\mathrm{i}\omega_2-\Gamma_{11}-\frac{1}{2}\Gamma_{22}\right)\rho_{24}+\Gamma_{23}\rho_{12}+\Gamma_{33}\rho_{13}+\left(\mathrm{i}\Omega_{21}-\frac{1}{2}\Gamma_{21}\right)\rho_{26}\\
&+\left(\mathrm{i}\Omega_{31}-\frac{1}{2}\Gamma_{31}\right)\rho_{27}+\left(-\mathrm{i}\Omega_{13}-\frac{1}{2}\Gamma_{13}\right)\rho_{54}+\left(-\mathrm{i}\Omega_{23}-\frac{1}{2}\Gamma_{23}\right)\rho_{34},
\end{aligned}
$$

$$
\begin{aligned}
\dot{\rho}_{26} &= \left(-\mathrm{i}\omega_1 - \Gamma_{22} - \frac{1}{2}\Gamma_{11}\right)\rho_{26} + \Gamma_{13}\rho_{12} + \Gamma_{33}\rho_{15} + \left(\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{24} \\
&\quad + \left(\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{27} + \left(-\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{56} + \left(-\mathrm{i}\Omega_{23} - \frac{1}{2}\Gamma_{23}\right)\rho_{36}, \\
\dot{\rho}_{27} &= \left(-\mathrm{i}\omega_1 - \mathrm{i}\omega_2 + \mathrm{i}\omega_3 - \frac{1}{2}\Gamma_{11} - \frac{1}{2}\Gamma_{22} - \frac{1}{2}\Gamma_{33}\right)\rho_{27} + \Gamma_{13}\rho_{13} + \Gamma_{23}\rho_{15} + \left(\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{24} \\
&\quad + \left(\mathrm{i}\Omega_{23} - \frac{1}{2}\Gamma_{23}\right)\rho_{26} + \left(-\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{57} + \left(-\mathrm{i}\Omega_{23} - \frac{1}{2}\Gamma_{23}\right)\rho_{37}, \\
\dot{\rho}_{34} &= \left(-\mathrm{i}\omega_3 - \Gamma_{11} - \frac{1}{2}\Gamma_{33}\right)\rho_{34} + \Gamma_{32}\rho_{13} + \Gamma_{22}\rho_{12} + \left(\mathrm{i}\Omega_{21} - \frac{1}{2}\Gamma_{21}\right)\rho_{36} \\
&\quad + \left(\mathrm{i}\Omega_{31} - \frac{1}{2}\Gamma_{31}\right)\rho_{37} + \left(-\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{54} + \left(-\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{24}, \\
\dot{\rho}_{36} &= \left(-\mathrm{i}\omega_1 + \mathrm{i}\omega_2 - \mathrm{i}\omega_3 - \frac{1}{2}\Gamma_{11} - \frac{1}{2}\Gamma_{22} - \frac{1}{2}\Gamma_{33}\right)\rho_{36} + \Gamma_{12}\rho_{12} + \Gamma_{32}\rho_{15} + \left(\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{34} \\
&\quad + \left(\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{37} + \left(-\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{56} + \left(-\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{26}, \\
\dot{\rho}_{37} &= \left(-\mathrm{i}\omega_1 - \Gamma_{33} - \frac{1}{2}\Gamma_{11}\right)\rho_{37} + \Gamma_{12}\rho_{13} + \Gamma_{22}\rho_{15} + \left(\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{34} \\
&\quad + \left(\mathrm{i}\Omega_{23} - \frac{1}{2}\Gamma_{23}\right)\rho_{36} + \left(-\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{57} + \left(-\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{27}, \\
\dot{\rho}_{45} &= \left(-\mathrm{i}\omega_1 + \mathrm{i}\omega_2 + \mathrm{i}\omega_3 - \frac{1}{2}\Gamma_{11} - \frac{1}{2}\Gamma_{22} - \frac{1}{2}\Gamma_{33}\right)\rho_{45} + \Gamma_{21}\rho_{21} + \Gamma_{13}\rho_{31} + \left(\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{43} \\
&\quad + \left(\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{42} + \left(-\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{65} + \left(-\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{75}, \\
\dot{\rho}_{56} &= \left(-\mathrm{i}\omega_3 - \Gamma_{22} - \frac{1}{2}\Gamma_{33}\right)\rho_{56} + \Gamma_{31}\rho_{15} + \Gamma_{11}\rho_{12} + \left(\mathrm{i}\Omega_{12} - \frac{1}{2}\Gamma_{12}\right)\rho_{54} \\
&\quad + \left(\mathrm{i}\Omega_{32} - \frac{1}{2}\Gamma_{32}\right)\rho_{57} + \left(-\mathrm{i}\Omega_{21} - \frac{1}{2}\Gamma_{21}\right)\rho_{36} + \left(-\mathrm{i}\Omega_{31} - \frac{1}{2}\Gamma_{31}\right)\rho_{26}, \\
\dot{\rho}_{57} &= \left(-\mathrm{i}\omega_2 - \Gamma_{33} - \frac{1}{2}\Gamma_{22}\right)\rho_{57} + \Gamma_{21}\rho_{15} + \Gamma_{11}\rho_{13} + \left(\mathrm{i}\Omega_{13} - \frac{1}{2}\Gamma_{13}\right)\rho_{54} \\
&\quad + \left(\mathrm{i}\Omega_{23} - \frac{1}{2}\Gamma_{23}\right)\rho_{56} + \left(-\mathrm{i}\Omega_{21} - \frac{1}{2}\Gamma_{21}\right)\rho_{37} + \left(-\mathrm{i}\Omega_{31} - \frac{1}{2}\Gamma_{31}\right)\rho_{27},
\end{aligned}
\tag{A1}
$$

with the condition $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} + \rho_{55} + \rho_{66} + \rho_{77} + \rho_{88} = 1$.

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