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Flexural vibration band gaps in thin plates with two-dimensional binary locally resonant structures*

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The complete flexural vibration band gaps are studied in the thin plates with two-dimensional binary locally resonant structures, i.e. the composite plate consisting of soft rubber cylindrical inclusions periodically placed in a host material. Numerical simulations show that the low-frequency gaps of flexural wave exist in the thin plates. The width of the first gap decreases monotonically as the matrix density increases. The frequency response of the finite periodic thin plates is simulated by the finite element method, which provides attenuations of over 20dB in the frequency range of the band gaps. The findings will be significant in the application of phononic crystals.

Keywords: phononic crystals, flexural vibration, band gaps, locally resonant
PACC: 0340K, 4600, 0260, 8160H

1. Introduction

In the last decade, the propagation of elastic or acoustic waves in periodic composite materials, called phononic crystals, has received much attention.[1−18] This is of interest for applications such as frequency filters, vibrationless environments for high-precision mechanical systems or design of new transducers and so on.

However, for the conventional phononic crystals band gap with the Bragg reflection mechanism, the spatial modulation must be of the same order as the wavelength in the gap. This would bring on structures with the size of outdoor sculptures in order to prevent environmental noises or a bigger size for the vibration isolation of common machines.[6]

The pioneering work of Liu *et al*[6] has opened a new field of phononic crystals. The authors of Ref.[6] studied three-dimensional (3D) phononic crystals consisting of cubic arrays of coated lead spheres (the coating is a thin film of a soft material) immersed in an epoxy matrix. They predicted the appearance of a gap in a frequency range of two orders of magnitude lower than that by the Bragg scattering. The two-dimensional (2D) counterpart, i.e. lattices of coated cylinders in epoxy, have also been studied.[7−9] However, it is not convenient for the structures to shield environmental noises and isolate the vibration because they are all too heavy with the high-density solid core.

The locally resonant phononic crystals with binary structures composed of periodic soft rubber cylinders immersed in epoxy host were recently researched.[10] The authors of Ref.[10] performed a comprehensive study on the 2D binary locally resonant phononic crystals. In the 2D system, only the in-plane mode in the phononic crystals can be excited when the incident waves have in-plane polarization. But the gap of the in-plane mode in the structure[10] is too narrow to be applied.

Vibration band gaps in phononic crystals have also been found.[11,12] The flexural waves propagating in composite plates consisting of cylindrical inclusions periodically placed in a host material had been studied earlier.[13,14] The flexural wave velocity is proportional

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to the in-plane wave vector and is not a constant as in an infinite medium. The difference makes the appearance of a full gap more difficult.[13]

In this paper, we study the flexural vibration band gaps in the thin plate with 2D binary locally resonant structures, i.e. the composite plates consisting of soft rubber cylinder, of square cross section, inclusions periodically placed in a host material.

2. Plane wave expansion method

Figure 1 shows a cell of the periodic binary phononic crystal thin plate. The system consists of an infinite repetition of cylinder, of square cross section, inclusions periodically placed in matrix. The thickness of the thin plate is h. The lattice constant is a, $2l$ is the width of the square cross section of the cylinder, and $F = 4l^2/a^2$ is the filling fraction.

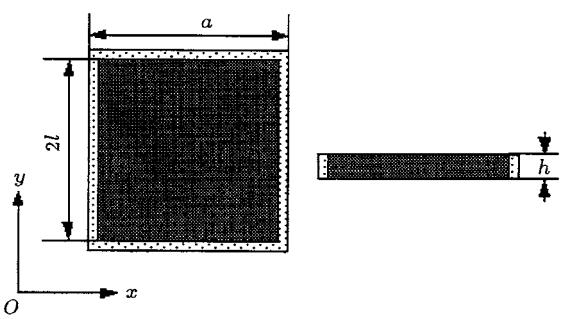


Fig.1. The sketch map of one cell of the binary periodic thin plate.

The wave equation of flexural wave for thin plates of thickness h is known to be[13,15]

$$\begin{aligned}-\rho h\frac{\partial^2 w}{\partial t^2} =&\frac{\partial^2}{\partial x^2}\left[D\left(\frac{\partial^2 w}{\partial x^2}+\sigma\frac{\partial^2 w}{\partial y^2}\right)\right]\\&+2\frac{\partial^2}{\partial x\partial y}\left[D\left(1-\sigma\right)\frac{\partial^2 w}{\partial x\partial y}\right]\\&+\frac{\partial^2}{\partial y^2}\left[D\left(\frac{\partial^2 w}{\partial y^2}+\sigma\frac{\partial^2 w}{\partial x^2}\right)\right],\end{aligned}\tag{1}$$

where w is the flexural displacement in the z direction, $D = Eh^3/12(1-\sigma^2)$ is the flexural rigidity and E is Young's modulus, σ is Poisson ratio.

Equation (1) can be rewritten as

$$\begin{aligned}-\alpha\frac{\partial^2 w}{\partial t^2} =&\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}+\beta\frac{\partial^2 w}{\partial y^2}\right)\\&+2\frac{\partial^2}{\partial x\partial y}\left(\gamma\frac{\partial^2 w}{\partial x\partial y}\right)\\&+\frac{\partial^2}{\partial y^2}\left(D\frac{\partial^2 w}{\partial y^2}+\beta\frac{\partial^2 w}{\partial x^2}\right),\end{aligned}\tag{2}$$

where $\alpha = \rho h$, $\beta = D\sigma$, $\gamma = D(1-\sigma)$, they are periodic function of $\boldsymbol{r} = (x, y)$.

Following the plane wave expansion methodology,[16] Eq.(2) can be derived in the form of a standard eigenvalue problem at hand:

$$\begin{aligned}&\omega^2\sum_{G'}\alpha(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'})\\=&\sum_{G'}(\boldsymbol{k}+\boldsymbol{G'})_x^2(\boldsymbol{k}+\boldsymbol{G})_x^2 D(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'})\\&+\sum_{G'}(\boldsymbol{k}+\boldsymbol{G'})_y^2(\boldsymbol{k}+\boldsymbol{G})_x^2\beta(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'})\\&+2\sum_{G'}(\boldsymbol{k}+\boldsymbol{G'})_x(\boldsymbol{k}+\boldsymbol{G'})_y\\&\times(\boldsymbol{k}+\boldsymbol{G})_x(\boldsymbol{k}+\boldsymbol{G})_y\gamma(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'})\\&+\sum_{G'}(\boldsymbol{k}+\boldsymbol{G'})_y^2(\boldsymbol{k}+\boldsymbol{G})_y^2 D(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'})\\&+\sum_{G'}(\boldsymbol{k}+\boldsymbol{G'})_x^2(\boldsymbol{k}+\boldsymbol{G})_y^2\beta(\boldsymbol{G}-\boldsymbol{G'})w_k(\boldsymbol{G'}),\end{aligned}\tag{3}$$

where $\boldsymbol{k}$ is the Bloch wave vector restricted within the first Brillouin zone, ω is the circular frequency of the wave, $\boldsymbol{G}$ and $\boldsymbol{G'}$ are reciprocal vectors.

If the infinite series is approximated by a sum of N reciprocal vectors, Eq.(3) is reduced to $N \times N$ matrix eigenvalue equations.

Equation (3) is based on the conventional plane wave expansion method, whose most serious problem is the slow convergence rate, especially for systems of either very high or very low filling ratios, or of large elastic mismatch.[17] The result obtained indicates that the slow convergence of conventional plane wave expansion method is due to the inappropriate formulation of the eigenproblem used in the calculation,[17] and then the plane wave expansion method has to be improved to provide good convergence.[17]

Now we describe simply the calculation detail. Assume $f(x)$ and $g(x)$ are two piecewise continuous, piecewise smooth, bounded, periodic functions, and $h(x)$ is a product of these two functions,

$$h(x) = f(x)g(x). \tag{4}$$

In the conventional plane wave expansion method, the Fourier coefficients of $h(x)$ can be obtained from the Fourier coefficients of $f(x)$ and $g(x)$ by Laurent's rule

$$h_n = \sum_{m=-\infty}^{+\infty} f_{n-m}g_m. \tag{5}$$

For a practical problem, the truncation of infinite series is inevitable. Equation (5) can be written as

$$h_n = \sum_{m=-M}^{M} f_{n-m}g_m. \tag{6}$$

However, for the new plane wave expansion method, instead of Laurent's rule, the inverse rule

$$h_n = \sum_{m=-M}^{M} \left[\frac{1}{f}\right]^{-1}_{n-m} g_m, \qquad (7)$$

should be used to obtain the Fourier coefficients of function $h(x)$, where h_n and g_m are the Fourier coefficients of corresponding functions, and $[f]$ denotes the Toplitz matrix of function f with the (n,m)th element being f_{n-m}, and -1 denotes the matrix inverse.

Thus the only difference between the new and the old formulations is the manner in which the elastic parameter functions are used in Eq.(3):[17] the new formulation uses $\left[\frac{1}{P}\right]^{-1}$ instead of $[P]$, where P can be one of α, D, β, γ. The new formulation can provide much more accurate numerical results than the conventional plane wave expansion method for the systems of either very high or very low filling ratios, or of large elastic mismatch.

3. Calculated results by plane wave expansion method

Here, for the binary locally resonant phononic crystal thin plate, we consider a square lattice of soft silicon rubber cylinder, with square cross section, in steel plate. Figure (2) shows the band structure corresponding to the 2D binary locally resonant phononic crystal thin plate with lattice constant a=20mm, filling fraction F=0.81, thickness h=2mm. The material properties are given in Table 1. In this paper, we use 2601 reciprocal vectors. The results show very good convergence.

Table 1. Physical parameters of the materials.

Material	Density ρ/kg·m^{-3}	Young's modulus E/Pa	Poisson ratio σ
Au	19500	8.5×10^{10}	0.42
Pb	11600	4.08×10^{10}	0.37
Cu	8960	12.71×10^{10}	0.35
Steel	7780	21.06×10^{10}	0.30
Al_2O_3	3986	40.27×10^{10}	0.23
Al	2730	7.76×10^{10}	0.35
Rubber	1300	7.6×10^{5}	0.47
Epoxy	1180	4.35×10^{9}	0.37

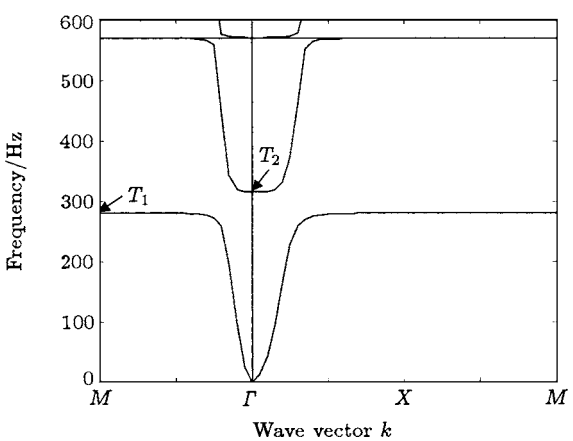


Fig.2. The band structure of binary locally resonant thin plate composed of square lattice of soft rubber cylinders, with square cross section, in steel matrix. The lattice constant a=20mm, filling fraction F=0.81, thickness h=2mm.

We can find one complete band gap within 600Hz. The gap extends from the frequency of 279.53 up to 314.29Hz. Since the flexural wave velocity is proportional to the in-plane wave vector and not a constant,[13] the flexural wave velocity is smaller at low frequencies. So the frequency of the gap in Fig.2 is just one order of magnitude lower than the one resulting from Bragg scattering. The band gap width is Δf =34.76Hz. And the normalized gap width is $\Delta f/f_{\rm g}$ =0.12, where $f_{\rm g}$ is the midgap frequency.

A remarkable feature in Fig.2 is the flat branches crossing the whole Brillouin zone. These flat branches are real and converged, corresponding to the resonant modes illustrated in Fig.3. It is similar to the band structure of z mode of 2D binary locally resonant phononic crystals.[10] At points T_1 and T_2, the amplitude of the vibrations is well localized in the region of rubber cylinders and it is very small in the host media, as illustrated in Figs.3(a) and (b), respectively. The rubber cylinder vibrates like a mass-spring oscillator. The time harmonic forces from the oscillators to the host structure split the original dispersion curves, and the band gap is generated. The rule used to judge whether a resonant mode in the phononic crystals can result in a corresponding subfrequency gap[10] is still available to the locally resonant phononic crystal thin plate.

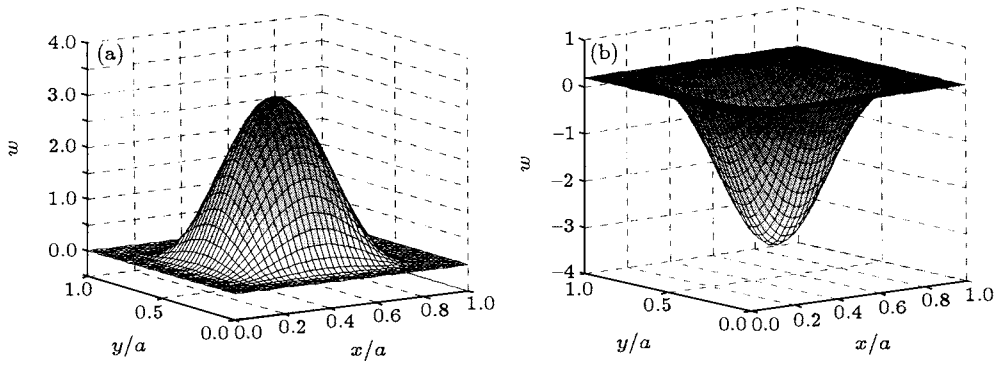


Fig.3. The flexural displacement in a lattice of the rubber/steel phononic crystal thin plate. The selected modes are corresponding to points (a) T_1, (b) T_2 in Fig.2, respectively.

As the next step, we consider the influence of material properties on the band gap width. Keeping the scatterer material to be rubber, we only replace the matrix with different materials. In the calculation, the geometric parameters are the lattice constant a=20mm, filling fraction F=0.81, thickness h=2mm. The material parameters are given in Table 1. In Fig.4(a), we plot the size of the first gap as a function of the density of the matrix material, with the upper (lower) line marking the upper (lower) edge of the first gap. We note that the lower edge of the gap is pinned by the highly localized vibration mode of the rubber. It has virtually no dependence on matrix material. But the upper edge of the gap becomes lower while the density of the matrix material increase. In Fig.4(b), we plot the normalized gap width of the first gap as matrix density. The normalized gap width of the first gap decreases monotonically as the matrix density increases.

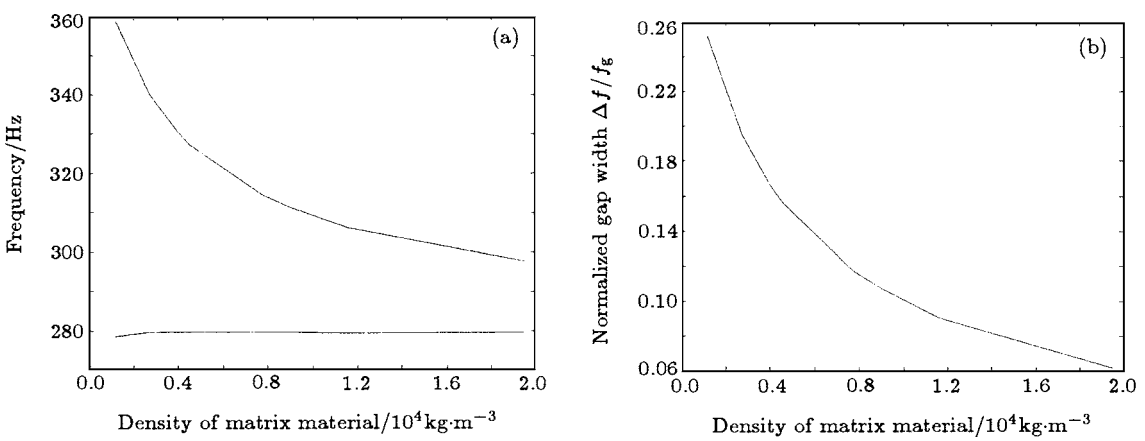


Fig.4. (a) The first gap frequency as a function of the density of the matrix material. (b) The dependence of the first band gap normalized width on the density of the matrix material.

4. Frequency response simulated by the finite element method

The finite element method has been used to calculate the transmission of the phononic crystals.[18] And the frequency response function of vibration has been used to describe the gaps effectively.[12] We calculate the frequency response function of the finite periodic binary locally resonant thin plate with the same material and geometric parameters as those in Fig.2 with the finite element method. The plate in the calculation is made of 8×8 periodic cells of rubber-steel in xOy plane.

The incident wave used in the calculation is the wideband acceleration white noise from 0 to 600Hz, whose spectrum is illustrated as the dash-dot line in Fig.5. The incident wave is applied to the left edge of the plate in z direction. The solid line in Fig.5 illustrates the frequency response of the right edge of the plate. There is one sharp drop in the frequency response curve with the frequency ranges from 274.5

to 309.0Hz. The band gap has a response drop-off over 20dB compared with the response outside the band gaps. The result calculated with the finite element method is in good agreement with the band structure. It also shows the improved plane wave expansion method has good convergence.

For comparing the effect of different locally resonant structures, we calculate the frequency response of the matrix plate, i.e. the steel framework without rubber inclusions, shown as the dashed line in Fig.5. There is not any attenuation in the frequency range of the gap.

From Fig.5, it is shown that the transmittance spectra of flexural waves across the thin plate structures made of rubber/steel present resonance features having a very asymmetric profile, which are explained as a Fano-like interference phenomenon.[7]

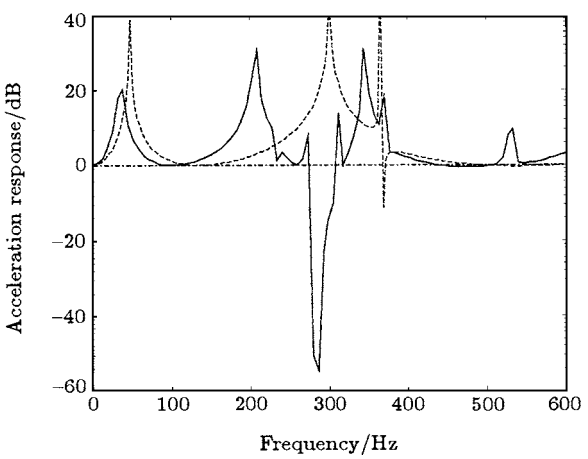


Fig.5. The response curves showing the acceleration response of the rubber/steel thin plate (solid line), the response curve of the matrix framework (dashed line) and the spectrum of incident wave (dash-dot line).

Moreover, in Fig.6 we show the calculated frequency response of the locally resonant thin plate composed of rubber/epoxy (dashed line), rubber/steel (solid line) and rubber/Pb (dash-dot line). The beginning frequency of the first gap for the three kinds of plate is almost the same. The width of the first gap decreases monotonically as the host density increases.

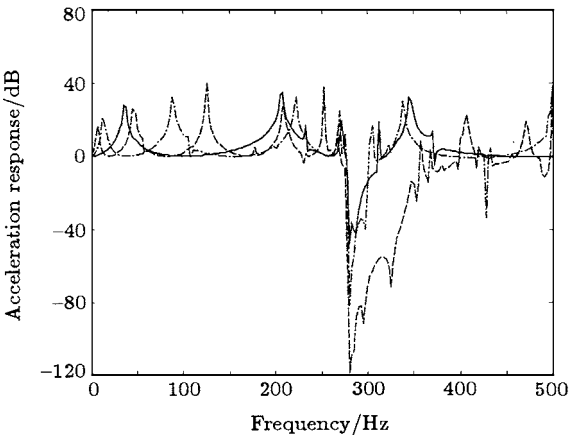


Fig.6. Frequency response of the thin plate composed of rubber/epoxy (dashed line), rubber/steel (solid line) and rubber/Pb (dash-dot line).

5. Conclusion

In conclusion, the flexural vibration band gap in 2D binary locally resonant phononic crystal thin plate is studied theoretically in this paper.

The band structure of the system is calculated with improved plane wave expansion method, which has a good convergence. Complete band gaps exist in the binary locally resonant phononic crystal thin plate. The vibration modes are calculated by plane wave expansion method to judge whether a resonant mode in the phononic crystals can result in a corresponding subfrequency gap.

Further keeping the embedded cylinders to be of rubber, we consider the effect of matrix material properties on the band gap width. The contrast of the matrix density and scatter density plays a fundamental role for the variation of the band gap width. The width of the first gap decreases monotonically as the host density increases.

Finally, the frequency response of the finite periodic binary locally resonant phononic crystal thin plate is simulated by using the finite element method, which provides an attenuation of over 20dB in the frequency range of the band gap. The existence of flexural vibration gaps in locally resonant phononic crystal thin plates gives a new idea in vibration control.

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