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Partially coherent nonparaxial modified Bessel–Gauss beams*

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The concept of partially coherent nonparaxial modified Bessel–Gauss (MBG) beams is proposed. Based on the generalized Rayleigh-Sommerfeld diffraction integral, the analytical propagation equations of nonparaxial MBG beams in free space are derived and analysed, and some special cases are discussed. In particular, under the paraxial approximation our results reduce to the corresponding paraxial ones. Numerical calculation examples are given to illustrate the dependence of intensity and spectral degree of coherence on the beam order m, ξ and f parameters, and to compare the difference between the paraxial and nonparaxial results.

Keywords: partially coherent nonparaxial modified Bessel–Gauss (MBG) beam, generalized Rayleigh-Sommerfeld diffraction integral, free-space propagation

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1. Introduction

In 2001 Ponomarenko introduced a new family of partially coherent beams carrying optical vortices, which is called the partially coherent modified Bessel–Gauss (MBG) beam.[1] The parametric characterization and propagation properties of MBGs have been studied extensively within the framework of the paraxial approximation.[1,2] However, the paraxial theory fails when the beam width and wavelength are comparable, and/or the far-field divergence angle becomes large.[3−8] In such a case the nonparaxial approach should be used. The present paper is aimed at studying partially coherent MBG beams beyond the paraxial approximation. In Section 2, starting from the generalized Rayleigh-Sommerfeld diffraction integral, analytical free-space propagation equations of nonparaxial MBG beams are derived and analysed. Some interesting cases, such as the on-axis and far-field expressions, as well as the paraxial ones are treated as special cases of our general results. Numerical illustrative examples and analysis are given in Section 3. Finally, in Section 4 a brief summary concludes the paper.

2. Theory

Let us consider a partially coherent MBG beam whose cross-spectral density function $W(\rho_{01}, \varphi_{01}, \rho_{02}, \varphi_{02}, 0)$ in the initial plane at z=0 in the cylindrical coordinate system is expressed as[1]

$$\begin{aligned} &W(\rho_{01}, \varphi_{01}, \rho_{02}, \varphi_{02}, 0) \\ =&\frac{\xi^{-m/2}}{1-\xi}\exp[-\mathrm{i}m(\varphi_{01}-\varphi_{02})] \\ &\times \exp\left(-\frac{1+\xi}{1-\xi}\frac{\rho_{01}^2+\rho_{02}^2}{w_0^2}\right) I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_{01}\rho_{02}}{w_0^2}\right), \quad (1) \end{aligned}$$

where $(\rho_{0j}, \varphi_{0j})$ (j=1, 2 unless otherwise stated) are coordinates in the plane at z=0, $I_m(\cdot)$ denotes the modified Bessel function of order m, w_0 is the waist width of the Gaussian part (m=0, ξ=0), ξ is the coherence parameter, $0 \le \xi \le 1$, and two limiting cases $\xi \to 0$ and $\xi \to 1$ correspond to the fully coherent and incoherent cases, respectively.

The propagation of partially coherent beams is characterized by the generalized Rayleigh-Sommerfeld diffraction integral,[9] which provides the cross-spectral density expression in the half space $z > 0$

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once the boundary condition in the plane at z=0 is given, i.e.

$$W(\rho_1,\varphi_1,\rho_2,\varphi_2,z) = \frac{1}{\lambda^2}\iint W(\rho_{01},\varphi_{01},\rho_{02},\varphi_{02},0)\frac{\cos\theta_1\cos\theta_2}{R_1R_2} \times \exp[\mathrm{i}k(R_2-R_1)]\mathrm{d}^2\rho_{01}\mathrm{d}^2\rho_{02}, \quad (2)$$

where (ρ_j,φ_j) are coordinates in the z plane, k is the wave number related to the wavelength λ by k=$2\pi/\lambda$, $\cos\theta_j = z/R_j$ and

$$R_j = \sqrt{\rho_{0j}^2+\rho_j^2-2\rho_{0j}\rho_j\cos(\varphi_j-\varphi_{0j})+z^2}. \quad (3)$$

R can be expanded into[10]

$$R_j = r_j + \frac{\rho_{0j}^2-2\rho_{0j}\rho_j\cos(\varphi_j-\varphi_{0j})}{2r_j}. \quad (4)$$

Equation (2) is valid provided that $R_j \gg \lambda$,[9] with $r_j=\sqrt{\rho_j^2+z^2}$. Replacing R_j in the exponential term of Eq.(2) by Eq.(4), and that in other terms by r_j, and substituting Eq.(1) into Eq.(2), after tedious integral calculations, the final result can be arranged as

$$W(\rho_1,\varphi_1,\rho_2,\varphi_2,z) = \frac{z^2k^2}{r_1^2r_2^2}\frac{\xi^{-m/2}}{1-\xi}\frac{1}{4\beta_1\beta_2-\alpha^2} \times \exp[\mathrm{i}k(r_2-r_1)]\exp[-\mathrm{i}m(\varphi_1-\varphi_2)] \times \exp\left(-\frac{\beta_1u_2^2+\beta_2u_1^2}{4\beta_1\beta_2-\alpha^2}\right) I_m\left(\frac{\alpha u_1u_2}{4\beta_1\beta_2-\alpha^2}\right), \quad (5)$$

where

$$\beta_j = \frac{1+\xi}{1-\xi}\frac{1}{w_0^2} + (-1)^{j+1}\frac{\mathrm{i}k}{2r_j}, \quad (6)$$

$$u_j = \frac{k}{r_j}\rho_j, \quad (7)$$

$$\alpha = \frac{4\sqrt{\xi}}{(1-\xi)w_0^2}. \quad (8)$$

In the derivation of Eq.(5) the following integral formulae of Bessel functions were used:[11]

$$\int_0^{2\pi}\exp[\mathrm{i}x\cos(\varphi-\phi)-\mathrm{i}m\phi]\mathrm{d}\phi = 2\pi\mathrm{i}^m\exp(-\mathrm{i}m\varphi)J_m(x), \quad (9)$$

$$\int_0^{\infty}\exp(-ux^2)J_m(\tau x)I_m(\nu x)x\mathrm{d}x = \frac{1}{2u}\exp\left(\frac{\nu^2-\tau^2}{4u}\right)J_m\left(\frac{\nu\tau}{2u}\right), \quad (10)$$

$$\int_0^{\infty}\exp(-ex^2)J_m(\vartheta x)J_m(gx)x\mathrm{d}x = \frac{1}{2e}\exp\left(-\frac{g^2+\vartheta^2}{4e}\right)I_m\left(\frac{g\vartheta}{2e}\right), \quad (11)$$

where $J_m(\cdot)$ denotes the Bessel function of order m.

On placing $r_1=r_2=r$, $\varphi_1=\varphi_2=\varphi$, $\rho_1=\rho_2=\rho$ into Eq.(5), the intensity in the z plane is given by

$$I(\rho,r) = \frac{z^2}{r^2}\frac{w_0^2}{w^2(r)}\frac{\xi^{-m/2}}{1-\xi}\exp\left[-\frac{1+\xi}{1-\xi}\frac{2\rho^2}{w^2(r)}\right] \times I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{w^2(r)}\right), \quad (12)$$

where

$$w^2(r) = w_0^2\left(1+\frac{4r^2}{k^2w_0^4}\right). \quad (13)$$

The spectral degree of coherence is defined as[1]

$$\mu(\rho_1,\rho_2,\varphi_1,\varphi_2,z) = \frac{W(\rho_1,\varphi_1,\rho_2,\varphi_2,z)}{\sqrt{I(\rho_1,\varphi_1,z)\,I(\rho_2,\varphi_2,z)}}. \quad (14)$$

From Eqs.(5), (12) and (14) we obtain

$$\mu(\rho_1,\varphi_1,\rho_2,\varphi_2,z) = \frac{w_1(r_1)w_2(r_2)k^2}{r_1r_2w_0^2(4\beta_1\beta_2-\alpha^2)}\exp[\mathrm{i}k(r_2-r_1)]\exp[-\mathrm{i}m(\varphi_1-\varphi_2)]I_m\left(\frac{\alpha u_1u_2}{4\beta_1\beta_2-\alpha^2}\right) \times \exp\left(-\frac{\beta_1u_2^2+\beta_2u_1^2}{4\beta_1\beta_2-\alpha^2}\right)\exp\left[\frac{1+\xi}{1-\xi}\left(\frac{\rho_1^2}{w_1^2(r_1)}+\frac{\rho_2^2}{w_2^2(r_2)}\right)\right] \times \left[I_m\left(\frac{w_0^2\alpha\rho_1^2}{w_1^2(r_1)}\right)I_m\left(\frac{w_0^2\alpha\rho_2^2}{w_2^2(r_2)}\right)\right]^{-1/2}, \quad (15)$$

where

$$w_j^2(r_j) = w_0^2\left(1+\frac{4r_j^2}{k^2w_0^4}\right). \quad (16)$$

Equations (5), (12) and (15) are the main analytical results obtained in this paper, and govern basic propagation properties of nonparaxial MBG beams in free space. Equation (5) indicates that the cross-spectral density function $W(\rho_1,\varphi_1,\rho_2,\varphi_2,z)$ depends on the order m, ξ parameters, waist width w_0, and wave number k (or, equivalently, f parameter (see Eq.(23))) and

relative orientation $\Delta\varphi = |\varphi_1 - \varphi_2|$. However, it turns out from Eqs.(12) and (15) that the intensity $I(\rho, r)$ and spectral degree of coherence $|\mu(\rho_1, \varphi_1, \rho_2, \varphi_2, z)|$ are independent of $\Delta\varphi$.

Some interesting special cases of Eqs.(5), (12) and (15) are as follows.

2.1. On-axis intensity

Letting m=0 and ρ=0 in Eq.(12), the on-axial intensity $I(0, z)$ is expressed as

$$I(0,z) = \frac{w_0^2}{w^2(z)} \frac{1}{1-\xi}, \quad (m = 0). \tag{17}$$

If $m \neq 0$ we have

$$I(0,z) = 0, \quad (m \neq 0), \tag{18}$$

because $I_m(0)$=0. Thus, a central dip appears in the intensity profile for $m \neq 0$.

2.2. Far-field expressions

If we use the far-field approximation

$$R_j = r_j - \frac{\rho_{0j}\rho_j \cos(\varphi_j - \varphi_{0j})}{r_j}. \tag{19}$$

Equations (5), (12) and (15) become

$$W_f(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \frac{z^2}{4r_1^2 r_2^2 f^4 k^2} \frac{\xi^{-m/2}}{1-\xi} \exp[\mathrm{i}k(r_2 - r_1)] \exp[-\mathrm{i}m(\varphi_1 - \varphi_2)] \times \exp\left[-\frac{1}{4f^2}\frac{1+\xi}{1-\xi}\left(\frac{\rho_1^2}{r_1^2} + \frac{\rho_2^2}{r_2^2}\right)\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1\rho_2}{4r_1 r_2 f^2}\right), \tag{20}$$

$$I_f(\rho, r) = \frac{z^2}{4r^4 f^4 k^2} \frac{\xi^{-m/2}}{1-\xi} \exp\left[-\frac{1}{2f^2 r^2}\frac{1+\xi}{1-\xi}\rho^2\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{4r^2 f^2}\right), \tag{21}$$

$$\mu_f(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \exp[\mathrm{i}k(r_2 - r_1)] \exp[-\mathrm{i}m(\varphi_1 - \varphi_2)] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1\rho_2}{4r_1 r_2 f^2}\right) \times \left[I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1^2}{4r_1^2 f^2}\right) I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_2^2}{4r_2^2 f^2}\right)\right]^{-1/2}, \tag{22}$$

respectively, where

$$f = \frac{1}{kw_0} \quad (f - \text{parameter}^{[3,4]}). \tag{23}$$

2.3. Paraxial expressions

Applying the paraxial approximation

$$r_j \approx z + \frac{\rho_j^2}{2z}, \tag{24}$$

Equations (5), (12), and (15) reduce to

$$W_p(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \frac{w_0^2}{w^2(z)} \frac{\xi^{-m/2}}{1-\xi} \exp\left[-\frac{1+\xi}{1-\xi}\frac{1}{w^2(z)}(\rho_2^2 + \rho_1^2)\right] \exp[-\mathrm{i}m(\varphi_1 - \varphi_2)] \times \exp\left[\frac{\mathrm{i}k}{2R(z)}(\rho_2^2 - \rho_1^2)\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1\rho_2}{w^2(z)}\right), \tag{25}$$

$$I_p(\rho, z) = \frac{w_0^2}{w^2(z)} \frac{\xi^{-m/2}}{1-\xi} \exp\left[-\frac{1+\xi}{1-\xi}\frac{2\rho^2}{w^2(z)}\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{w^2(z)}\right), \tag{26}$$

$$\mu_p(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \exp\left[\frac{\mathrm{i}k}{2R(z)}(\rho_2^2 - \rho_1^2)\right] \exp[-\mathrm{i}m(\varphi_1 - \varphi_2)] I_m\left(\frac{2\rho_1\rho_2}{\sigma_c^2}\right) \times \left[I_m\left(\frac{2\rho_1^2}{\sigma_c^2}\right) I_m\left(\frac{2\rho_2^2}{\sigma_c^2}\right)\right]^{-1/2}, \tag{27}$$

where

$$w^2(z) = w_0^2\left(1 + \frac{4z^2}{k^2 w_0^4}\right), \tag{28}$$

$$R(z) = z + \frac{k^2 w_0^4}{4z}, \tag{29}$$

$$\frac{1}{\sigma_c^2} = \frac{2\sqrt{\xi}}{(1-\xi)w^2(z)}. \tag{30}$$

Equations (25), (26), and (27) are essentially consistent with Eqs.(35), (9) and (11) in Ref.[1], respectively.

Moreover, if the paraxial far-field approximation

$$R_j = z - \frac{\rho_{0j}\rho_j \cos(\varphi_j - \varphi_{0j})}{z} \tag{31}$$

is further applied to Eqs.(25)–(27), we have

$$W_{pf}(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \frac{1}{4z^2 f^4 k^2}\frac{\xi^{-m/2}}{1-\xi}\exp\left[\frac{ik}{2z}(\rho_2^2 - \rho_1^2)\right]\exp[-im(\varphi_1 - \varphi_2)]$$
$$\times \exp\left[-\frac{1}{4f^2 z^2}\frac{1+\xi}{1-\xi}\left(\rho_1^2 + \rho_2^2\right)\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1\rho_2}{4z^2 f^2}\right), \tag{32}$$

$$I_{pf}(\rho, r) = \frac{1}{4z^2 f^4 k^2}\frac{\xi^{-m/2}}{1-\xi}\exp\left[-\frac{1}{2f^2 z^2}\frac{1+\xi}{1-\xi}\rho^2\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{4z^2 f^2}\right), \tag{33}$$

$$\mu_{pf}(\rho_1, \varphi_1, \rho_2, \varphi_2, z) = \exp\left[\frac{ik}{2z}(\rho_2^2 - \rho_1^2)\right]\exp[-im(\varphi_1 - \varphi_2)] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1\rho_2}{4z^2 f^2}\right)$$
$$\times \left[I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_1^2}{4z^2 f^2}\right) I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho_2^2}{4z^2 f^2}\right)\right]^{-1/2}. \tag{34}$$

3. Numerical illustrative examples and analysis

To illustrate the propagation properties of partially coherent MBG beams in free space and to compare the results with those in the paraxial case, numerical calculations were made by using the formulae derived in Sec.2. Figures 1 and 2 show the normalized radial intensity distributions $I/I_{\max}$ ($I_{\max}$: maximum intensity) at z=20z_R ($z_R = \frac{\pi w_0^2}{\lambda}$: Rayleigh length of the Gaussian beam) of partially coherent MBG beams of order m=0 in Fig.1 and order m=2 in Fig.2 for different values of ξ=0.1, 0.9, and f=0.01 in (a) and f=0.3 in (b). The solid curves denote the results by using Eq.(12) and the circles represent the corresponding paraxial results by using Eq.(26). It is seen that for a small value of f=0.01 in Figs.1(a) and 2(a) there is no difference between the paraxial and nonparaxial results, whereas for a large value of f=0.3 in Figs.1(b) and 2(b) the discrepancies between them appear. This means that the paraxial approach for such a case is no longer applicable. The radial intensity distribution in the z plane depends on m, ξ and f. The intensity reaches its maximum at ρ=0 for m=0, but there is a central dip in the intensity profile for m=2. Addition-

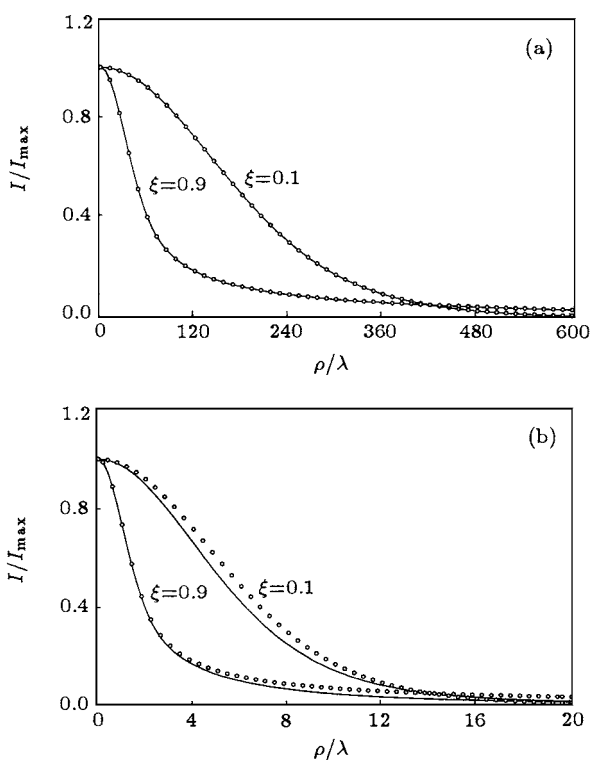


Fig.1. Normalized radial intensity distributions $I/I_{\max}$ in the plane z=20z_R of partially coherent MBG beams of order m=0 for different values of ξ =0.1 and 0.9. (a) f=0.01; (b) f=0.3 "—": by using Eq.(12), "o o o": by using Eq.(26).

ally, Fig.2 indicates that for the almost coherent case of ξ=0.1 the intensity profile is fairly symmetric about its maximum, and the symmetry becomes worse for f=0.3 than for f=0.01. However, the symmetry disappears and a long tail instead comes out in the intensity profile for the almost incoherent case of ξ=0.9.

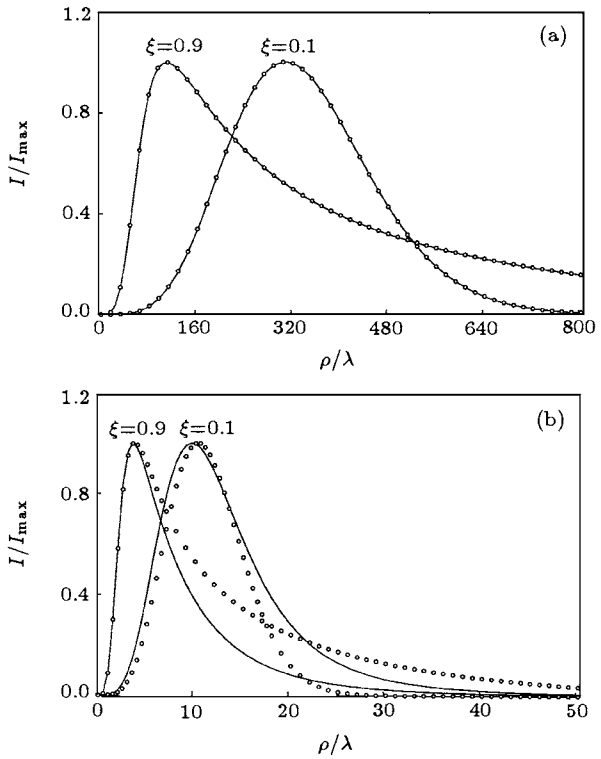


Fig.2. Normalized radial intensity distributions $I/I_{\max}$ in the plane z=20z_{R} of partially coherent MBG beams of order m=2 for different values of ξ =0.1 and 0.9. (a) f=0.01; (b) f=0.3. "—": by using Eq.(12), "o o o": by using Eq.(26).

The spectral degree of coherence $|\mu|$ in the plane z=20z_{R} of partially coherent MBG beams of order m=0 for different values of ξ=0.1, 0.9 and f=0.01 in (a) and f=0.3 in (b) are given in Fig.3. The other calculation parameters are ρ_1=50λ and ρ_2=ρ. From Fig.3 we see that $|\mu|$ is dependent on parameters m, ξ and f, and0 $\leq$ $|\mu|$ $\leq$ 1. As can be expected, for f=0.01 in Fig.3(a) the paraxial approximation is applicable, whereas for f=0.3 in Fig.3(b) the paraxial approximation fails, and the nonparaxial approach should be used.

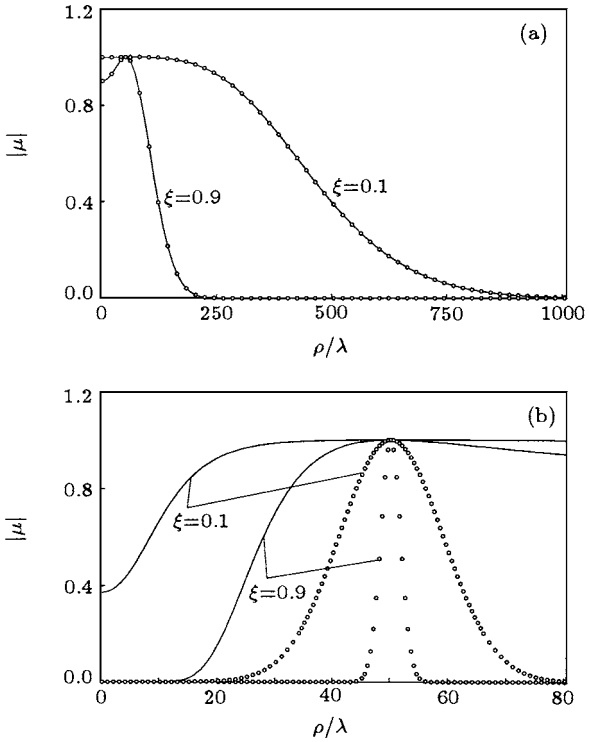


Fig.3. Spectral degree of coherence $|\mu|$ in the plane z=20z_{R} of partially coherent MBG beams of order m=0 for different values of ξ =0.1 and 0.9. (a) f=0.01; (b) f=0.3. "—" — by using Eq.(15), "o o o" — by using Eq.(27).

4. Conclusion

In this study the concept of partially coherent MBG beams has been extended to the nonparaxial regime. The closed-form propagation equations of nonparaxial MBG beams in free space have been derived, and the on-axis intensity, far-field and paraxial expressions have been given as special cases of our general results. Theoretical and numerical results have shown that the intensity and spectral degree of coherence of nonparaxial MBG beams depend on the order m, ξ and f parameters, whereas their cross-spectral density function is additionally dependent on the orientation $\Delta\varphi$. The nonparaxial approach has to be used when the f parameter is not small. Finally, we would like to point out that, instead of the generalized Rayleigh-Sommerfeld diffraction integral, the Wigner distribution function recently introduced into the nonparaxial regime in Ref.[12] can be also used to treat nonparaxial MBG beams, and deliver the same results.

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