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Effect of disturbance in perfect state transfer*

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The influence of the disturbance caused by the imperfection of the engineering coupling constants in the perfect state transfer is calculated. The results show that the fidelity for the perfect state transfer is seriously affected by the errors occurring near the input and output spins. Such results are helpful for the realization of the perfect state transfer in the case where there exist errors in experiments.

Keywords: XY-chain, perfect state transfer, disturbance
PACC: 0365, 0365B

1. Introduction

The quantum state transfer from one location to another is an important task in quantum information, such as quantum computing,[1,2] quantum key distribution,[3] teleportation[4,5] and so on.[6−22] Bose proposed a scheme using an unmodulated spin chain as a channel through which the desired state was input and later output over a reasonable distance.[8] Portes *et al* studied the time evolution of statistical distribution and the transfer of the distribution in coupled systems.[9] Yang and Chu established the quantum state transfer with two 3-level atoms in cavity QED systems.[10] Subrahmanyam investigated the transmission across a channel of qubits in the spin chain using the Heisenberg XY dynamics.[11]

Recently, Christandl *et al* proposed a novel scheme for the perfect state transfer(PST) of an arbitrary quantum state.[15] Using XY-couplings, one can transfer a quantum state from the input qubit A to the output qubit B, by choosing a fixed period of time. The fidelity of this protocol is unity theoretically. One-link one-dimensional hypercube with no more than three spin-$\frac{1}{2}$ qubits is established for the perfect transmission. For a general multipartite system, the PST can be implemented in the one-link d-dimensional hypercube with spin-$\frac{1}{2}$ particles, where the vertices in such a graph are arranged in columns. The edges are only between the vertices in adjacent columns, i.e. there is no interaction along vertices in the same column. In such an N-column graph G, the vertices in the ith column, denoted by G_i, are marked by G_{ij}, where $j = \{1, \cdots, b_i\}$, and $b_i = C_{N-1}^{i-1}$. G can be expressed as an N-qubit one-dimensional chain with spin-m particles, where $m > \frac{1}{2}$, and is determined by N. Zhang *et al* implemented the PST experimentally by using a nuclear magnetic resonance (NMR) quantum computer.[16]

The previous work only discussed the ideal cases.[15,17] In experiments, however, errors are unavoidable, and the disturbance caused by such errors will surely reduce the fidelity.[21] For the PST, one of the most essential ideas is to engineer the coupling constants in the modulated XY-chain. The imperfection of the engineering coupling constants is the main error to reduce the fidelity of the state transfer in experiments. In this paper, we focus on the influence of the errors caused by such imperfection. We take the 4-qubit chain and 5-qubit chain as two examples to show the fidelity changing with the disturbance.

2. Errors in PST systems

The Hamiltonian used for the PST in the modu-

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lated chain for a multipartite system is described as

$$H_G = \sum_{(n,n+1)\in E(G)} \frac{J_n}{2}[\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y] \quad (1)$$

by setting $\hbar = 1$, where σ_n^x and σ_n^y are the Pauli matrices. $E(G)$ is the set of edges. When H_G is restricted in the subspace S_G, it is equivalent to the expression of the Hamiltonian of a fictitious spin $S = \frac{1}{2}(N-1)$ particle, i.e. $H = \lambda S_x$, where S_x is the x- component of S, and λ is a constant. J_n in Eq.(1) is represented as $J_n = \frac{\lambda}{2}\sqrt{n(N-n)}$, and J has the form

$$\begin{pmatrix} 0 & J_1 & 0 & \cdots & 0 \\ J_1 & 0 & J_2 & \cdots & 0 \\ 0 & J_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & J_{N-1} \\ 0 & 0 & 0 & J_{N-1} & 0 \end{pmatrix}. \quad (2)$$

The evolution caused by H_G can be expressed as

$$U(t) = \exp(-\mathrm{i}\lambda t S_x). \quad (3)$$

The fidelity for state transfer is defined as

$$F(t) = \langle N|U(t)|1\rangle, \quad (4)$$

where $|1\rangle \equiv |1_A 0\cdots 00_B\rangle$ and $|N\rangle \equiv |0_A 0\cdots 01_B\rangle$. The initial state for the PST is $|\psi_I\rangle = \alpha|0_A 0\cdots 00_B\rangle + \beta|1_A 0\cdots 00_B\rangle = \alpha|\underline{0}\rangle + \beta|1\rangle$, by setting the qubit A in state $\alpha|0\rangle + \beta|1\rangle$ and the other qubits in $|0\rangle$, where α and β are both complex numbers and $|\alpha|^2 + |\beta|^2 = 1$. The final state is $|\psi_F\rangle = \alpha|\underline{0}\rangle + \sum_{n=1}^{N}\beta_n(t)|n\rangle$, where $|n\rangle$ means that the nth qubit is in $|1\rangle$ and the other qubits are all in $|0\rangle$. According to the work of Christandl *et al*,[15] when $t = \pi/\lambda$, $|F(t)| = 1$, so that the state transfer is perfect. The state $\alpha|0\rangle + \beta|1\rangle$ can be obtained from the output point B just through an additional operation. $F(t)$ is a periodic function. When $t = 2\pi/\lambda$, the output state returns to the initial state $|\psi_I\rangle$; when $t = 3\pi/\lambda$, the PST occurs once again. Such phenomena have been observed in the NMR system.[16] The details of the PST can be found in Ref.[15].

We introduce the error terms into the Hamiltonian to discuss the effect of disturbance caused by such errors. The Hamiltonian with errors is

$$H_G(i) = \sum_{(n,n+1)\in E(G), n\neq i} \frac{J_n}{2}[\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y] + \frac{J_i(1+\delta_i)}{2}[\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y], \quad (5)$$

where the error term $\delta_i \geq -1$ for $1 \leq i \leq n$. For convenience, we only consider the disturbance on the ith edge. $H_G(i)$ is symmetric for $\delta_i \geq -1$ and $\delta_i \leq -1$ cases. Obviously, δ_i also expresses the disturbance occurring on the ith edge in the graph of XY-chain. When $\delta_i = 0$, the state transfer is perfect on the ith edge. We study the disturbance on each edge to show the corresponding influence on the probability amplitude for state transfer.

3. Two examples

We take four- and five-qubit chains as two examples for analysis. When the disturbance exists in a four-qubit XY-chain, the fidelity is not equal to 1. The transform caused by $U(\pi/\lambda)$ can be expressed as

$$\begin{aligned} |\psi\rangle =& \alpha|00\cdots 00\rangle + \beta|10\cdots 00\rangle \longrightarrow, \\ |\psi'\rangle =& \alpha|00\cdots 00\rangle + \beta'|00\cdots 01\rangle + \text{other terms}. \end{aligned} \quad (6)$$

The first term $\alpha|00\cdots 00\rangle$ on the right-hand side of the above is unchanged during the transfer. However, for the second term, when $t = \pi/\lambda$, $|\beta| \neq |\beta'|$, which means that the state transfer is not perfect. The other terms in $|\psi'\rangle$ are caused by the disturbance in $H_G(i)$. Through Eq.(6), we define the fidelity for the spin chain with disturbance as

$$P(t) = \frac{|\beta^*\beta'|}{|\beta|^2}, \quad (7)$$

where β^* is the complex conjugate of β. Obviously, when $\delta_i = 0$, then $P(t) = 1$ and the state transfer is perfect. The changes of $P(t)$ vs. δ_1, δ_2 and δ_3 are shown in Figs.1–3, respectively.

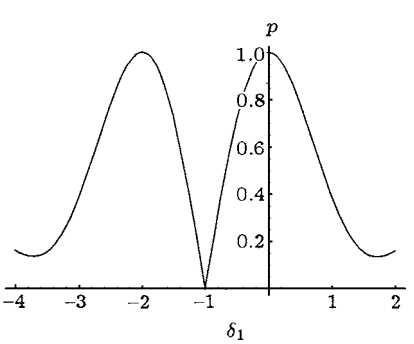


Fig.1. Curve of P changing with δ_1 for a 4-qubit state transfer.

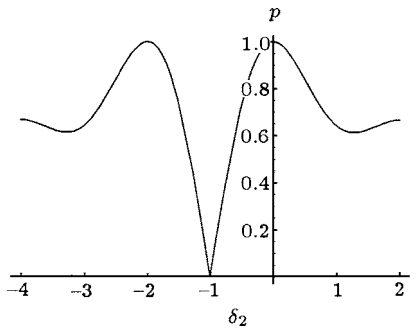


Fig.2. Curve of P changing with δ_2 for a 4-qubit state transfer.

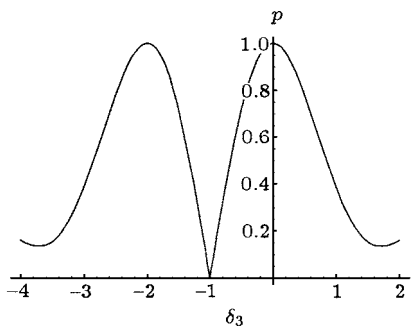


Fig.3. Curve of P changing with δ_3 for a 4-qubit state transfer.

Through comparing Fig.1 with Fig.3, one finds that the influences caused by δ_1 and δ_3 are equivalent, because the graph expressing the 4-qubit state transfer is symmetric. The influence caused by δ_2 is smaller than that caused by δ_1 or δ_3. When $\delta_i = -1$, $P = 0$, which means the state transfer fails completely because the XY-coupling is damaged on the ith edge. When δ_i is nearly 0 or -2 (the symmetric point of 0), P deviates from the fidelity of PST very slightly. Hence it is not difficult to accomplish the state transfer because slight disturbance does not introduce large deviation from $P = 1$. The attenuation of P is slower in Fig.2 than in Figs.1 and 3. The case of δ_i near -1 should be avoided in experiments, because the PST is destroyed greatly in such a case.

For the sake of contrast, we also discuss the 5-qubit chain with disturbance. Because the graph of the 5-qubit chain is symmetric, we only give the change of P versus δ_1 or δ_2. The results are presented in Fig.4 and Fig.5, respectively. Through comparing Fig.4 with Fig.5, one can find that the influence of disturbance near the input and output points is greater than that in the middle of the chain.

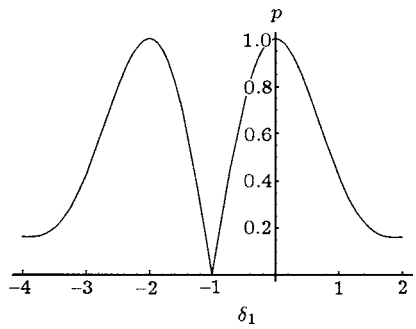


Fig.4. Curve of P changing with δ_1 for a 5-qubit state transfer.

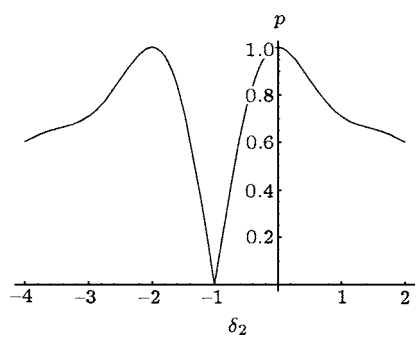


Fig.5. Curve of P changing with δ_2 for a 5-qubit state transfer.

4. Conclusion

We have shown that the probability amplitude for the transfer of quantum states fluctuates with the increase of disturbance on different edges of the graph of the XY-chain by discussing the 4-qubit and 5-qubit cases as examples. The results show that the influence of disturbance near the input and output spins is greater than that in the middle of the chain. The XY-coupling for PST is relatively stable because slight disturbance does not reduce the fidelity too much. The results can be generalized to a more than five spins chain, and are helpful for the implementation of the perfect state transfer in experiments.

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