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An NLMS algorithm with optimized preparatory step-size parameters for SQUID-based MCG data processing*

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We present a new least-mean-square algorithm of adaptive filtering to improve the signal to noise ratio for magnetocardiography data collected with high-temperature SQUID-based magnetometers. By frequently adjusting the adaptive parameter α to systematic optimum values in the course of the programmed procedure, the convergence is accelerated with a highest speed and the minimum steady-state error is obtained simultaneously. This algorithm may be applied to eliminate other non-steady relevant noises as well.

Keywords: magnetocardiography, optimal step size
PACC: 0650D, 0755

1. Introduction

The progress of high-temperature Josephson junction technology provides broader applications for high-temperature SQUIDs. Recently, more and more attention have been focused on SQUID-based magnetocardiography (MCG)—the measurement of biomagnetic and bioelectric effects in human heart, one of the most important SQUID applications. Normally, the field amplitude of MCG is about 100pT. In order to obtain clearer signals, the SQUIDs have to be operated in a shielded room with very high quality to acquire better performance, which is expensive. Researchers have done many tests in using a simple-shielded room to reduce the noise, making MCG a technique much cheaper and more widely applicable. However, the procedure of signal collection is disturbed by strong background noise under an unsatisfactory electromagnetic shielding condition. Therefore, to improve the output signal-to-noise (S/N) ratio of the systems has become one of the important tasks in popularizing the MCG measurement.

Since the adaptive filtering automatically changes its weight parameters during signal treatment according to the ambient noise, it is widely used to reduce the non-steady ambient noise in the MCG. The two fundamental algorithms, least-mean-square (LMS) and recursive-least-square (RLS) algorithms, have been applied in most cases. Comparatively speaking, the LMS algorithm can perform quite well in tracking to obtain smaller steady-state errors, while the RLS one could converge with a faster speed. In SQUID-based MCG, one intends to obtain the track ability as good as possible so as to obtain a higher S/N ratio. Therefore, the LMS algorithm is the more popular one. And now, Zhu *et al*[1] have improved the track ability by using a modified LMS algorithm with a new error function, in which a certain component of the instantaneous signal represented approximately by an average of the error data just gained has been eliminated. On the other hand, the convergence course is usually getting slower in finding the best track ability at the same time.[2] Thus, it is difficult to implement a fast real-time tracking of MCG signals. In order to solve

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this problem, Park and Song[3] have presented a new LMS algorithm that can appropriately shift the update step parameter from a large size to a smaller one. Based on their work, we have developed a complementary LMS algorithm.[4] It is designed to improve the speed of convergence by adjusting the step size back and forth between two constant values according to the concrete conditions of the ambient noise.

In order to keep the best track ability as well as the fastest speed of convergence, a newly improved LMS algorithm is presented in this paper. We implement the optimal step size in each adaptive period to gain the fastest adaptive course. In addition, the algorithm is also used to deal with the high-temperature SQUID-based MCG data in this paper.

2. Data collection

In order to collect both the MCG signal and the ambient noise at the same time, we arrange two high-T_c SQUID sensors 10cm apart on a holder. One sensor for signal channel is fixed near the man's chest ($\leq$1cm) to measure the original MCG field. The other one placed a little farther from the chest works in parallel with the first sensor to measure the background noise. Both of them are immersed in liquid nitrogen at 77K.

The MCG experiments are made inside a magnetically shielded room that attenuates the earth's steady field to less than 100nT, and the environmental interference down to 20pT that comes mainly from the power line. In comparison with the environmental variation, the intrinsic noise due to sensors themselves with a typical value of 2pT is negligible in this situation. In order to make the noise components in the two channels correlated to a great extent, we fix the two SQUIDs in parallel to form a vertical axial gradiometer. With this configuration the SQUIDs may respond only the z-components of the signal and noise fields. When the distance between the noise source and the sensors is much larger than the baseline of the SQUID gradiometer, the source can be regarded as a remote one and the major parts of the noises in the two channels are correlated. In our experiment the sampling frequency is 2kHz and the sampling time is about 30s.

3. LMS algorithm of optimized step-size

3.1. Improved LMS algorithm

Our work is based on the improved NLMS algorithm,[1] which can be expressed by the following equations. The first two equations are

$$y(k) = W^{\mathrm{T}}(k)X(k), \tag{1}$$

$$\varepsilon(k) = d(k) - y(k), \tag{2}$$

where $X(k) = [x_k, x_{k-1}, \ldots, x_{k-M+1}]^{\mathrm{T}}$, $W(k) = [w_k, w_{k-1}, w_{k-M+1}]^{\mathrm{T}}$ and $d(k)$ are the sequences of input reference vector, filter weight vector and input signal respectively. Number k denotes time sequence of the data acquisition. The superscript T stands for transposition of vectors and M is the filter length. From Eq.(2) we note that the error function $\varepsilon(k)$ depends on $d(k)$. In another words it contains the signal component.

The update equation of the weight vector W is given by

$$W(k+1) = W(k) + \mu(k)X(k)S(k), \tag{3}$$

where $S(k)$ can be regarded as signal-independent error function written as

$$S(k) = \varepsilon(k) - \langle\varepsilon\rangle. \tag{4}$$

In Eq.(4)

$$\langle\varepsilon\rangle = \sum_{i=1}^{n} p(i)\varepsilon(k-i) / \sum_{i=1}^{n} p(i), \tag{5}$$

is the average of the error data just gained and can be regarded as the instantaneous signal. The distribution of average weights is an exponential function expressed by

$$p(i) = \mathrm{e}^{-\mathrm{i}/l}, \tag{6}$$

where l is the correlation length of the signal.

As usual, in the algorithm the step-size parameter of the adaptive filter at time sequence k is defined as

$$\mu(k) = \frac{\alpha}{X(k)^{\mathrm{T}}X(k) + \gamma}, \tag{7}$$

where α is a normalized adaptive constant and γ is a very small positive number used to avoid the singularity of Eq.(7) when $X^{\mathrm{T}}(k)X(k)$ is approaching zero. From Eq.(7) we can see that the choice of α and γ values for a proper step-size parameter μ is important, because the parameter μ controls the convergence speed and the track ability of the algorithm.

How to properly choose the parameters α, γ and the filter length M in LMS algorithm has been discussed by many researchers.[5,6] Correlation length l is related to data acquisition rate and correlation time of

the signal, namely it may be determined by the sampling number within the correlation time. Generally, the frequency bandwidth of MCG signal ranges from 0.1 to 100Hz. The signal should not have significant change in one millisecond. So it is reasonable to set l between 1 and 2 for the sampling frequency of 2kHz. The positive integer n should be chosen large enough so that the terms after $e^{-n/l}$ become negligibly small. In our calculation, we simply choose l=2 and n=5.

3.2. Determination of optimal step-size parameter

The adaptive parameter α is an important value in the adaptive algorithm. It has an immediate relationship with step-size μ according to Eq.(7). Generally, the speed of convergence is getting faster with a bigger α. In Fig.1, it shows three different speeds of convergence with different α. On the other hand,

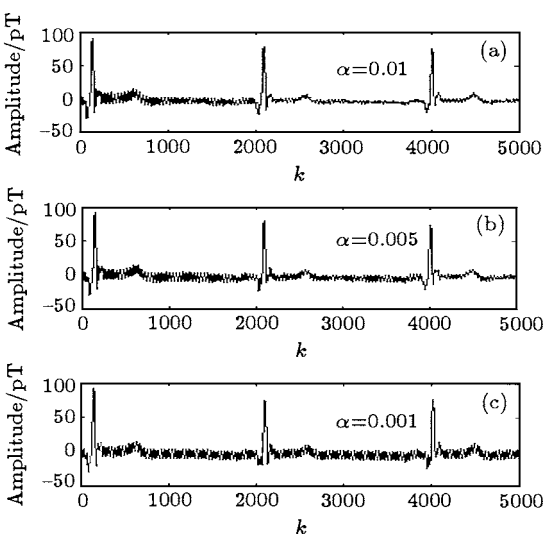


Fig.1. The real time traces of MCG signal at the adaptive filtering output, for the comparison of the convergence speeds with parameters of (a) α=0.01, (b) α=0.005 and (c) α=0.001.

Fig.2 shows three steady-state errors, which demonstrates a better error corresponding to a smaller α. In the algorithm the filter length M is chosen to be 40 and the γ parameter 10^{-10}.

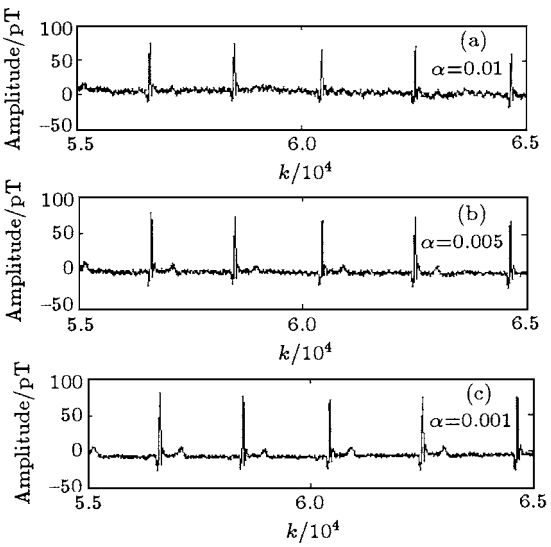


Fig.2. The real time traces of MCG signal at the adaptive filtering output, for the comparison of the steady-state errors with parameters of (a) α=0.01, (b) α=0.005 and (c) α=0.001.

Statistically, during the adaptive process the peak-to-peak value $S_{\mathrm{p-p}}(k)$ of the error decays with k. It can be expressed approximately by the following equation:

$$S_{\mathrm{p-p}}(k) = (A - a) \times \mathrm{e}^{-(k-1)/k_0} + a, \qquad (8)$$

where a is the steady-state error and k_0 is the time constant of convergence in which the error decays to $1/e$ of its original value, they are all functions of the adaptive parameter α, and can be determined experimentally. In Eq.(8) A is the initial error. A set of experimental data is shown in Table 1.

Table 1. The experimentally determined initial error A, the steady-state error a and the time constant k_0 for different parameters α.

Sequence number	α	A	a	k_0
1	$\alpha_1 = 0.02$	1700	600	800
2	$\alpha_2 = 0.01$	1700	500	1050
3	$\alpha_3 = 0.008$	1700	450	1500
4	$\alpha_4 = 0.003$	1700	300	4000
5	$\alpha_5 = 0.002$	1700	230	7000
6	$\alpha_6 = 0.001$	1700	200	10000

In Fig.3(a), a set of declining curves show the peak-to-peak values of $S(k)$ as functions of time with different α. It can be found that at the beginning the speed of $S_{\mathrm{p-p}}(k)$ is decreased faster for a bigger α. As the time elapses, the decreasing speed for a smaller α catches up with that for a larger α at a certain moment. Finally, the decreasing speed for the smallest α becomes the biggest. This indicates that the α value with the fastest-decreasing speed drops continuously with k.

From Eq.(8), we can write the decreasing speed of peak-to-peak value $S(k)$ as a function of k by the following equation:

$$\frac{\mathrm{d}S_{\mathrm{p-p}}(k)}{\mathrm{d}k} = -\frac{A-a}{k_0}\mathrm{e}^{-(k-1)/k_0}. \tag{9}$$

Taking $S_{\mathrm{p-p}}(k)$as a variable, we can rewrite the above equation as

$$\frac{\mathrm{d}S_{\mathrm{p-p}}(k)}{\mathrm{d}k} = -\frac{[S_{\mathrm{p-p}}(k)-a]}{k_0}. \tag{10}$$

The decreasing speed of the peak-to-peak value of $S(k)$ with three different α (0.01, 0.003, 0.001) as functions of k and $S_{\mathrm{p-p}}(k)$ is depicted in Figs.3(b) and 3(c) respectively. At the beginning, the curve with α=0.01 decreases with the fastest speed. After a while, the decreasing speed of the curve with α=0.003 becomes the largest. Finally, the curve with α=0.001 take over the leading position with the fastest decreasing speed of all curves.

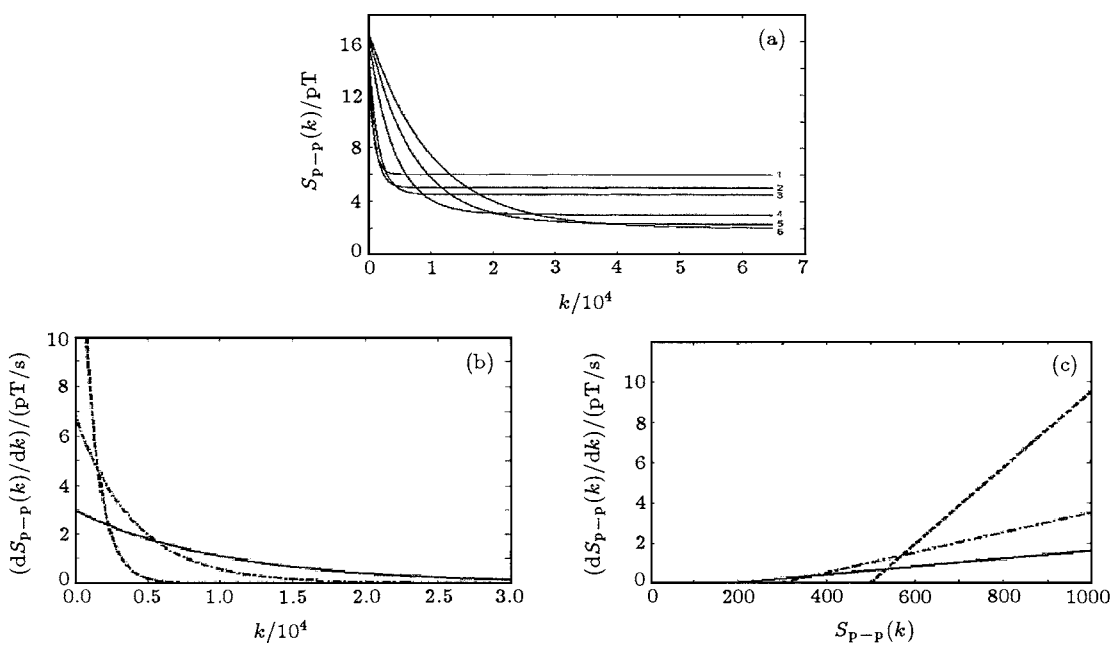


Fig.3. The declining curves of peak-to-peak value of $S(k)$ (a) numbered with 1, 2, 3, 4, 5, and 6 in the adaptive course indicated in Table 1 accordingly; the decreasing speed of the steady-state error $\mathrm{d}S_{\mathrm{p-p}}(k)/\mathrm{d}k$ as a function of (b) k and (c) $S_{\mathrm{p-p}}(k)$. (Note: dashed lines represent α=0.01, dot-dashed lines α=0.003 and solid lines α=0.001).

In the whole course of the adaptive filtering, the α corresponding to the fastest decreasing speed of the peak-to-peak value of the error can be obtained as a monotonic function of $S_{\mathrm{p-p}}(k)$. For simplicity, the whole error-reducing procedure is divided into six successive sections. And the optimal α in each section can be represented by a single averaged value in the section. The six values of the optimal α are listed in Table 2.

Table 2. The six error developing zones corresponding to the optimal α values.

S	$S \geq$920	920$> S \geq$620	620$> S \geq$540	540$> S \geq$380	380$> S \geq$333	$S <$333
α	0.02	0.01	0.008	0.003	0.002	0.001

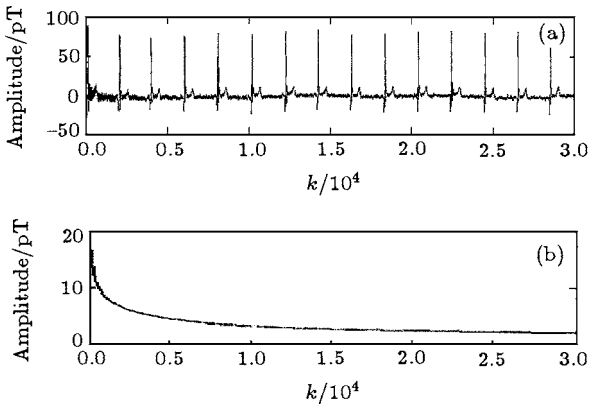


Fig.4. The MCG output data (a) after adaptive filtering, and the theoretical curve (b) of peak-to-peak value of $S(k)$ as functions of time k.

In our adaptive filtering, the representative parameter α is kept switching from time to time according to the values listed in Table 2 by recognizing the range of the error $S_{\mathrm{p-p}}(k)$ in the whole adaptive process. By using the method a set of real MCG data has been treated. The resulting MCG output in the course of adaptive filtering is shown in Fig.4(a). It should be noted that both the curves drawn in Fig.3 and the figures listed in Table 2 are correct only in statistical meaning. Therefore, α is selected from Table 2 by recognizing the average of consecutive peak-to-peak values of error data for m times. Here we choose m=5. To simplify the whole calculation, the recognition is taken once every 0.05 second (100 data points). The convergence process exhibited in the time-dependent noise part of the MCG output is in good agreement with the theoretical curve given in Fig.4(b) calculated from Eq.(8).

4. Conclusions

In this paper, we implemented an improved algorithm with adjusting step-size parameter of adaptive filtering to eliminate ambient noise involved in MCG signals. This algorithm has fundamentally settled the competition between the steady-state error and the convergence speed in NLMS algorithms. In this way, both the fastest convergence and the best tracking ability are realized at the same time. The peak-to-peak value of the noise in MCG output reduces from 17pT to 5pT in a time interval of a cardiac cycle. The MCG experiment given in Fig.4 shows that a real time trace for MCG signals measured in a simply shielded room can be realized by using the algorithm. The method has practical significance in regard to popularizing the SQUID-based MCG applications.

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