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A simple asymptotic trajectory control of full states of a unified chaotic system

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A simple full-state asymptotic trajectory control (FSATC) scheme is proposed to asymptotically drive full states of a unified chaotic system (UCS) to arbitrary desired trajectories. The FSATC uses only information, i.e. one state of the UCS. A sinusoidal wave and two chaotic variables are taken as illustrative tracking trajectories to verify that using the proposed FSATC can make full UCS states track desired trajectories with high tracking accuracy in a finite time.

Keywords: chaotic control, a unified chaotic system, trajectory control
PACC: 0545

1. Introduction

During the last two decades, chaotic phenomena were observed in numerous fields, such as electronic devices and systems, laser systems, chemical reactions, bio-systems, power converters, information processing, neural networks, and life and medical sciences. Controlling those chaotic systems for engineering applications is an attractive and hot subject recently. Many methods of controlling the systems have been proposed, such as linear and nonlinear feedback controls,[1,2] adaptive control,[3,4] neural network control,[5] fuzzy control[6] and variable structure control.[7−9] Depending on all system states, some tracking control approaches (e.g. neural network control, fuzzy control and variable structure control) bears some drawbacks since partial system states are difficult to measure or even cannot be measured in practical applications. As to tracking trajectory, most tracking control methods aim to drive a chaotic system to its equilibrium point e.g. its original one. Even though some approaches can track arbitrary desired trajectories, most of them control only a system state to an arbitrary desired trajectory.[7,8] As far as we know, up to data, driving all states of a chaotic system to arbitrary desired trajectories has not been fully reported yet.

A unified chaotic system (UCS), namely

$$\begin{aligned}\dot{x} &= (25\alpha + 10)(y - x),\\ \dot{y} &= (28 - 35\alpha)x - xz + (29\alpha - 1)y,\\ \dot{z} &= xy - (\alpha + 8)z/3,\end{aligned}$$

was created by Lü *et al*[10] which was controlled by an important parameter α and displayed chaotic behaviour when $\alpha \in [0,\ 1]$. It is interesting that the UCS actually belongs to an extended Lorenz system when $0 \leq \alpha < 0.8$, Lü system when α=0.8, and extended Chen system when $0.8 < \alpha \leq 1$. Very recently, how to control the UCS becomes attractive and some research results have been reported.[11−13] However, as mentioned above, the same drawbacks such as dependance on all states and the driving of the UCS only to an equilibrium point, still exist in those trajectory control strategies proposed previously.

In the paper, a simple full-state asymptotic trajectory control (FSATC) scheme is presented to asymptotically drive full states of the UCS to arbitrary desired trajectories. The FSATC uses only information, i.e. one state of the UCS. A sinusoidal wave and two chaotic variables are taken as illustrative tracking trajectories to verify that using FSATC can make full UCS states track desired trajectories with high tracking accuracy in a finite time.

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2. The FSATC for a UCS

The controlled UCS can be described as

$$\begin{aligned}
\dot{x}_1 &= (25\alpha + 10)(x_2 - x_1) + u_1,\\
\dot{x}_2 &= (28 - 35\alpha)x_1 - x_1x_3 + (29\alpha - 1)x_2 + u_2,\\
\dot{x}_3 &= x_1x_2 - (\alpha + 8)x_3/3 + u_3,\\
y &= x_2, \qquad (1)
\end{aligned}$$

where u_1, u_2, u_3 are control inputs.

To drive full states of the system (1) to arbitrary desired trajectories, the FSATC method is designed as follows:

$$\begin{aligned}
u_1 =& \dot{x}_{1d} - (25\alpha + 10)(x_{2d} - x_{1d})\\
&- (25\alpha + 10)(y - x_{2d}),\\
u_2 =& \dot{x}_{2d} - (29\alpha - 1)x_{2d} - (28 - 35\alpha)x_{1d}\\
&+ x_{1d}x_{3d} - (K + 29\alpha - 1)(y - x_{2d}),\\
u_3 =& \dot{x}_{3d} - x_{1d}x_{2d} + (\alpha + 8)x_{3d}/3 - x_{1d}(y - x_{2d}), \qquad (2)
\end{aligned}$$

where x_{1d}, x_{2d}, x_{3d} are assumed to be bounded and smooth trajectories; $K > 0$ is a parameter that is to be chosen.

The next Theorem is used to confirm that full-states of the system (1) can asymptotically track desired trajectories, i.e. x_{1d}, x_{2d} and x_{3d}, by using the controller (2).

Theorem Adopt control law (2) in the system (1), then full-states of the system (1) can asymptotically track the given desired trajectories, i.e.

$$\begin{aligned}
\lim_{t\to\infty}(x_1 - x_{1d}) &= \lim_{t\to\infty}(x_2 - x_{2d})\\
&= \lim_{t\to\infty}(x_3 - x_{3d}) = 0. \qquad (3)
\end{aligned}$$

Proof Define

$$\begin{aligned}
&e_1 = x_1 - x_{1d}, e_2 = x_2 - x_{2d},\\
&e_3 = x_3 - x_{3d}, E = [e_1, e_2, e_3]^{\mathrm{T}}. \qquad (4)
\end{aligned}$$

Substituting the expression (2) into the system (1) and using the definition (4) yield

$$\dot{e}_1 = -(25\alpha + 10)e_1, \qquad (5a)$$

$$\dot{e}_2 = (28 - 35\alpha - x_{3d})e_1 - e_1e_3 - x_{1d}e_3 - K\,e_2, \qquad (5b)$$

$$\dot{e}_3 = x_{2d}e_1 + e_1e_2 - (\alpha + 8)e_3/3. \qquad (5c)$$

Since $25\alpha + 10 > 0$ for all $\alpha \in [0\ 1]$, it is clear from the expression (5a) that

$$\lim_{t\to\infty} e_1(t) = 0 \qquad (6)$$

is satisfied.

Since x_{2d} is bounded and $(8 + \alpha)/3 > 0$ for all $\alpha \in [0\ 1]$, substituting the expression (6) into the expression (5c) produces

$$\dot{e}_3 = -(8 + \alpha)e_3/3, \qquad (7)$$

which implies that

$$\lim_{t\to\infty} e_3(t) = 0 \qquad (8)$$

is fulfilled.

Since x_{1d} and x_{3d} are bounded and $(8 + \alpha)/3 > 0$ for all $\alpha \in [0\ 1]$, substituting expressions (6) and (8) into the expression (5b) gives rise to

$$\dot{e}_2 = -K\,e_2. \qquad (9)$$

Since $K > 0$ is designed,

$$\lim_{t\to\infty} e_2(t) = 0 \qquad (10)$$

is guaranteed.

Based on the results (6), (8) and (10), we know

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} e_2(t) = \lim_{t\to\infty} e_3(t) = 0. \qquad (11)$$

Therefore, the system (5) can be globally asymptotically stable at the original point. According to the definition (5), which implies that the expression (3) is guaranteed by using the control law (2).

Q.E.D.

Remark 1 It is obvious from expressions (5a), (7) and (9) that exponential convergence of synchronization error e_1, e_2 and e_3 can be obtained.

Remark 2 When partial or full states of a chaotic system are taken as desired trajectories, using the FSATC can make the UCS synchronized with the chosen chaotic states, and also can generate a new chaotic system as desired.

Remark 3 It is clear that the controller (2) uses the derivatives of desired trajectories. Therefore, desired trajectories must be differentiable. That is the reason why we assume that desired trajectories are smooth. In addition, in the case of the desired trajectories that are differentiable almost everywhere, the same results can be guaranteed except those singular points.

3. Simulation

Parameters of the controlled UCS are chosen as

$$\alpha = 0.9, \quad x_1(0) = x_3(0) = 1, x_2(0) = -1, K = 1. \tag{12}$$

The desired trajectories are taken as $x_{1d} = \sin(t)$, $x_{2d} = z_1$ and $x_{3d} = \theta_1$, respectively, where

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= \bar{a}\, z_1 + \bar{b} z_1^3 + \bar{c} z_2 + f\cos(\bar{w}t), \\ x_{2d} &= z_1, \end{aligned} \tag{13a}$$

with parameters: $\bar{a} = 1$, $\bar{b} = -1$, $\bar{c} = -0.25$, $f = 0.4$, $\bar{w} = 1$, $z_1(0) = 0$, $z_2(0) = 1$; and

$$\begin{aligned} \dot{\theta}_1 &= -\theta_2 - \theta_3, \\ \theta_2 &= \theta_1 + \bar{\bar{a}}\theta_2, \\ \theta_3 &= \bar{\bar{b}} + (x_1 - \bar{\bar{c}})\theta_3, \\ x_{3d} &= \theta_1, \end{aligned} \tag{13b}$$

with parameters:

$$\bar{\bar{a}} = 0.2, \bar{\bar{b}} = 0.2, \bar{\bar{c}} = 5.7, \theta_1(0) = 0, \theta_2(0) = \theta_3(0) = 1.$$

Obviously, desired trajectories x_{2d} and x_{3d} are states of well-known Duffing and Rössler systems, respectively. In addition, systems (13a) and (13b) display chaotic behaviours under the chosen parameters. It is clear that desired trajectories x_{1d}, x_{2d} and x_{3d} are typical. Simulation results are plotted in Figs.1–3. It is easily found from Figs.1–3 that system states x_1, x_2 and x_3 can track x_{1d}, x_{2d} and x_{3d}, respectively, with high accuracy in a finite time.

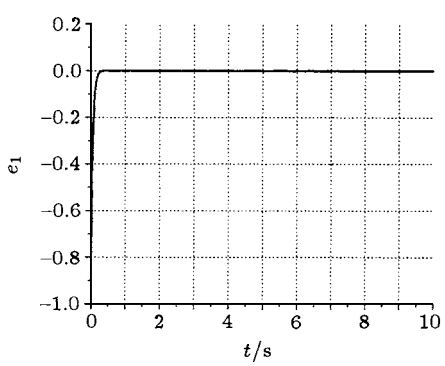


Fig.1. Time response of tracking error $e_1(t)$.

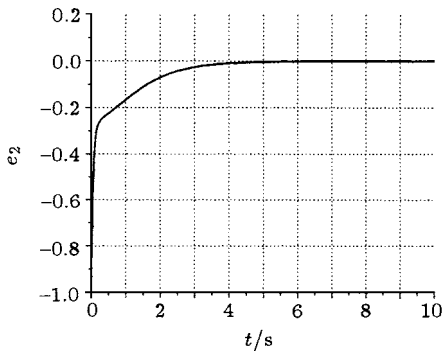


Fig.2. Time response of tracking error $e_2(t)$.

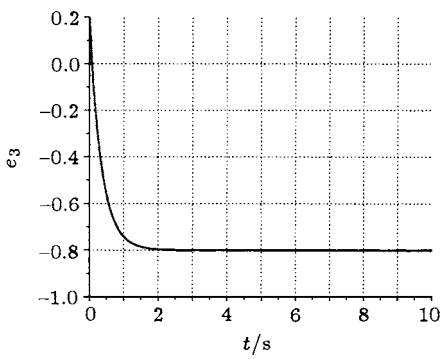


Fig.3. Time response of tracking error $e_3(t)$.

4. Conclusion

In the paper, a simple FSATC method is presented to asymptotically drive full states of the UCS to arbitrary desired trajectories. The FSATC uses only information, i.e. one state of the UCS. A sinusoidal wave and two chaotic variables are taken as illustrative tracking trajectories to verify that using the proposed FSATC can make full UCS states track desired trajectories with high tracking accuracy in a finite time. Since chaotic synchronization and anti-chaos problems can be changed into the state-tracking ones, the proposed FSATC can also be applied to chaotic synchronization and anti-chaos. In fact, when partial or full states of a chaotic system are taken as desired trajectories, using the proposed FSATC can make the UCS synchronized with the chosen chaotic system, and also can generate a new chaotic system as desired.

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