Vol 15 No 2, February 2006
1009-1963/2006/15(02)/0296-05

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

The propagation of solitary waves in clouds of Bose–Einstein condensed atoms*

Ao Sheng-Mei(敖胜美) and Yan Jia-Ren(颜家壬)

Department of Physics, Hunan Normal University, Changsha 410081, China

(Received 6 February 2005; revised manuscript received 7 April 2005)

The Gross–Pitaevskii equation which describes the motion of a Bose–Einstein condensed (BEC) atom in an elongated trap is solved analytically, and a solitary-wave solution is obtained in the low-density case without neglecting the effect of the interatomic interaction on the transverse function. It is shown that this effect leads to the velocity of the solitary wave slowing down and the profile of the solitary wave widening.

Keywords: solitary wave, BEC atom, Gross–Pitaevskii equation
PACC: 0530J, 0340, 0290

In recent years, the propagation of solitary waves in clouds of Bose–Einstein condensed (BEC) atoms has attracted much attention of all in theory as well as in experiment.[1−8] It is known that Bose–Einstein condensed atoms obey the Gross–Pitaevskii (G-P) equation which is a nonlinear Shrödinger (NLS) equation. It is possible experimentally to make very elongated clouds of Bose–Einstein condensed atoms in a magnetic or optical trap, which provides excellent conditions for investigating the propagation of essentially one-dimensional sound pulses.[1] In the previous work, the propagation of pulses in such a trap was considered in the Thomas–Fermi approximation (TFA), and it was demonstrated that pulses propagated at a speed of $\left(\frac{\bar{n}U_0}{m}\right)^{1/2}$ in the linear regime.[2,3] Here $U_0 = 4\pi\hbar^2 a_{\rm sc}/m$ is a constant corresponding to the effective two-body interaction, $\bar{n}$ is the particle density averaged over the cross-section of the cloud, m is the atomic mass, and $a_{\rm sc}$ is the scattering length for an atom-atom collision. Morgan *et al*[4] studied the cloud of BEC atoms in a trap and considered a class of solitary wave solutions that are, in fact, centre-of-mass oscillations of the stationary states of the time-independent G-P equation. Jackson *et al*[6] examined the possibility of solitary waves propagating along the axis of such a cloud, and sought the solitary wave solutions under the low-density limit and the high-density limit separately. Burger *et al*[7] and Denschlag *et al*[8] performed the relevant experiments, separately, to study the generation and the propagation of dark solitons in BEC, and they found that dark soliton velocities are smaller than the speed of sound.

In this paper, we solve the G-P equation, which describes the motion of a BEC atom in an elongated trap, in the low-density case without neglecting the effect of the interatomic interaction on the transverse function.

The condensate wavefunction $\psi = \psi(\boldsymbol{r}, \boldsymbol{t})$ satisfies the G-P equation,

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + U_0 \mid \psi \mid^2 + V\right)\psi, \qquad (1)$$

where V is the trapping potential, which is harmonic in the transverse direction and does not contain the restoring force along the axis (taken as the z axis) of the trap, that is,

$$V = \frac{1}{2}m\omega_\perp^2(x^2 + y^2) \qquad (2)$$

with $\omega_\perp$ being harmonic frequency, x and y being coordinates perpendicular to the z axis. We assume that the transverse dimension of the cloud is so small that the time scale for adjustment of the transverse profile of the particle density to the equilibrium form appropriate for the instantaneous number of particle per unit length is small compared with the time for the

*Project supported by the National Natural Science Foundation of China (Grant No 10375022).

pulse to pass a given point. Then, the problem becomes quasi-one-dimensional, and with this assumption, the wavefunction may be written in the form

$$\psi(\boldsymbol{r},t)=f(z,t)g(x,y,\sigma), \tag{3}$$

where $g=g(x,y,\sigma)$ is the equilibrium wavefunction for the transverse motion depending on time expressed implicitly through σ, and

$$\sigma=\sigma(z,t)=\int \mathrm{d}x\mathrm{d}y \mid \psi \mid^2 = \mid f(z,t) \mid^2 = \mid f \mid^2 \tag{4}$$

is the local density per unit length. Here, we choose g to be normalized so that $\int \mid g \mid^2 \mathrm{d}x\mathrm{d}y=\int g^2\mathrm{d}x\mathrm{d}y=1$.

It is convenient to derive the equations for f and g based on a variational principle. G-P equation can be derived by requiring stationarity of the action

$$\begin{aligned} S=&-\int \frac{\mathrm{i}\hbar}{2}\left(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi^*}{\partial t}\right)\mathrm{d}\boldsymbol{r}\mathrm{d}t \\ &+\int\left(\frac{\hbar^2}{2m}\mid\nabla\psi\mid^2+\frac{1}{2}U_0\mid\psi\mid^4+V\mid\psi\mid^2\right)\mathrm{d}\boldsymbol{r}\mathrm{d}t. \end{aligned} \tag{5}$$

Substituting Eq.(3) into Eq.(5), S can be rewritten as

$$\begin{aligned} S=&-\int\frac{\mathrm{i}\hbar}{2}\left[(fg)^*\frac{\partial(fg)}{\partial t}-fg\frac{\partial(fg)^*}{\partial t}\right]\mathrm{d}\boldsymbol{r}\mathrm{d}t \\ &+\int\frac{\hbar^2}{2m}\left(|g\frac{\partial f}{\partial z}|^2+|f\nabla g|^2\right)\mathrm{d}\boldsymbol{r}\mathrm{d}t \\ &+\int\left(\frac{1}{2}U_0|fg|^4+V|fg|^2\right)\mathrm{d}\boldsymbol{r}\mathrm{d}t. \end{aligned} \tag{6}$$

Minimizing Eq.(6) with respect to f^* and g^*, the equations governing f and g can be given by[6]

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial f}{\partial t}=&-\frac{\hbar^2}{2m}\frac{\partial^2 f}{\partial z^2}+\left(\frac{\hbar^2}{2m}\int|\nabla_\perp g|^2\mathrm{d}x\mathrm{d}y\right)f \\ &+U_0\left(\int|g|^4\mathrm{d}x\mathrm{d}y\right)|f|^2f \\ &+\left(\int|g|^2V\mathrm{d}x\mathrm{d}y\right)f, \end{aligned} \tag{7}$$

$$-\frac{\hbar^2\nabla_\perp^2}{2m}g+Vg+U_0|fg|^2g=\mu(\sigma)g, \tag{8}$$

respectively, where $\mu(\sigma)$ is the chemical potential. In order to obtain Eqs.(7) and (8), the normalization condition on g is employed, and the characteristic length of the pulse is considered to be sufficiently long that in all cases of interest, the term containing $\dfrac{1}{\mid f\mid^2}\mid\dfrac{\partial f}{\partial z}\mid^2$ is negligible and the term $f\nabla g$ in Eq.(6) is approximately equal to $f\nabla_\perp g$, where $\nabla=\nabla_\perp+\boldsymbol{e}_z\dfrac{\partial}{\partial z}=\boldsymbol{e}_x\dfrac{\partial}{\partial x}+\boldsymbol{e}_y\dfrac{\partial}{\partial y}+\boldsymbol{e}_z\dfrac{\partial}{\partial z}$ with $\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$ being the unit vectors in the directions x, y and z, respectively.

It is clear that Eq.(7) is a one-dimensional G-P equation for f and Eq.(8) is a two-dimensional stationary G-P equation for g. To obtain the equations resembling those of hydrodynamics for f, we assume that $f=\sqrt{\sigma}\mathrm{e}^{\mathrm{i}\phi}$, and the superfluid velocity associated with f is $v=\dfrac{\hbar}{m}\dfrac{\partial\phi}{\partial z}$. Thus, Eq.(7) can be recast as a pair of quantum hydrodynamic equations:

$$\frac{\partial\sigma}{\partial t}=-\frac{\partial(\sigma v)}{\partial z}, \tag{9}$$

$$\hbar\frac{\partial\phi}{\partial t}=-\mu(\sigma)-\frac{1}{2}mv^2+\frac{\hbar^2}{2m\sqrt{\sigma}}\frac{\partial^2\sqrt{\sigma}}{\partial z^2}, \tag{10}$$

where

$$\begin{aligned} \mu(\sigma)=&\frac{\hbar^2}{2m}\int|\nabla_\perp g|^2\mathrm{d}x\mathrm{d}y \\ &+\int|g|^2V\mathrm{d}x\mathrm{d}y+\sigma U_0\int|g|^4\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{11}$$

When differentiated with respect to z, Eq.(10) gives a generalized Euler equation

$$m\frac{\partial v}{\partial t}=-\frac{\partial}{\partial z}\left[\mu(\sigma)+\frac{1}{2}mv^2-\frac{\hbar^2}{2m\sqrt{\sigma}}\frac{\partial^2\sqrt{\sigma}}{\partial z^2}\right]. \tag{12}$$

We seek solutions of these equations for which the local fluid velocity, $v(z)$, and the local particle density per unit length, $\sigma(z)$, both propagate at a uniform velocity, u, without change in form, i.e. we assume that v and σ are only functions of $(z-ut)$ with u being constant. Thus, one may write $\dfrac{\partial\sigma}{\partial t}=-u\dfrac{\partial\sigma}{\partial z}$ and $\dfrac{\partial v}{\partial t}=-u\dfrac{\partial v}{\partial z}$. The condensate far from the solitary wave is at rest and has its equilibrium density σ_0. With these boundary conditions, the continuity equation (9) gives

$$v=u\left(1-\frac{\sigma_0}{\sigma}\right). \tag{13}$$

Also, Eq.(12) can be written as

$$mu\frac{\partial v}{\partial z}=\frac{\partial}{\partial z}\left[\mu(\sigma)+\frac{1}{2}mv^2-\frac{\hbar^2}{2m\sqrt{\sigma}}\frac{\partial^2\sqrt{\sigma}}{\partial z^2}\right]. \tag{14}$$

Integrating this equation with respect to z, we have

$$\frac{\hbar^2}{2m\sqrt{\sigma}}\frac{\partial^2\sqrt{\sigma}}{\partial z^2}=\mu(\sigma)-\mu(\sigma_0)+\frac{1}{2}mv^2-mvu. \tag{15}$$

Substituting Eq.(13) into Eq.(15), multiplying Eq.(15) by $\dfrac{\partial\sqrt{\sigma}}{\partial z}$ and integrating with respect to z, we find

$$\frac{\hbar^2}{2m}\left(\frac{\partial\sqrt{\sigma}}{\partial z}\right)^2=\epsilon(\sigma)-\mu(\sigma_0)(\sigma-\sigma_0)-mu^2\frac{(\sigma-\sigma_0)^2}{2\sigma}, \tag{16}$$

where $\epsilon(\sigma) = \int_{\sigma_0}^{\sigma} \mu(\sigma')\mathrm{d}\sigma'$. In Eq.(15) and Eq.(16), we have imposed the boundary conditions $\sigma = \sigma_0$ and $v = 0$ in the case far from disturbance.

There is no simple expression of $\epsilon(\sigma)$ for general values of σ. To obtain analytical results, Jackson *et al*[6] explored the low-density limit case ($\sigma \ll \sigma_c$) in which the interaction-energy term (the third term on the left-hand side) in Eq.(8) was neglected and the hight-density limit case ($\sigma \gg \sigma_c$) in which the kinetic energy term (the first term on the left-hand side) in Eq.(8) was neglected, where $\sigma_c \sim a_{sc}^{-1}$.

In this paper, for the low-density case we directly solve the partial differential equation (8) that $g(x, y, \sigma)$ satisfies, without neglecting any term, by use of a method similar to that in Ref.[9]. Thus, the expression of $g(x, y, \sigma)$ is obtained as (for further details see the Appendix)

$$g(x, y) \approx \left[\left(1 - \frac{1}{2}a_{sc}\sigma\right) + \frac{a_{sc}\sigma}{2a_\perp^2}r^2\right]\frac{\mathrm{e}^{-r^2/2a_\perp^2}}{\sqrt{\pi}a_\perp}. \quad (17)$$

From Eq.(11) the chemical potential $\mu(\sigma)$ can be calculated as follows:

$$\mu(\sigma) \approx \hbar\omega_\perp\left(1 + 2a_{sc}\sigma - \frac{5}{4}a_{sc}^2\sigma^2 + \frac{3}{2}a_{sc}^3\sigma^3 - \frac{1}{4}a_{sc}^4\sigma^4\right), \quad (18)$$

and Eq.(16) can be taken as the following form:

$$\frac{\hbar^2}{2m}\left(\frac{\partial\sigma^{1/2}}{\partial z}\right)^2 = \left[\frac{\sigma U_0}{2S}\left(1 - \frac{5}{6}a_{sc}\sigma_0\right) - mu^2\right]\frac{(\sigma - \sigma_0)^2}{2\sigma}, \quad (19)$$

where $S = \pi a_\perp^2$ is the cross-section of the atom cloud. Here, we have considered the condition $\sigma \leq \frac{\sigma_c}{5}$ which is satisfied in some relevant experimental situations[1] and must be satisfied for structures to be observable.[6] We see from Eq.(19) that the condition $\frac{\sigma U_0}{2S}\left(1 - \frac{5}{6}a_{sc}\sigma_0\right) - mu^2 \geq 0$ must be satisfied in order to obtain real solutions. In other words, the density of particles per unit length must exceed the minimum value σ_{min}, which determines the velocity u of the dark solitary wave

$$u^2 = \frac{\sigma_{min}U_0}{2mS}\left(1 - \frac{5}{6}a_{sc}\sigma_0\right) = u_0^2\left(1 - \frac{5}{6}a_{sc}\sigma_0\right). \quad (20)$$

We find also from Eq.(19) that the profile of the dark solitary wave can be given by

$$\sigma(z) = \sigma_{min} + (\sigma_0 - \sigma_{min})\tanh^2(z/\xi), \quad (21)$$

$$\xi \equiv \xi(n_0) = \xi_0(n_0)(1 - \frac{5}{6}a_{sc}\sigma_0)^{-1/2}, \quad (22)$$

where $\xi_0(n_0) = \left[2\pi n_0 a_{sc}\left(1 - \frac{\sigma_{min}}{\sigma_0}\right)\right]^{-1/2}$ with $n_0 = \frac{\sigma_0}{S}$. Here u_0 and ξ_0 are, respectively, the velocity and the width of the dark solitary wave in Ref.[6] where the transverse structure $g(x, y, \sigma)$ was determined without considering the interatomic interaction. Thus, compared with the results in Ref.[6], the factor of $\left(1 - \frac{5}{6}a_{sc}\sigma_0\right)$ in the above results is due to the effect of the interatomic interaction on the transverse function $g(x, y, \sigma)$.

We see from Eqs.(20) and (22) that the effect of the interatomic interaction on the transverse function can lead to the velocity of the dark solitary wave slowing down and the profile of the solitary wave widening. We can interpret the above results in physics: the interatomic interaction (being repulsive for the case of $a_{sc} > 0$) smoothes the transverse function $g(x, y, \sigma)$ and decreases the density of atoms $n(0, 0, z)$ in the z axis, which can lead to the repulsive force along the z axis weakening and the dark solitary wave widening. On the other hand, this weakened repulsive force can be effectively characterized by an interaction constant U_0' ($< U_0$) in one-dimensional G-P equation for $f(z, t)$, which determines a speed of sound $c_s\left(= \sqrt{\frac{\sigma_0 U_0'}{mS}}\right)$ and a velocity of the dark solitary wave $u\left(= c_s\sqrt{\frac{\sigma_{min}}{\sigma_0}}\right)$ slower than those in the case without transverse interaction.

Let us now estimate the correction for Jackson's theory.[6] For the case of $\sigma_0 = \frac{\sigma_c}{5} \sim \frac{1}{5a_{sc}}$ (e.g. $\sigma_0 \sim 3.8 \times 10^5\mathrm{cm}^{-1}$, $7.3 \times 10^5\mathrm{cm}^{-1}$ and $9.2 \times 10^5\mathrm{cm}^{-1}$ for ^{87}Rb, Na and H atoms, respectively[8,10]), the correction ratios for the velocity and the width of the solitary wave are estimated as

$$\frac{u - u_0}{u_0} = (1 - \frac{1}{6}a_{sc}\sigma_c)^{1/2} - 1 \sim -0.088, \quad (23)$$

$$\frac{\xi(n_0) - \xi_0(n_0)}{\xi_0(n_0)} = (1 - \frac{1}{6}a_{sc}\sigma_c)^{-1/2} - 1 \sim 0.096, \quad (24)$$

respectively. In the low-density limit ($a_{sc}\sigma_0 \ll 1$), our results are the same as those in Ref.[6].

For the cases of $\sigma > \frac{\sigma_c}{5}$ and the non-high-density limit, the problem of pulse propagation cannot be solved by this method because the motion is no longer quasi-one-dimensional. In order to be able to observe quasi-one-dimensional dark solitary waves, the densities of atoms per unit volume must

be less than $n_{\rm obs} \approx 10^{13}$ cm^{-3}.[6] From the formula $N_{\rm obs} = \frac{8\pi^{3/2}}{3} n_{\rm obs}^{3/2} (a_z a_\perp)^2 a_{\rm sc}^{1/2}$ with $a_z = \sqrt{\frac{\hbar}{m\omega_z}}$[6] and ω_z being frequency in the z axis, we estimate the total number of atoms $N_{\rm obs} \sim 2 \times 10^3$ for structures to be observable in the typical trap in Ref.[7]. In the experiment and the numerical simulations of dark solitons performed by Burger *et al*,[7] $N(= 1.5 \times 10^5$ and $5 \times 10^4) > N_{\rm obs}$, our method is not valid, but the simulations show that the dark soliton velocity drops down and its width grows up when the trap is switched off and the condensate expands in the radial direction, which qualitatively demonstrates our results.

In the case of the high-density limit, the kinetic energy is very small compared with the interaction energy and the TFA may be used to calculate the structure of the cloud in the transverse direction (seeing Refs.[5] and [6]).

In this paper we have discussed the propagation of solitary waves in elongated traps in the low-density case without neglecting particle interactions, and examined the effect of the structure in the transverse direction on the velocity and the profile of solitary waves.

Appendix: Solving the transverse function $g(x, y, \sigma)$

In this appendix, by use of a method similar to that in Ref.[9], we solve in detail Eq.(8) and obtain the expression of transverse function $g(x, y, \sigma)$.

Substituting Eq.(2) into Eq.(8) and introducing the following transformations:

$$r = a_\perp r', \quad \mu(\sigma) = \hbar\omega_\perp \mu'(\sigma),$$

$$g(x, y, \sigma) = g'(x', y', \sigma)/a_\perp \tag{25}$$

with $r = \sqrt{x^2 + y^2}$ and $a_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$, we can rewrite Eq.(8) in a dimensionless form

$$\nabla_\perp'^2 g' - r'^2 g' + 2\mu' g' = F g'^3, \tag{26}$$

where $F \equiv F(z, t) = 8\pi a_{\rm sc}\sigma$, $\nabla_\perp'^2 = \partial_{x'}^2 + \partial_{y'}^2$ and $g'(x', y', \sigma)$ satisfies the normalization condition $\int g'^2 {\rm d}x'{\rm d}y' = 1$. For the experimentally relevant situation, F is evidently larger than 1[1,6,8], and we can assume that $F = \varepsilon^{-1} F_0$ with ε being a small positive parameter and $|F_0|$ being of the order of 1. Expanding g' and μ' in a power series of ε as follows

$$g' = g_0 + \varepsilon g_1 + \varepsilon^2 g_2 + \cdots, \tag{27}$$

$$\mu' = \mu_0 + \varepsilon\mu_1 + \varepsilon^2\mu_2 + \cdots. \tag{28}$$

substituting them into Eq.(26), and equating the coefficients of each power of ε on the left-hand side of the resulting equation correspondingly to those on the right-hand side, we easily obtain $g_0 = 0$ and linear equations governing g_1, g_2, $\cdots$:

$$(\nabla_\perp'^2 - r'^2 + 2\mu_0) g_1 = 0, \tag{29}$$

$$(\nabla_\perp'^2 - r'^2 + 2\mu_0) g_2 = -2\mu_1 g_1 + F_0 g_1^3, \quad \cdots, \tag{30}$$

where μ_0 is the dimensionless chemical potential of the condensate without interaction. Equation (29) can be easily solved

$$g_1' = A{\rm e}^{-r'^2/2}, \quad \mu_0 = 1, \tag{31}$$

where A is a constant to be determined later. In order to solve Eq.(30), we put $g_2 = w(x', y'){\rm e}^{-r'^2/2}$. Thus, Eq.(30) reduces to an equation governing w:

$$\nabla_\perp'^2 w - 2\boldsymbol{r}' \cdot \nabla_\perp' w = -2\mu_1 A + F_0 A^3 {\rm e}^{-r'^2}. \tag{32}$$

Expanding w in Hermite polynomial as

$$w(x', y') = {\sum_{n_1, n_2}}' C_{n_1 n_2} H_{n_1}(x') H_{n_2}(y') \tag{33}$$

and inserting it into Eq.(32), we obtain

$$2 {\sum_{n_1, n_2}}' c_{n_1 n_2}(n_1 + n_2) H_{n_1}(x') H_{n_2}(y') = 2\mu_1 A - F_0 A^3 {\rm e}^{-r'^2}, \tag{34}$$

where $\sum'_{n_1, n_2}$ means the summation over all positive integers except for $n_1 = n_2 = 0$. Here, we have employed the following Hermite equation:

$$\frac{{\rm d}^2}{{\rm d}x^2} H_n(x) - 2x \frac{\rm d}{{\rm d}x} H_n(x) + 2n H_n(x) = 0 \tag{35}$$

governing the Hermite polynomial $H_n(x)$. Multiplying Eq.(34) by $H_0(x')H_0(y'){\rm e}^{-r'^2}$, integrating with respect to x', y' over the whole $x'oy'$ plane and employing the orthonormal relation of the Hermite polynomials

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) {\rm e}^{-x^2} {\rm d}x = 2^m m! \sqrt{\pi} \delta_{mn}, \tag{36}$$

we obtain the first-order correction for the chemical potential $\mu_1 = F_0 A^2/4$. Similarly, multiplying Eq.(34) by $H_{m_1}(x')H_{m_2}(y'){\rm e}^{-r'^2}$, integrating with respect to x', y' and employing Eq.(36), we have

$$C_{m_1 m_2} = -\frac{2\mu_1 A I_{m_1} I_{m_2}}{2^{m_1+m_2}(m_1 + m_2) m_1! m_2! \pi}, \tag{37}$$

where

$$I_{m_i} = \int_{\infty}^{\infty} H_{m_i}(x)\mathrm{e}^{-2x^2}\mathrm{d}x, \quad i = 1, 2. \tag{38}$$

All I_{m_i} can be calculated directly as follows:

$$I_0 = \sqrt{\pi/2}, \quad I_2 = -\sqrt{\pi/2}, \quad I_4 = 3\sqrt{\pi/2}, \quad \cdots,$$
$$I_{2k+1} = 0, \quad k = 0, 1, 2, \cdots. \tag{39}$$

Then, we can easily calculate all expansion coefficients from Eq.(37):

$$C_{20} = C_{02} = \mu_1 A/2^4, \quad C_{22} = -\mu_1 A/2^8,$$
$$C_{40} = C_{04} = -\mu_1 A/2^9, \quad \cdots,$$
$$C_{m_1 m_2} = 0, \quad \text{for odd integers } (m_1 \text{ and } m_2). \tag{40}$$

As a primary approximation, we neglect all terms with $n_1 + n_2 > 2$ in Eq.(33), and obtain

$$w(x', y') \approx C_{20}H_2(x') + C_{02}H_2(y') = \mu_1 A(r'^2 - 1)/4, \tag{41}$$
$$g'(x', y') \approx \epsilon g_1 + \epsilon^2 g_2 = \epsilon A[1 - F(\epsilon A)^2(1 - r'^2)/2^4]\mathrm{e}^{-r'^2/2} \tag{42}$$

from Eq.(33) and Eq.(27), where $\epsilon A \approx \dfrac{1}{\sqrt{\pi}}$ determined by the normalization condition of g'. Consequently, we obtain the transverse wavefunction expressed by Eq.(17).

References

[1] Andrews M R, Kurn D M, Miesner H J, Durfee D S, Townsend C G, Inouye S and Ketterle W 1997 *Phys. Rev. Lett.* **79** 553
Andrews M R, Kurn D M, Miesner H J, Durfee D S, Townsend C G, Inouye S and Ketterle W 1998 *Phys. Rev. Lett.* **80** 2967

[2] Zaremba E 1998 *Phys. Rev.* A **57** 518

[3] Kaavoulakis G M and Pethick C J 1998 *Phys. Rev.* A **58** 1563

[4] Morgan S A, Ballagh R J and Burnett K 1997 *Phys. Rev.* A **55** 4338

[5] Baym G and Pethick C J 1996 *Phys. Rev. Lett.* **76** 6

[6] Jackson A D, Kaavoulakis G M and Pethick C J 1998 *Phys. Rev.* A **58** 2417

[7] Burger S, Bongs K, Dettmer S, Ertmer W and Sengstock K 1999 *Phys. Rev. Lett.* **83** 5198

[8] Denschlag J, Simsarian J E, Feder D L, Clark Charles W, Collins L A, Cubizolles J, Deng L, Hagley E W, Helmerson K, Reinhardt W P, Rolston S L, Schneider B I and Phillps W D 2000 *Science* **287** 97

[9] Yan J R, Lu J, Ao S M and Cao D B 2002 *Chin. Phys. Lett.* **19** 1245

[10] Wang H L, Yan X H and Tang Y 2004 *Chin. Phys.* **13** 2030