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The scattering fields for a spherical target irradiated by a plane electromagnetic wave in an arbitrary direction*

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The relation between corresponding trigonometric functions in two rotating coordinate systems is presented. The transformation formula for a vector in the two rotating spherical coordinate systems is obtained. The scattering fields for a spherical target irradiated by a plane electromagnetic wave in an arbitrary direction are derived. These fields in a particular case retrogress to those available in the literature. The obtained results have great potential in practical applications.

Keywords: scattering, spherical target, polarization in an arbitrary direction, electromagnetic wave
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1. Introduction

The spherical target is the most typical scattering target of radar targets.[1] Its scattering fields can be considered as a benchmark to evaluate the scattering characteristics of targets with other shapes and estimate the numerical computations related to target scattering, etc. So, the spherical target is extensively used and researched.[2−7] The investigation on the scattering fields from a sphere has been presented in the literature[8−11] on the condition that the incident wave propagates in the z-direction and polarizes in the x-direction in an original coordinate system Σ. However, the incident wave in engineering, such as the dual station radar system, propagates and polarizes in arbitrary directions.[12] These results given by the literature must cause a lot of inconvenience of applications of the scattering characteristics of the relevant objects such as light scattering, raindrops, fog and dust storms. Obviously, a good knowledge of the scattering fields from a spherical target in the case of the wave being incident in an arbitrary direction and polarizing arbitrarily[13,14] is of theoretical significance and of great applied value.

In this paper, the scattering field of a sphere is studied in detail by coordinate transformation when the incident wave propagates in an arbitrary direction and polarizes arbitrarily. The expressions of scattering fields are obtained. First, a new rectangular coordinate system is introduced by rotating the original coordinate system, and the relation between trigonometric functions in the two systems is derived. Then the relation between vectors in the two spherical coordinate systems is obtained. Finally the scattering field of a spherical target is acquired when the incident wave propagates in an arbitrary direction and polarizes arbitrarily.

2. Translations of vectors and angles

2.1. Transformation of trigonometric functions

It is assumed that there are two coordinate systems Σ and Σ' as illustrated in Fig.1.

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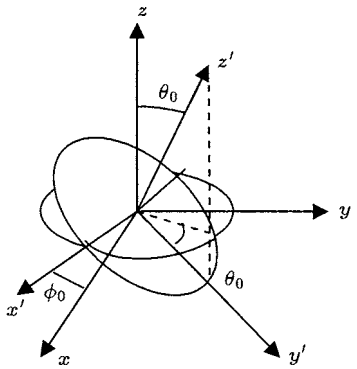


Fig.1. Relations between coordinate axes in Σ system and Σ' system.

The coordinates for the rectangular system and the spherical system are denoted respectively by x, y, z and r, θ, φ for the system Σ. The unit vectors in the system are denoted by u_x, u_y, u_z and $\hat{r}$, $\hat{\theta}$, $\hat{\varphi}$ respectively.

Correspondingly, the coordinates x', y', z' and r', θ', φ' are for the system Σ', and their unit vectors are represented as e_x, e_y, e_z and $\hat{r}', \hat{\theta}', \hat{\varphi}'$ respectively. θ_0 is the angle between the z'-axis and the z-axis, φ_0 is the angle between the x'-axis and the x-axis. We first rotate the rectangular coordinate system Σ around the z-axis by an angle φ_0 in the counter clockwise sense, thus obtain a transitional coordinate system Σ''. We then rotate the coordinate system around its x''-axis by an angle θ_0 in the clockwise sense, and obtain the system Σ. The transformation of the unit vectors between the systems Σ and Σ' is obtained in the above process as follows:

$$\boldsymbol{e} = \boldsymbol{A}_0 \boldsymbol{u}, \quad \boldsymbol{u} = \boldsymbol{A}_0^{-1} \boldsymbol{e}. \tag{1a}$$

Where

$$\boldsymbol{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}, \quad \text{and} \quad \boldsymbol{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix},$$

$$\boldsymbol{A}_0 = \begin{bmatrix} \cos\varphi_0 & -\sin\varphi_0 & 0 \\ \cos\theta_0 \sin\varphi_0 & \cos\theta_0 \cos\varphi_0 & -\sin\theta_0 \\ \sin\theta_0 \sin\varphi_0 & \sin\theta_0 \cos\varphi_0 & \cos\theta_0 \end{bmatrix},$$

$$\boldsymbol{A}_0^{-1} = \begin{bmatrix} \cos\varphi_0 & \cos\theta_0 \sin\varphi_0 & \sin\theta_0 \sin\varphi_0 \\ -\sin\varphi_0 & \cos\theta_0 \cos\varphi_0 & \sin\theta_0 \cos\varphi_0 \\ 0 & -\sin\theta_0 & \cos\theta_0 \end{bmatrix}.$$

Let V be a vector that can be expressed in the two systems, respectively, as

$$V = v_x u_x + v_y u_y + v_z u_z,$$

and

$$V' = v'_x e_x + v'_y e_y + v'_z e_z.$$

Obviously, $V = V'$, we thus obtain the following expressions:

$$v'_x = V' \cdot e_x = V \cdot e_x = e_x \cdot u_x v_x + e_x \cdot u_y v_y + e_x \cdot u_z v_z,$$

$$v'_y = V' \cdot e_y = V \cdot e_y = e_y \cdot u_x v_x + e_y \cdot u_y v_y + e_y \cdot u_z v_z,$$

$$v'_z = V' \cdot e_z = V \cdot e_z = e_z \cdot u_x v_x + e_z \cdot u_y v_y + e_z \cdot u_z v_z.$$

Applying Eq.(1a) to the above yields

$$V' = \boldsymbol{A}_0 V, \tag{1b}$$

which implies that the transformation of a vector is the same as that of the unit vector. Let α_0 and β_0 be the angles between the z'-axis and the x-axis, and between the z'-axis and y-axis respectively. The following expressions are thus obtained by utilizing the dot product of the unit vectors in three coordinate axes:

$$\begin{aligned} &\cos\alpha_0 = \sin\theta_0 \sin\varphi_0 \\ &\cos\beta_0 = \sin\theta_0 \cos\varphi_0, \\ &\cos^2\alpha_0 + \cos^2\beta_0 + \cos^2\theta_0 = 1. \end{aligned} \tag{2}$$

It is concluded that the direction of z'-axis in the system Σ is entirely determined by both angles of θ_0 and φ_0.

It is well known that the position vectors $\boldsymbol{r}$ and $\boldsymbol{r}'$ for a point p in the two systems are the same: $\boldsymbol{r} = \boldsymbol{r}'$. The relation between the spherical coordinate system and the rectangular coordinate system is written as

$$\begin{aligned} x &= r\sin\theta\cos\varphi, \\ y &= r\sin\theta\sin\varphi, \\ z &= r\cos\theta, \\ x' &= r'\sin\theta'\cos\varphi', \\ y' &= r'\sin\theta'\sin\varphi', \\ z' &= r'\cos\theta'. \end{aligned} \tag{3}$$

Substituting the expression (3) into Eq.(1) yields

$$\begin{aligned} \sin\theta'\cos\varphi' &= \cos\varphi_0 \sin\theta\cos\varphi - \sin\varphi_0 \sin\theta\sin\varphi, \\ \sin\theta'\sin\varphi' &= \cos\theta_0 \sin\varphi_0 \sin\theta\cos\varphi - \sin\theta_0 \cos\theta \\ &\quad + \cos\theta_0 \cos\varphi_0 \sin\theta\sin\varphi, \\ \cos\theta' &= \sin\theta_0 \sin\theta \sin(\varphi_0 + \varphi) + \cos\theta_0 \cos\theta. \end{aligned} \tag{4a}$$

In the above expression, we have to express the trigonometric functions in the system Σ' as those in

the system Σ for the sake of application. There appears no change in the third expression in the above after the transformation. In order to exclusively obtain the sinusoidal function, the following transformations are introduced:

$$\sin\theta'\cos\varphi' = A,$$
$$\sin\theta'\sin\varphi' = B,$$

$$\begin{aligned} A &= \cos\varphi_0\sin\theta\cos\varphi - \sin\varphi_0\sin\theta\sin\varphi, \\ B &= \cos\theta_0\sin\varphi_0\sin\theta\cos\varphi - \sin\theta_0\cos\theta \\ &\quad + \cos\theta_0\cos\varphi_0\sin\theta\sin\varphi. \end{aligned} \tag{4b}$$

Since $\sin\theta' \geq 0$, we thus obtain the following expressions:

$$\begin{aligned} &\sin\theta' = \sqrt{A^2+B^2}, \\ &\cos\varphi' = A/\sqrt{A^2+B^2}, \\ &\sin\varphi' = B/\sqrt{A^2+B^2}, \end{aligned} \tag{5a}$$

$$\cos\theta' = \sin\theta_0\sin\theta\sin(\varphi_0+\varphi) + \cos\theta_0\cos\theta. \tag{5b}$$

Up to now, all the trigonometric functions in the system Σ' are completely expressed as those in the system Σ, which establishes a base of using the coordinate transformation to investigate the scattering field from a target irradiated by a plane electromagnetic wave in an arbitrary direction. Obviously, it is impossible to change the above components into a simple expression, which implies that obtaining the general scattering field from a target is very difficult when the incident wave irradiates a target in an arbitrary direction.

2.2. The transformation of a vector in spherical coordinate systems

Let V be a vector in the system Σ, so V_s is used to express it in the spherical coordinate system and V_r is a vector in the rectangular coordinate system. Its relation with V_s can be written as

$$\boldsymbol{V}_\mathrm{r} = \boldsymbol{P}\boldsymbol{V}_\mathrm{s}, \tag{6}$$

where

$$\boldsymbol{P} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}.$$

We can easily obtain a relationship corresponding to the relationship (6), in the system Σ'

$$\boldsymbol{V}'_\mathrm{r} = \boldsymbol{P}_1\boldsymbol{V}'_\mathrm{s}, \tag{7}$$

where

$$\boldsymbol{P}_1 = \begin{bmatrix} \sin\theta'\cos\varphi' & \cos\theta'\cos\varphi' & -\sin\varphi' \\ \sin\theta'\sin\varphi' & \cos\theta'\sin\varphi' & \cos\varphi' \\ \cos\theta' & -\sin\theta' & 0 \end{bmatrix}.$$

Substituting the relationships (6) and (7) into the expression (1b) yields

$$\begin{aligned} &\boldsymbol{V}_\mathrm{s} = \boldsymbol{P}_\mathrm{s}\boldsymbol{V}'_\mathrm{s}, \\ &\boldsymbol{P}_\mathrm{s} = \boldsymbol{P}^{-1}\boldsymbol{A}_0^{-1}\boldsymbol{P}_1. \end{aligned} \tag{8}$$

The transformation (8) presents the relation between vectors in the two spherical coordinate systems, one of which is rotating around another one arbitrarily. Since the scattering field is often expressed in terms of spherical form, the expression (8) is the base of obtaining the transformation of scattering field. For convenience, the components of the above are expressed in the following forms:

$$\begin{aligned} p_{s11} &= \cos\theta\cos\theta'\cos\theta_0 + \cos\varphi_0\sin\theta'\sin\varphi'\sin\theta\sin\varphi\cos\theta_0 - \sin\theta_0\cos\theta\sin\theta'\sin\varphi' \\ &\quad + \sin\theta'\sin\varphi'\sin\theta\cos\varphi\cos\theta_0\sin\varphi_0 + \sin\theta_0\cos\theta'\sin\theta\cos\varphi\sin\varphi_0 \\ &\quad + \sin\theta_0\cos\varphi_0\cos\theta'\sin\theta\sin\varphi - \sin\theta\sin\theta'\cos\varphi'\sin\varphi\sin\varphi_0 + \cos\varphi_0\sin\theta\sin\theta'\cos\varphi'\cos\varphi, \\ p_{s12} &= -\cos\theta\sin\theta'\cos\theta_0 - \sin\theta_0\cos\theta\cos\theta'\sin\varphi' + \cos\varphi_0\sin\theta\cos\theta'\cos\varphi'\cos\varphi \\ &\quad - \sin\theta_0\cos\varphi_0\sin\theta'\sin\theta\sin\varphi - \sin\theta_0\sin\theta'\sin\theta\cos\varphi\sin\varphi_0 - \sin\theta\cos\theta'\cos\varphi'\sin\varphi\sin\varphi_0 \\ &\quad + \cos\theta_0\sin\varphi_0\cos\theta'\sin\varphi'\sin\theta\cos\varphi + \cos\varphi_0\cos\theta'\sin\varphi'\sin\theta\sin\varphi\cos\theta_0, \\ p_{s13} &= \sin\theta'\sin\theta\sin\varphi\sin\varphi_0 - \cos\varphi_0\sin\varphi'\sin\theta\cos\varphi - \sin\theta_0\cos\theta\cos\varphi' \\ &\quad + \cos\varphi_0\cos\varphi'\sin\theta\sin\varphi\cos\theta_0 + \cos\varphi'\sin\theta\cos\varphi\cos\theta_0\sin\varphi_0, \end{aligned}$$

$$
\begin{aligned}
p_{s21} =& -\cos\theta'\sin\theta\cos\theta_0 + \sin\theta_0\sin\theta'\sin\varphi'\sin\theta + \sin\theta_0\cos\theta\cos\theta'\cos\varphi\sin\varphi_0 \\
&+ \sin\theta_0\cos\varphi_0\cos\theta\cos\theta'\sin\varphi + \cos\varphi_0\cos\theta\sin\theta'\sin\varphi'\sin\varphi\cos\theta_0 \\
&+ \cos\theta\sin\theta'\sin\varphi'\cos\varphi\cos\theta_0\sin\varphi_0 - \cos\theta\sin\theta'\cos\varphi'\sin\varphi\sin\varphi_0 + \cos\varphi_0\cos\theta\sin\theta'\cos\varphi'\cos\varphi, \\
p_{s22} =& \sin\theta'\sin\theta\cos\theta_0 - \sin\theta_0\cos\varphi_0\cos\theta\sin\theta'\sin\varphi + \cos\varphi_0\cos\theta\cos\theta'\sin\varphi'\sin\varphi\cos\theta_0 \\
&+ \cos\theta\cos\theta'\sin\varphi'\cos\varphi\cos\theta_0\sin\varphi_0 - \sin\theta_0\cos\theta\sin\theta'\cos\varphi\sin\varphi_0 + \cos\varphi_0\cos\theta\cos\theta'\cos\varphi'\cos\varphi \\
&+ \sin\theta_0\cos\theta'\sin\varphi'\sin\theta - \cos\theta\cos\theta'\cos\varphi'\sin\varphi\sin\varphi_0, \\
p_{s23} =& \cos\theta\sin\varphi'\sin\varphi\sin\varphi_0 - \cos\varphi_0\cos\theta\sin\varphi'\cos\varphi + \sin\theta_0\cos\varphi'\sin\theta \\
&+ \cos\varphi_0\cos\theta\cos\varphi'\sin\varphi\cos\theta_0 + \cos\theta\cos\varphi'\cos\varphi\cos\theta_0\sin\varphi_0, \\
p_{s31} =& \cos\varphi_0\cos\theta_0\sin\theta'\sin\varphi'\cos\varphi + \cos\varphi_0\sin\theta_0\cos\theta'\cos\varphi - \cos\varphi_0\sin\theta'\cos\varphi'\sin\varphi \\
&- \sin\theta_0\cos\theta'\sin\varphi\sin\varphi_0 - \sin\theta'\cos\varphi'\cos\varphi\sin\varphi_0 - \cos\theta_0\sin\theta'\sin\varphi'\sin\varphi\sin\varphi_0, \\
p_{s32} =& \cos\varphi_0\cos\theta_0\cos\theta'\sin\varphi'\cos\varphi - \cos\varphi_0\sin\theta_0\sin\theta'\cos\varphi - \cos\varphi_0\cos\theta'\cos\varphi'\sin\varphi \\
&+ \sin\theta_0\sin\theta'\sin\varphi\sin\varphi_0 - \cos\theta'\cos\varphi'\cos\varphi\sin\varphi_0 - \cos\theta_0\cos\theta'\sin\varphi'\sin\varphi\sin\varphi_0, \\
p_{s33} =& \cos\varphi_0\cos\varphi'\cos\theta_0\cos\varphi + \cos\varphi_0\sin\varphi'\sin\varphi + \sin\varphi'\cos\varphi\sin\varphi_0 - \cos\varphi'\cos\theta_0\sin\varphi\sin\varphi_0.
\end{aligned}
$$

There is no need here to derive the inverse transformation of the expression (8) since it is easily derived. After substituting Eq.(5) into the above formula, all the components can be entirely expressed by the parameters θ_0, φ_0 and angles θ, φ.

2.3. Discussion

When the above parameters are changed into $\theta_0 = \varphi_0 = 0$, the transformation matrix becomes the following form:

$$
\boldsymbol{A}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},
$$

which demonstrates that the corresponding vectors in the two rectangular coordinate systems are identical with each other. This conclusion must be drawn because the corresponding axes are parallel to each other. The expression (4a) is changed into

$$
\begin{aligned}
&\sin\theta'\cos\varphi' = \sin\theta\cos\varphi, \\
&\sin\theta'\sin\varphi' = \sin\theta\sin\varphi, \\
&\cos\theta' = \cos\theta.
\end{aligned}
$$

It is concluded from the above results that the matrix $\boldsymbol{P}$ and matrix $\boldsymbol{P}_1$ are the same. It is thus obtained from the expression (8) that the transformation matrix of a vector in the spherical coordinates is changed into the following form:

$$
\boldsymbol{P}_{\mathrm{s}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

All the results obtained in this section demonstrate the validity of the investigation in this paper.

3. Application

It is assumed that there is a spherical target, say, a conductor sphere or a medium sphere, located at the origin of the coordinate system Σ'. An electromagnetic wave polarizing in the x'-direction and propagating in the z'-direction irradiates the target. It is observed in the system Σ that the incident wave propagates in the direction at angles of α_0, β_0 and θ_0 respectively with three axes. These angles are completely determined by the expression (2). Meanwhile, the incident wave has a determined polarization. The scattering field of a target in the system Σ' is obtained as[8−10]

$$
\boldsymbol{E}' = \begin{bmatrix} 0 \\ E'_{s\theta} \\ E'_{s\varphi} \end{bmatrix}.
$$

Substituting the above expression into Eq. (8) yields the scattering field in the system Σ

$$
\begin{aligned}
E_\theta &= p_{s22}E'_{s\theta} + p_{s23}E'_{s\varphi}, \\
E_\varphi &= p_{s32}E'_{s\theta} + p_{s33}E'_{s\varphi},
\end{aligned} \tag{9}
$$

which is the general expression of the scattering field for a target when the incident wave propagates in an arbitrary direction. It is remarkable that the trigonometric functions must be replaced with the formula (5). Under this condition we can conclude from the Mie theory that obtaining the scattering parameters such as the scattering cross-section is difficult. However numerically computing those parameters is feasible. The process of obtaining the expression (9) holds truce for ordinary targets in reality.

4. Conclusion

The scattering field from a spherical target is studied in detail for the first time utilizing the coordinate transformation, when an incident wave propagates and polarizes arbitrarily. The expression of the scattering field is obtained. First, a new coordinate system is introduced by rotating the original one arbitrarily. The relation between the trigonometric functions in the two systems is presented. Then, the transformation of a vector between two spherical coordinate systems is obtained. Finally, the scattering fields from a sphere are obtained utilizing this transformation when the incident wave propagates and polarizes arbitrarily. The obtained results in this paper are universal in application. The present work provides a new approach to investigating the scattering fields from targets with other shapes for what is considered herein. This paper is the fact that the wave is incident upon a target in an arbitrary direction. This work has not only the theoretical significance, but also extensive applications in several fields such as radar target identification, target concealment, etc.

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