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Quantum logic gates operation using SQUID qubits in bimodal cavity

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We present a scheme to realize the basic two-qubit logic gates such as the quantum phase gate and SWAP gate using a detuned microwave cavity interacting with three-level superconducting-quantum-interference-device (SQUID) qubit(s), by placing SQUID(s) in a two-mode microwave cavity and using adiabatic passage methods. In this scheme, the two logical states of the qubit are represented by the two lowest levels of the SQUID, and the cavity fields are treated as quantized. Compared with the previous method, the complex procedures of adjusting the level spacing of the SQUID and applying the resonant microwave pulse to the SQUID to create transformation are not required. Based on superconducting device with relatively long decoherence time and simplified operation procedure, the gates operate at a high speed, which is important in view of decoherence.

Keywords: superconducting-quantum-interference-device, bimodal cavity, adiabatic passage, quantum logic gate

PACC: 0367, 8525, 8970, 4250

1. Introduction

Quantum phase gate (QPG) (or controlled-NOT (CNOT) gate) and the SWAP gate are the basic two-qubit gates, since this set of operations makes a universal quantum computing. All the great effort that has been made is to attain the goal of building corresponding quantum computers. The biggest obstacle to realizing the quantum computers is the decoherence of the system and the environment. Hence, people do their best to find out some good system to realize the quantum computers. Recently, a number of groups have proposed how to perform quantum logic using superconducting devices such as Josephson-junction circuits,[1−3] Josephson junctions,[4−7] Cooper pair boxes,[8−11] and superconducting-quantum-interference devices (SQUIDS).[12−16] Because these systems can be easily fabricated to implement a large-scale quantum computing and have been demonstrated to have relatively long decoherence time,[17,18] they will play an important role in building up quantum computers.

For the three-level SQUID qubits, quantum logic gate operation can be accomplished[12,14] by applying two microwave fields to the SQUID, namely, the fields are usually treated as being classical (classical microwave pulse). In fact, when the fields are treated as quantized, the quantum logic operation can be finished between the fields and the SQUID. The quantization of the field may bring about some new possibilities. Hence, in this paper, we consider two situations where one or two SQUID qubits in the Λ configuration interact with a bimodal quantized cavity field, where the modes are highly detuned from the corresponding one-photon transition. We focus on how to realize the operation of the QPG and SWAP gate by using the two-photon Raman transitions. The two-photon transitions are especially attractive in this case as they can work with the two lower flux states $|0\rangle$ and $|1\rangle$ of the SQUID system. In such a situation, the upper level state does not participate in the transition and thus it is possible to minimize the effect of decoherence associated with the finite lifetime of the upper level state.[19]

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This paper is organized as follows. In Sec. 2, we consider an SQUID qubit interacting with a bimodal field to realize QPG and SWAP gate operation. These operations are all based on the idea in Ref. [19]. In Sec. 3, we show that a faster QPG scheme can be performed using two identical SQUID qubits interacting with a bimodal field in the large single-photon detuning. This operation can be realized by taking only one step. A brief discussion and summary are given in Sec. 4.

2. The interaction of an rf SQUID qubit with a bimodal field and quantum logic gates operation

Let us consider a system composed of an rf SQUID coupled to a bimodal field. We assume that all other cavity modes are well decoupled to the three energy levels of the SQUID (see Fig.1).

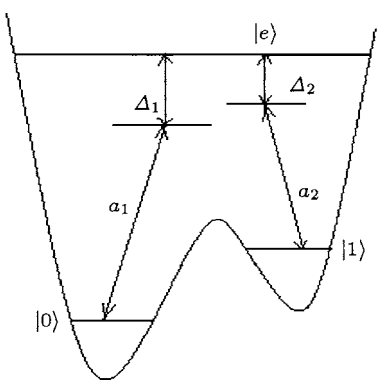


Fig.1. Simplified level diagram of an rf SQUID with Λ-type three levels $|0\rangle$,$|1\rangle$and $|e\rangle$ interacting with a bimodal cavity field, described by annihilation operations a_1 and a_2, and Δ_i, (i=1, 2) are the corresponding single-photon detunings.

The SQUID consists of Josephson tunnel junction, which is enclosed by a superconducting loop (the size of an rf SQUID is on the order of 10μm–100μm). The Hamiltonian for an rf SQUID plus the bimodal field is

$$H = H_{\rm c} + H_{\rm s} + V_{\rm i}, \tag{1}$$

where $H_{\rm c}$, $H_{\rm s}$ and $V_{\rm i}$ are the Hamiltonian of the bimodal field, the Hamiltonian of the SQUID, and the interaction energy, respectively.

The Hamiltonian of the bimodal cavity field can be written as

$$H_{\rm c} = \sum_{j=1}^{2} \hbar\omega_{cj} \left(a_j^{+} a_j + \frac{1}{2} \right), \tag{2}$$

where a_j^{+} and a_j ($j = 1, 2$) are the creation and annihilation operators of the modes with the frequencies of ω_{c1} and ω_{c2}. The modes with annihilation operators a_1 and a_2 interact with the transitions $|e\rangle \Leftrightarrow |0\rangle$ and $|e\rangle \Leftrightarrow |1\rangle$, respectively, where $|e\rangle$ is the upper level state of the SQUID qubit.

For an rf SQUID with junction capacitance C and loop inductance L, the Hamiltonian is[20,21]

$$H_{\rm s} = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} - E_{\rm J} \cos\left(2\pi \frac{\Phi}{\Phi_0} \right), \tag{3}$$

where Φ is the magnetic flux threading the ring and Q is the total charge on the capacitor, Φ and Q are the conjugate variables of the system with the commutation relation $[\Phi, Q] = {\rm i}\hbar$, Φ_x is the static (or quasistatic) flux externally applied to the ring, and $E_{\rm J} \equiv I_{\rm c}\Phi_0/2\pi$ is the Josephson coupling energy with $I_{\rm c}$ being the critical current of the junction and $\Phi_0 = h/2e$ being the flux quantum.

The cavity fields and the SQUID ring are coupled together inductively with a coupling energy given by[13]

$$V_{\rm i} = \sum_{j=1}^{2} \lambda_j (\Phi - \Phi_x) \Phi_j^c, \tag{4}$$

where $\lambda_j = -1/L$, $j = 1, 2$ are the coupling parameters linking their corresponding cavity mode to the SQUID ring; and Φ_j^c, ($\Phi_j^c = \int_{\rm S} \boldsymbol{B}_j(\boldsymbol{r}, t) \cdot {\rm d}\boldsymbol{S}_j$, $j = 1, 2$) are their corresponding magnetic flux threading the ring by $\boldsymbol{B}(\boldsymbol{r}, t) = \sqrt{\hbar\omega_{\rm c}/(2\mu_0)}\,[a(t) + a^{+}(t)]\,\boldsymbol{B}(\boldsymbol{r})$, and $\boldsymbol{S}$ is a surface bounded by the ring, $\boldsymbol{r}$ is the position vector of a point on $\boldsymbol{S}$.

Assume that the magnetic fields of the cavity fields are perpendicular to the plane of the SQUID ring, set $H_{\rm S} = \sum_n E_n |n\rangle\langle n|$, $|n\rangle$ to be the eigenstate of $H_{\rm S}$ with an eigenvalue $E_{\rm n}$. Use the interaction picture and apply the rotating wave approximation, then the Hamiltonian of the system can be expressed as[19]

$$H_{\rm i} = \hbar \left[g_1 |e\rangle\langle 0| a_1 {\rm e}^{{\rm i}\Delta_1 t} + g_2 |e\rangle\langle 1| a_2 {\rm e}^{{\rm i}\Delta_2 t} + {\rm H.c.} \right], \tag{5}$$

where $g_j = \lambda_j (\hbar\omega_{cj}/2\mu_0)^{1/2} \langle e|m\rangle \Phi_j^c$, ($j = 1, 2; m = 0, 1$) are coupling constants, $\Delta_1 = \omega_e - \omega_0 - \omega_{c1}$ and $\Delta_2 = \omega_e - \omega_1 - \omega_{c2}$ are the cavity modes detuning from resonance with the $|m\rangle \Leftrightarrow |e\rangle$, $\hbar\omega_0$ and $\hbar\omega_1$ denote the energies of the two lower flux states $|0\rangle$ and $|1\rangle$ of the SQUID system, respectively. We now work within the limits of large one-photon detuning on the assumption of $g_1 = g_2 = g$, and $(\Delta_1 - \Delta_2) \ll g$. In this limit, the

upper level state $|e\rangle$ can be adiabatically eliminated, and the effective Hamiltonian of this system is given by[19]

$$\begin{aligned} H_{\text{eff}} =& -\frac{\hbar g^2}{\Delta_1}\left(|0\rangle\langle 0|\, a_1^+ a_1 + |1\rangle\langle 1|\, a_2^+ a_2\right) \\ & -\frac{\hbar g^2}{\Delta_1}\left(|0\rangle\langle 1|\, a_1^+ a_2 + |1\rangle\langle 0|\, a_2^+ a_1\right) \\ & +\hbar\left(\Delta_1-\Delta_2\right)|1\rangle\langle 1|. \end{aligned} \tag{6}$$

Here the first two terms on the right-hand side of Eq.(6) represent the photon-number dependent Stark-shifts and the next two terms give the interaction leading to a transition from the initial state to the finial state. The last term represents a shifting of the level $|1\rangle$ due to the two-photon detuning. In the following, we will discuss three cases of the quantum logic gate operation separately.

2.1. The case of Stark-shifts and $(\Delta_1-\Delta_2)\neq 0$

In this subsection, we consider the case of Stark-shifts and $(\Delta_1-\Delta_2)\neq 0$, and realize the QPG operation in this case. The QPG operation is defined through the following input-output relations: $|i\rangle_1|j\rangle_2 \xrightarrow{U} \exp\{\mathrm{i}\varphi_{ij}\}|i\rangle_1|j\rangle_2$, where i and j denote the qubit basis. In order to do so, we suppose that the two levels $|0\rangle$ and $|1\rangle$ of the SQUID qubit, $|0\rangle_i$and $|1\rangle_i, (i=1,2)$ of the two modes represent two logical states of the qubit itself. Here we discuss the QPG with $\varphi=\pi$.

According to Ref.[19], the evolution of the system can be described by

$$|\Psi\rangle = A(t)|0\rangle|n_1,n_2\rangle + B(t)|1\rangle|(n_1-1),(n_2+1)\rangle, \tag{7}$$

where n_1 and n_2 are the photon numbers of Modes 1 and 2, respectively, and the coefficients $A(t)$ and $B(t)$, respectively, are

$$\begin{aligned} A(t) =& \mathrm{e}^{\mathrm{i}\nu t/2}\left\{\left[\cos\left(\frac{\Omega t}{2}\right)\right.\right. \\ & \left.+\frac{\mathrm{i}}{\Omega}(\Delta_1-\Delta_2)\sin\left(\frac{\Omega t}{2}\right)\right]A(0) \\ & \left.+\frac{2\mathrm{i}g^2}{\Delta_1\Omega}\sin\left(\frac{\Omega t}{2}\right)B(0)\right\}, \end{aligned} \tag{8a}$$

$$\begin{aligned} B(t) =& \mathrm{e}^{-\mathrm{i}(\Delta_2-\Delta_1)t}\mathrm{e}^{\mathrm{i}\nu t/2}\left\{\frac{2\mathrm{i}g^2}{\Delta_1\Omega}\sin\left(\frac{\Omega t}{2}\right)A(0)\right. \\ & +\left[\cos\left(\frac{\Omega t}{2}\right)-\frac{\mathrm{i}}{\Omega}(\Delta_1-\Delta_2)\right. \\ & \left.\left.\times\sin\left(\frac{\Omega t}{2}\right)\right]B(0)\right\}, \end{aligned} \tag{8b}$$

where

$$\nu = \frac{2g^2}{\Delta_1}+(\Delta_2-\Delta_1), \tag{9a}$$

$$\Omega = \left[\left(\frac{2g^2}{\Delta_1}\right)^2+(\Delta_2-\Delta_1)^2\right]^{1/2}. \tag{9b}$$

Set $\nu = 2(\Delta_2-\Delta_1)$, i.e. $(\Delta_2-\Delta_1)/g = 2/(\Delta_1/g)$. According to Eqs.(8) and (9), it is easy for us to obtain the time evolution of the system as follows:

$$|0_1\rangle|1_2\rangle|1\rangle \xrightarrow{gt=\pi\Delta_1/(\sqrt{2}g)} -|0_1\rangle|1_2\rangle|1\rangle, \tag{10a}$$

$$|0_1\rangle|1_2\rangle|0\rangle \to |0_1\rangle|1_2\rangle|0\rangle. \tag{10b}$$

For the cavity modes being in all the vacuum state and the SQUID in the state $|0\rangle$ or $|1\rangle$, the evolution of the system does not take place. For the case of Mode 1 in the state $|0_1\rangle$ and with this mode regarded as a quantum data bus, from Eq.(10) one can see that the following QPG can be obtained:

$$\begin{cases} |0_2\rangle|0\rangle|0_1\rangle \\ |0_2\rangle|1\rangle|0_1\rangle \\ |1_2\rangle|0\rangle|0_1\rangle \\ |1_2\rangle|1\rangle|0_1\rangle \end{cases} \xrightarrow{gt=\pi\Delta_1/(\sqrt{2}g)} \begin{cases} |0_2\rangle|0\rangle|0_1\rangle, \\ |0_2\rangle|1\rangle|0_1\rangle, \\ |1_2\rangle|0\rangle|0_1\rangle, \\ -|1_2\rangle|1\rangle|0_1\rangle. \end{cases} \tag{11}$$

which clearly involves the state basis of the SQUID qubit and the Fock state basis in Mode 2.

2.2. The case of Stark-shifts and $(\Delta_1-\Delta_2)=0$

Setting $(\Delta_1-\Delta_2)=0$, then Eq.(6) reduces to

$$\begin{aligned} H_{\text{eff}} =& H_0+H_1, \\ H_0 =& -\frac{\hbar g^2}{\Delta_1}\left(|0\rangle\langle 0|\, a_1^+ a_1 + |1\rangle\langle 1|\, a_2^+ a_2\right), \\ H_1 =& -\frac{\hbar g^2}{\Delta_1}\left(|0\rangle\langle 1|\, a_1^+ a_2 + |1\rangle\langle 0|\, a_2^+ a_1\right). \end{aligned} \tag{12}$$

In the basis vectors $\{|0_1,0_2\rangle;\ |0_1,1_1\rangle;|1_1,0_2\rangle\}$, we have the commutation relation: $[H_0,H_1]=0$. Thus, the time evolution of the system is determined by Schrödinger's equation,

$$\mathrm{i}\frac{\mathrm{d}|\Psi(t)\rangle}{\mathrm{d}t} = H_{\text{eff}}|\Psi(t)\rangle. \tag{13}$$

Perform the unitary transformation

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_0 t}|\Psi'(t)\rangle, \tag{14}$$

Then the state $|\Psi'(t)\rangle$ satisfy the following relation:

$$\mathrm{i}\frac{\mathrm{d}|\Psi'(t)\rangle}{\mathrm{d}t} = H_1|\Psi'(t)\rangle. \tag{15}$$

According to Eqs.(14) and (15), one can easily obtain

$$|0_1,1_1\rangle|1\rangle \to e^{ig^2t/\Delta_1}\left[\cos\left(\frac{g^2t}{\Delta_1}\right)|0_1,1_1\rangle|1\rangle + i\sin\left(\frac{g^2t}{\Delta_1}\right)|1_1,0_1\rangle|0\rangle\right], \quad (16)$$

$$|1_1,0_1\rangle|0\rangle \to e^{ig^2t/\Delta_1}\left[\cos\left(\frac{g^2t}{\Delta_1}\right)|1_1,0_1\rangle|0\rangle + i\sin\left(\frac{g^2t}{\Delta_1}\right)|0_1,1_1\rangle|1\rangle\right]. \quad (17)$$

We take the interaction time as $t=\pi\Delta_1/2g^2$, the following SWAP gate can be performed:

$$\begin{cases} |0_1,1_2\rangle|0\rangle \\ |1_1,0_2\rangle|0\rangle \\ |0_1,1_2\rangle|1\rangle \\ |1_1,0_2\rangle|1\rangle \end{cases} \xrightarrow{gt=\pi\Delta_1/2g^2} \begin{cases} |0_1,1_2\rangle|0\rangle, \\ -|0_1,1_2\rangle|1\rangle, \\ -|1_1,0_2\rangle|0\rangle, \\ |1_1,0_2\rangle|1\rangle. \end{cases}, \quad (18)$$

It is clear that the SWAP operation can be accomplished between the two-mode photon number basis and the SQUID qubit.

2.3. The case of $\Delta_1 \gg g, (\Delta_1-\Delta_2)=0$ and neglecting Stark-shifts

Setting $(\Delta_1-\Delta_2)=0$, and considering no Stark-shifts, we have the effective Hamiltonian

$$H'_{\rm eff} = -\frac{\hbar g^2}{\Delta_1}\left(|0\rangle\langle 1|a_1^+a_2 + |1\rangle\langle 0|a_2^+a_1\right). \quad (19)$$

When the state of Mode 1 is regarded as a quantum data bus, we can easy perform the QPG operation as follows:

$$\begin{cases} |0_2\rangle|0\rangle|0_1\rangle \\ |0_2\rangle|1\rangle|0_1\rangle \\ |1_2\rangle|0\rangle|0_1\rangle \\ |1_2\rangle|1\rangle|0_1\rangle \end{cases} \xrightarrow{gt=\pi\Delta_1/g^2} \begin{cases} |0_2\rangle|0\rangle|0_1\rangle, \\ |0_2\rangle|1\rangle|0_1\rangle, \\ |1_2\rangle|0\rangle|0_1\rangle, \\ -|1_2\rangle|1\rangle|0_1\rangle. \end{cases} \quad (20)$$

Similarly, if we regard Mode 2 as a data-bus, one can also perform the QPG operation.

3. The interaction of two SQUID qubits with a bimodal field and quantum logic gate operation

We now place two identical SQUIDs denoted A and B in a two-mode microwave cavity. The separation of the two SQUIDs is assumed to be much larger than the linear dimension of each SQUID ring so that the interaction between the two SQUIDs is negligible. Also, the couplings of SQUIDs to their corresponding cavity modes are assumed to be identical, namely, the transitions $|1_j\rangle \to |e_j\rangle$ with $j=A,B$ are coupled to the cavity mode 2, with the coupling constant g_1, which is assumed real. The other transitions $|0_j\rangle \to |e_j\rangle$ with $j=A,B$ are coupled to the cavity mode 1, with the coupling constant g_0, which is also assumed real. The same coupling can be readily obtained by setting the two SQUIDs in two different positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ of the cavity axis where the cavity-mode magnetic components $\boldsymbol{B}(\boldsymbol{r}_1,t)$ and $\boldsymbol{B}(\boldsymbol{r}_2,t)$ are the same.[13] In order to analyse our system further, we use the interaction picture and apply the rotating wave approximation. Thus the Hamiltonian of the system can be expressed as

$$H^i_{AB} = \hbar\sum_{j=A,B}[g_0e^{-i\Delta_1 t}|0_j\rangle\langle e_j|a_1^+ + g_1e^{-i\Delta_2 t}|1_j\rangle\langle e_j|a_2^+ + \text{H.c.}]. \quad (21)$$

Since Mode 1 (2) couples simultaneously the transitions $|0_j\rangle \Leftrightarrow |e_j\rangle$ $(|1_j\rangle \Leftrightarrow |e_j\rangle$ $(j=A,B)$, and if the initial numbers of photon in Modes 1 and 2 are n and μ, respectively, then the states of the two SQUIDs and two cavity-mode system can be expanded in terms of the possible basis vectors in the following way:

$$|\Psi(t)\rangle = A(t)|0_A\rangle|0_B\rangle|n,\mu\rangle + B(t)|1_A\rangle|1_B\rangle|(n-1),(\mu+1)\rangle + C(t)|e_A\rangle|e_B\rangle|(n-1),\mu\rangle. \quad (22)$$

Assume that $g_0=g_1=g, \Delta_i \gg g, (i=1,2)$, and $|\Delta_1-\Delta_2| \ll g$, then the upper level states $|e_j\rangle$ are adiabatically dissipated. Therefore, it is not difficult to obtain the effective Hamiltonian as follows:

$$H^e_{A,B} = -\frac{\hbar g^2}{\Delta_1}\sum_{j=A,B}\left(|0_j\rangle\langle 0_j|a_1^+a_1 + |1_j\rangle\langle 1_j|a_2^+a_2\right) - \frac{\hbar g^2}{\Delta_1}\sum_{j=A,B}\left(|0_j\rangle\langle 1_j|a_1^+a_2 + |1_j\rangle\langle 0_j|a_2^+a_1\right) + \hbar(\Delta_1-\Delta_2)\sum_{j=A,B}|1_j\rangle\langle_j 1|. \quad (23)$$

Adopting the same approach as that in Ref.[19], we obtain the same coefficients $A(t)$ and $B(t)$ as those in Eq.(8).

(i) If the two SQUIDs are assumed to be initially in the state $|1_A\rangle|1_B\rangle$ then $A(0)=0$ and $B(0)=1$. Further, the bimodal cavity-field is assumed to be initially in the state $|0_1,1_2\rangle$. If $(\Delta_2-\Delta_1)/g = 2/(\Delta_1/g)$ and an interaction time $t=\pi\Delta_1/\left(\sqrt{2}g^2\right)$, then it is easy from Eqs.(8) and (23) to obtain

$$|1_A\rangle|1_B\rangle|0_1,1_2\rangle \to -|1_A\rangle|1_B\rangle|0_1,1_2\rangle. \quad (24)$$

For the other cases of $|0_A\rangle|1_B\rangle, |1_A\rangle|0_B\rangle$ and $|0_A\rangle|0_B\rangle$, and the bimodal cavity-field being initially in the state $|0_1, 1_2\rangle$, the evolution of the state of the system does not take place. If we regard the bimodal cavity-field as a quantum data-bus, we can perform the following QPG operation corresponding to the two SQUIDs:

$$\begin{cases} |0_A\rangle|0_B\rangle \\ |0_A\rangle|1_B\rangle \\ |1_A\rangle|0_B\rangle \\ |1_A\rangle|1_B\rangle \end{cases} \xrightarrow{gt=\pi\Delta_1/(\sqrt{2}g^2)} \begin{cases} |0_A\rangle|0_B\rangle, \\ |0_A\rangle|1_B\rangle, \\ |1_A\rangle|0_B\rangle, \\ -|1_A\rangle|1_B\rangle. \end{cases} \tag{25}$$

(ii) If the two SQUIDs are assumed to be initially in the state $|0_A\rangle|0_B\rangle$ then $A(0) = 1$, and $B(0) = 0$. Further, the bimodal cavity-field is assumed to be initially in the state $|1_1, 0_2\rangle$. If $(\Delta_1 - \Delta_2)/g = 2/(\Delta_1/g)$ and an interaction time $t = \pi\Delta_1/(\sqrt{2}g^2)$, it is not difficult to obtain

$$|0_A\rangle|0_B\rangle|1_1, 0_2\rangle \rightarrow -|0_A\rangle|0_B\rangle|1_1, 0_2\rangle. \tag{26}$$

For their being initially in $|0_A\rangle|1_B\rangle|1_1, 0_2\rangle$, $|1_A\rangle|0_B\rangle|1_1, 0_2\rangle$ and $|1_A\rangle|1_B\rangle|1_1, 0_2\rangle$, the states of the system will not be changed. Thus, we have

$$\begin{cases} |0_A\rangle|0_B\rangle \\ |0_A\rangle|1_B\rangle \\ |1_A\rangle|0_B\rangle \\ |1_A\rangle|1_B\rangle \end{cases} \xrightarrow{gt=\pi\Delta_1/(\sqrt{2}g^2)} \begin{cases} -|0_A\rangle|0_B\rangle, \\ |0_A\rangle|1_B\rangle, \\ |1_A\rangle|0_B\rangle, \\ |1_A\rangle|1_B\rangle. \end{cases} \tag{27}$$

It is clear that the operations accomplish a two-SQUID-qubit QPG by taking only one step.

4. Discussion and conclusions

We have presented a scheme for basic two-qubit gates operation such as QPG and SWAP gate using a detuned microwave cavity interacting with three-level SQUID qubit(s), by placing SQUID(s) in a two-mode microwave cavity and using the adiabatic passage method. Consider the relation between the operation time and the cavity decay time. If the cavity decay constant $\kappa_a = \kappa_b = \kappa$ for each mode is 10^5Hz (we take $Q_c = 2 \times 10^6$, where Q_c is the cavity quality factor,$\omega_{c1} = \omega_{c2} = 33.3$GHz,[22]) the radiative time of the cavity is about $T_r = 1/\kappa \approx 10^{-5}$s, and the coupling constant is $g \sim 1.8 \times 10^8$/s,[22] the required SQUID-qubit-cavity-mode interaction time (the gate time) is on the order of 10^{-7}s, much shorter than T_r. Thus all operations such as the storage of the photons after initial preparation and detection of the photonic qubit can be accomplished within the cavity lifetime. On the other hand, if $|\Delta_1 - \Delta_2|/g = 2/(\Delta_1/g)$, then for the large one-photon detuning $\Delta_i \gg g, (i = 1, 2)$ we have $|\Delta_1 - \Delta_2| \ll g$. The condition of the adiabatic approximation is satisfied.

Our proposal is possessed of sixfold distinct advantages: (i) The upper level state $|e\rangle$ is adiabatically eliminated during the gate operation so that decoherence due to energy relaxation from the level $|e\rangle$ is minimized. (ii) The operations discussed above involve either Mode 1 or Mode 2 or both Mode 1 and Mode 2 as well as the two lower flux states $|0\rangle$ and $|1\rangle$ of the SQUID system. Because the transition $|0\rangle \Leftrightarrow |1\rangle$is dipole-forbidden, one can further increase the potential barrier between the qubit levels. Therefore, the two-qubit gates operation discussed in the present paper is not affected by decoherence due to spontaneous emission of the SQUID levels. (iii) Compared with Ref. [13], complex procedures of adjusting the level spacing between the SQUIDs and applying the resonant microwave pulse to the SQUID to create transformation are not required. Our gate operation is performed by taking only one step, and the gate operation speed is greatly improved, which is important in view of decoherence. (iv) In Ref. [16], the QPG operation is performed by using three identical SQUIDs coupled to a single-mode cavity, while our present scheme can be realized only by using two SQUIDs coupled to a two-mode cavity, the task of data-bus is undertaken by two-mode cavity. (v) The present scheme is possible to be extended in order to perform quantum computing on lots of SQUID qubits inside a bimodal cavity (see Sec. 3). (vi) Our model (see Sec. 3) can be used to prepare entanglement of many SQUID qubits by measuring the cavity-field. Based on the simple step and faster operation, this method has robust ability of overcoming decoherence. Therefore, based on SQUID-qubit in cavity quantum electrodynamics, our proposed approach might be realized now or in the near future.

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