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A method to measure the nonlinear force caused emittance growth in a RF photoinjector*

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Based on the multi-slit method, a new method is introduced to measure the non linear force caused emittance growth in a RF photoinjector. It is possible to reconstruct the phase space of a beam under some conditions by the multi-slit method. Based on the reconstructed phase space, besides the emittance, the emittance growth from the distortion of the phase space can also be measured. The emittance growth results from the effects of nonlinear force acting on electron, which is very important for the high quality beam in a RF photoinjector.

Keywords: thermal emittance, RF photoinjector, FEL (free electron laser), multi-slit method
PACC: 2900, 2915D, 2925B

1. Introduction

The research on RF photoinjector is promoted mainly due to practical requirements for free electron lasers (FELs). Beams with pulse duration of less than 1μm and peak current of several kA are required in the x-ray FEL SASE projects.[1,2] High quality beams are crucial to optimizing the system performance of many x-ray FEL projects such as TESLA[1] and LCLS.[2,3]

Some new technologies need developing as the short-wavelength FEL[4] in their practical applications requires the beams to have low emittance and low energy spread. The RF photoinjector can produce such high-quality beams, as well as maintain the beam quality along their propagation path by matching components carefully.[5,6] Many investigations have been made especially by use of simulations. Beam simulations show that the nonlinear force can affect the final emittance significantly. For example, the emittance can reach a very low value when a uniform distribution of beam is chosen as its initial distribution. This phenomenon has been demonstrated in the experiment on the Telsa Test Photoinjector (TTF).[2]

2. Physical model

The transverse movement of electron in a RF photoinjector can be described[7] by the following equation:

$$\frac{\mathrm{d}p_r}{\mathrm{d}z} = N(z,\varphi)r + \alpha r^2, \tag{1}$$

where $p_r = \beta\gamma r'$ is the normalized transverse momentum of electron, $N(z,\varphi)r$ denotes the linear force acting on an electron, and αr^2 the nonlinear force.

A beam can be separated into several small beam slices at different times. The movement of electron in a single beam slice can be described by[8−10]

$$\frac{\mathrm{d}p_{r0}}{\mathrm{d}z} = N(z,\varphi)r_0 + \alpha r_0^2, \tag{2a}$$

$$\frac{\mathrm{d}\Delta p_r}{\mathrm{d}z} = (N(z,\varphi) + 2\alpha r_0)\Delta r, \tag{2b}$$

where (p_{r0}, r_0) is the phase space centre of the beam slice, and $(\Delta p_r, \Delta r)$ is the phase state of the electron in the beam slice, which are related by the following relations hips:

$$\begin{aligned} p_r &= p_{r0} + \Delta p_r, \\ r &= r_0 + \Delta r. \end{aligned} \tag{3}$$

Equations (2) and (3) show that the nonlinear force does not affect the phase space of the beam slice, but changes its centre position. Because Eq.(2b) is a linear equation, the movement of electron in the beam slice can be simplified as

$$\begin{aligned} \Delta P_r &\approx A\Delta r_0, \\ \Delta r &\approx B\Delta r_0, \end{aligned} \tag{4}$$

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where Δr_0 is the initial position of the electron inside the beam slice.

The solution of the Eq.(2a) can be expressed as

$$P_{r0} \approx Ar_0 + A'r_0^2,$$
$$r_0 \approx Br_0, \tag{5}$$

where $A' \approx A^2 \int \alpha \mathrm{d}z$ describes the effect of the nonlinear force.

Based on the definition of the rms emittance[11,12], the rms emittance of the beam slice $\varepsilon_{\mathrm{slice}}$ can be obtained from Eq.(5), then

$$\varepsilon_{\mathrm{slice}}^2 \approx \varepsilon_0^2 + A'^2 B^2 \left\langle r_0^2 \right\rangle \left\langle r_0^4 \right\rangle, \tag{6}$$

where ε_0 is the initial emittance of the beam.

The rms emittance of the whole beam $\varepsilon_{\mathrm{rms}}$ can be obtained by integrating Eq.(6), and it is

$$\varepsilon_{\mathrm{rms}}^2 \approx \varepsilon_0^2 + \left(\left\langle A^2 \right\rangle \left\langle B^2 \right\rangle - \left\langle AB \right\rangle^2 \right) \left\langle r_0^2 \right\rangle^2 + \left\langle A'^2 \right\rangle \left\langle B^2 \right\rangle \left\langle r_0^4 \right\rangle \left\langle r_0^2 \right\rangle. \tag{7}$$

It can be concluded from Eq.(7) that the first part of the beam emittance is due to thermal emittance, the second part originates from linear-force-induced emittance growth, and the third part stems from nonlinear force. When the nonlinear force is ignored, then $A' = 0$ in Eqs.(6) and (7), and the third part in Eq.(7) vanishes.

In order to show how the nonlinear force makes the emittance grow, the thermal emittance and the emittance growth due to the linear force are ignored. On the assumption that $N(z, \varphi) = -1.0$ for the whole beam the initial phase space is shown in Fig.1. The phase space of the beam after the acceleration without the action of nonlinear force at $\alpha = 0.0$ is shown in Fig.2. There appears no distortion of the phase space under the above condition as shown in Fig.2. The phase space of the beam that is accelerated under the action of nonlinear force at $\alpha = 0.05$ is shown in Fig.3. Though the area of the phase space is also kept to be zero, the rms emittance increases significantly due to the distortion of the phase space as shown in Fig.3.

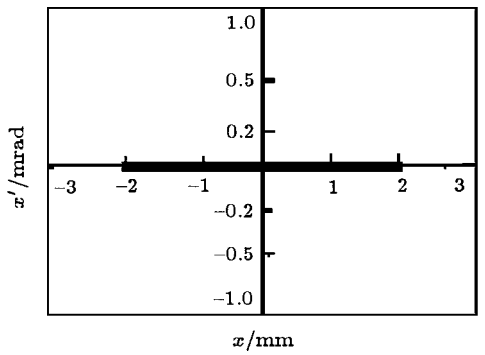


Fig.1. Initial phase space.

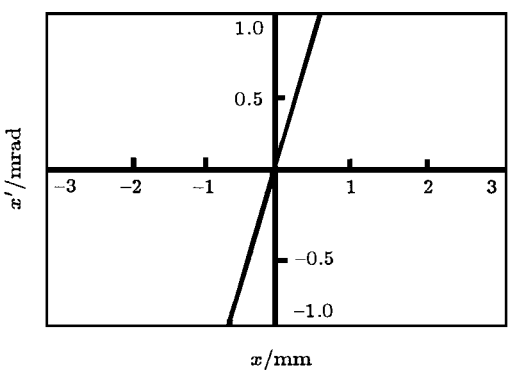


Fig.2. Phase space of accelerated beam without the action of nonlinear force.

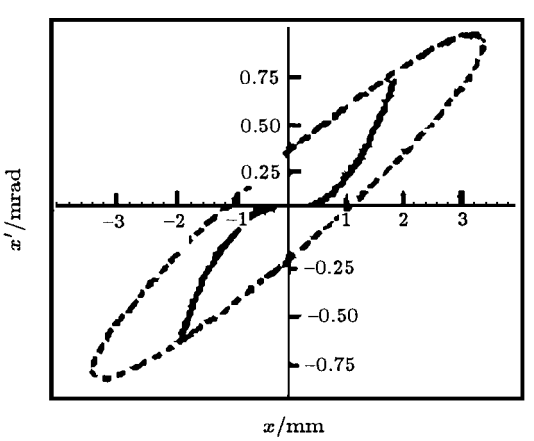


Fig.3. Phase space of accelerated beam with the action of nonlinear force. dotted: phase ellipse, solid: phase space.

3. Simulation

The nonlinear force acting on electron results mainly from the space charge effects in a RF photoinjector especially near the cathode. The nonlinear space charge force is dependent upon beam distribution. The nonlinear part of space charge force is considerably small compared with the linear part when the beam charge is of uniform distribution, and the

influence of the nonlinear part is significantly obvious when the beam charge is of Gaussian distribution. The simulated phase space of the beam (beam charge: 1nC) with the initial distribution being uniform distribution on the software ASTRA is shown in Fig.4, the final emittance is only 1.6π·mm·mrad. The simulated phase space of the beam (beam charge: 1nC) with the initial distribution being Gaussian distribution is shown in Fig.5. It can be seen that the phase space is significantly distorted, and the rms emittance is 9.4π·mm·mrad. Their phase space centres are shown in Fig.6 and Fig.7 respectively. The phase space centres are in line for the case where the initial distribution is uniform distribution, but out of line for the case where the initial distribution is a Gaussian distribution. The phase ellipse of the beam is shown in Fig.7, and the corresponding emittance growth from the distortion is 8.8π·mm·mrad. So the emittance growth is 9.4π·mm·mrad with the initial distribution being a Gaussian distribution, which is much bigger than that with the initial distribution being a uniform distribution.

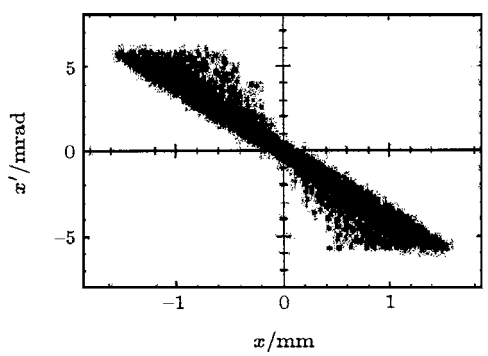


Fig.4. Phase space of the accelerated beam with the initial distribution being a uniform distribution.

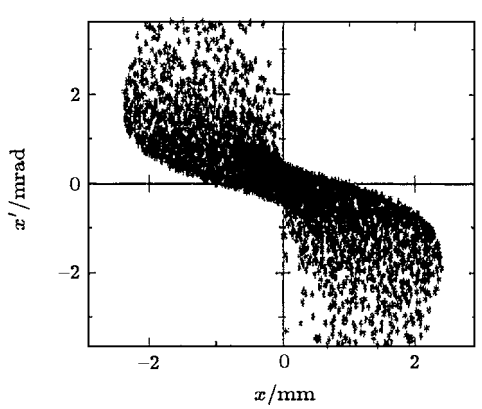


Fig.5. Phase space of the accelerated beam with the initial distribution being a Gaussian distribution.

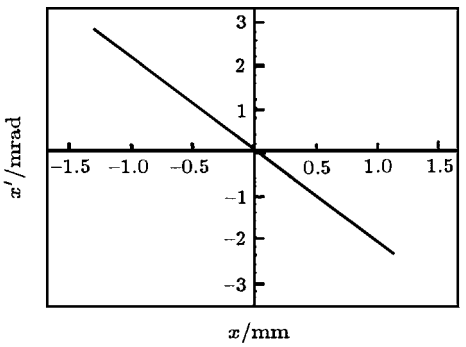


Fig.6. Phase space centre of the accelerated beam with the initial distribution being a uniform distribution.

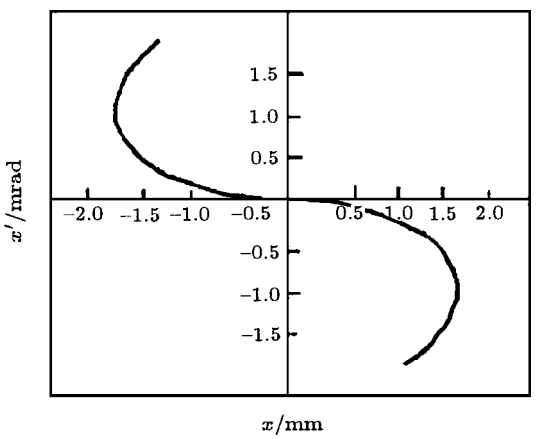


Fig.7. Phase space centre of the accelerated beam with the initial distribution being a Gaussian distribution.

4. Measurement method

The multi-slit method is widely used in the emittance measurement in a RF photoinjector.[12,13] The emittance measurement by the multi-slit method is usually arranged as shown in Fig.8, in which the metal with many slits can stop most of electrons out of the beam, but let other electrons pass through the slits to form many beamlets, drift and then hit on the screen behind the slits. The images of beamlets on the screen can be used to reconstruct the phase-space and measure the emittance of the beam. After the slits, the space charge effect is significantly mitigated. When ignoring the space charge effect, the movement of electron can be expressed as

$$x = x_0 + x_0' z, \quad x' = x_0'. \tag{8}$$

On the screen after the slits, when

$$x_0 \ll x_0' L, \tag{9}$$

where L is the distance between the slits and the screen, then

$$x_0' \approx x/L. \tag{10}$$

From Eq.(10), the images of beamlets on the screen can be used to construct the phase space (x_0, x_0') of the beam on the slits with the positions of the slits (as shown in Fig.8).

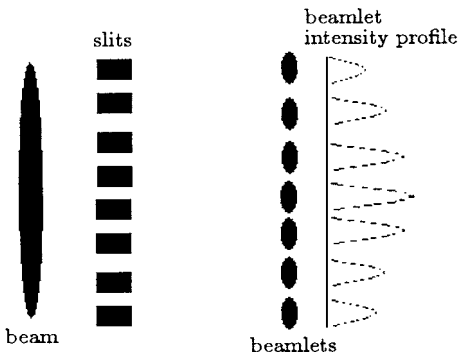


Fig.8. Arrangement of multislit-emittance measurement.

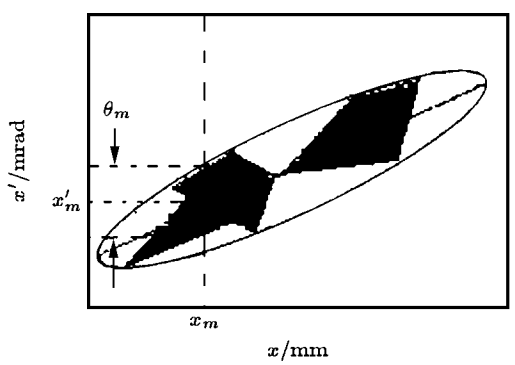


Fig.9. Schematic phase space of the beam.

According to the definition of the rms emittance, the measured emittance is

$$\varepsilon_{\rm rms}^2 = \frac{\sum\limits_m I_m\theta_m^2}{\sum\limits_m I_m}\frac{\sum\limits_m I_m x_m^2}{\sum\limits_m I_m} + \left[\frac{\sum\limits_m I_m x_m'^2}{\sum\limits_m I_m}\frac{\sum\limits_m I_m x_m^2}{\sum\limits_m I_m} - \left(\frac{\sum\limits_m I_m x'_m x_m}{\sum\limits_m I_m}\right)^2\right] \tag{11}$$

where I_m is the integrated detected intensity of the mth beamlet, (x_m, x'_m) is the phase centre of the beamlet, and θ_m is the divergence of the beamlet.

In Eq.(11), the first part comes from the area of the phase space, the second part originates from the position (x_m, x'_m) of the beamlet phase space. The second part means the distortion of the phase space due to the effects of the nonlinear force.

If the centres of the beamlets' phase space are in line, the second part of rms emittance vanishes. Thus the rms emittance stems completely from the area of the phase space, which is expressed as

$$\varepsilon_{\rm phase}^2 = \frac{\sum\limits_m I_m\theta_m^2}{\sum\limits_m I_m}\frac{\sum\limits_m I_m x_m^2}{\sum\limits_m I_m}. \tag{12}$$

If there exists distortion, even when $\theta_m = 0$, which means the area of the phase space is zero, the rms emittance is not zero due to the distortion, and the emittance growth from the distortion is

$$\varepsilon_{\rm distorts}^2 = \left[\frac{\sum\limits_m I_m x_m'^2}{\sum\limits_m I_m}\frac{\sum\limits_m I_m x_m^2}{\sum\limits_m I_m} - \left(\frac{\sum\limits_m I_m x'_m x_m}{\sum\limits_m I_m}\right)^2\right]. \tag{13}$$

5 Conclusion

With the linear force acting on the beam, the phase space centres of the beamlets are in line, and no distortion of the phase space happens.[14,15] But with nonlinear force, the phase space centres of the beam slices are not in line, and the phase space is distorted, so is the corresponding emittance growth, which is due to the nonlinear force.[16] This can be measured by the multi-slit method as defined in Eq.(13).

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