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A hybrid quantum encoding algorithm of vector quantization for image compression*

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Many classical encoding algorithms of vector quantization (VQ) of image compression that can obtain global optimal solution have computational complexity $O(N)$. A pure quantum VQ encoding algorithm with probability of success near 100% has been proposed, that performs operations $45\sqrt{N}$ times approximately. In this paper, a hybrid quantum VQ encoding algorithm between the classical method and the quantum algorithm is presented. The number of its operations is less than $\sqrt{N}$ for most images, and it is more efficient than the pure quantum algorithm.

Keywords: vector quantization, Grover's algorithm, image compression, quantum algorithm

PACC: 9265S, 9575M, 4230, 2980C

1. Introduction

The two open research problem of vector quantization (VQ) are to design a real-time encoding algorithm and to design the codebook generation algorithm that can generate an approximately local optimal solution (finding the global optimal solution is a non-deterministic polynomial complete (NPC) problem).[1] The property of local clustering is revealed by Pang[1] and it is used to design a fast codebook generation algorithm, that is faster than the renowned Linde, Buzo, and Gray (LBG) algorithm by a factor of 4–13.[2] But the other open problem, i.e. fast encoding of VQ with global optimal solution, has not been solved. The encoding is described as:[1]

Let codebook $C = \{c[i]|0 \leq i \leq N, c[i] \in R^k, k \geq 1\}$. For an arbitrary input vector $x \in R^k$, we have to find the index i_0 such that distance $d(x, c[i_0])$ is minimum, where R^k is K-dimensional Euclidean space. Vector $c[i]$ is called the codevector.

Grover's algorithm[3] is a renowned quantum search algorithm, and its accuracy of rotation is improved recently.[4] Long proposes a modified Grover algorithm.[5] Long's algorithm has 100% probability of success even for the case that the size of database N is very small. This property of Long's algorithm is suitable for quantum image compression because N is not a giant number in image compression in general. Some hybrid search algorithms have also been proposed to improve efficiency of algorithms,[6,7] in which more physical resources are used to commute the efficiency.

Boyer, Brassard, Høyer and Tap propose an iteration algorithm, named BBHT algorithm in this paper, in which the Grover iteration is applied repeatedly.[8] The BBHT algorithm (Boyer, Brassard, Høyer, Tap)[8] is designed specially to solve the searching case that the number of solutions is unknown.

Quantum image compression is possible.[9−11] Pang *et al* present a quantum discrete cosine transformation (QDCT) for image compression.[9] The method of QDCT can be applied to other transformation such as Fourier transformation and classical output will be brought. Latorre presents a nice and maybe hopeful quantum image compression method. Latorre's algorithm requires that the quantization method (scalar quantization or VQ) is incorporated with it. Otherwise, it is not competitive with classical methods.[10] Pang *et al* present a pure quantum VQ encoding algorithm with probability of success near 100%, that performs operations $45\sqrt{N}$ times

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approximately.[11]

This paper presents a hybrid quantum VQ algorithm between the classical method and the quantum algorithm to solve the open problem of fast encoding of VQ. The number of its operations is less than $\sqrt{N}$ and its probability of success is 100% approximately.

Notation x: input vector

Notation $\delta_0 = \min\{d(c[i], c[j]) \quad | \quad i \neq j, \quad 0 \leq i, j < N\}$;

Notation I: whole encoding space

Notation $S = \{x \quad | \quad d(x, c[i_0]) < \frac{\delta_0}{2}\}$ and $T = \{x \quad | \quad d(x, c[i_0]) < \hat{\delta}\}$, where $\hat{\delta} \geq \frac{\delta_0}{2}$;

Notation $\Omega(c[i]) = \{c[j] \quad | \quad d(c[i], c[j]) < 2\hat{\delta}, c[j] \in C\}$;

Notation $\inf_\Omega = \min\{|\Omega[c[i]] \quad || \quad 0 \leq i < N\}$.

All of the notations are diagrammatized in Fig.2.

Parameter δ_0 can be calculated before encoding. The classical data structure of $\Omega(c[i])$ can be constructed by using classical method before encoding (see Fig.3).

2. Quantum VQ inequality iteration and quantum VQ encoding

Pang *et al* present a quantum representation of image to which database technique is applied,[9] and this quantum method is also used to represent data of image in this paper.

Definition Oracle $O_{\rm d}$:

$$\delta|x\rangle|i\rangle|c[i]\rangle|0\rangle \xrightarrow{O_{\rm d}} |\delta\rangle|x\rangle|i\rangle|c[i]\rangle|d(x, c[i])\rangle,$$

$O_{\rm d}$ is used to compute the distance $d(x, c[i])$ and $(O_{\rm d})^{-1}$ is the inverse transformation of it.

Definition Oracle $O_{\rm f}$:

$$|\delta\rangle|x\rangle|i\rangle|c[i]\rangle|d(x, c[i])\rangle \xrightarrow{O_{\rm f}} (-1)^{f(i)}|\delta\rangle|x\rangle|i\rangle|c[i]\rangle|d(x, c[i])\rangle,$$

where function $f(i) = 1$ if $d(x, c[i]) < \delta$, otherwise $f(i) = 0$.

Definition quantum VQ inequality iteration $G_{\rm inequality}$:

$$G_{\rm inequality} = (2|\xi\rangle\langle\xi| - I)(U_{\rm L})^{-1}(O_{\rm d})^{-1} \times O_{\rm f} O_{\rm d} U_{\rm L},$$

where $|\xi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$ and the unitary operation $U_{\rm L}$ and $(U_{\rm L})^{-1}$ is LOAD operation that loads data into registers from memory.[9]

The operation $U_{\rm L}$ is defined as $|\delta\rangle|x\rangle|i\rangle|0\rangle|0\rangle \xrightarrow{U_{\rm L}} |\delta\rangle|x\rangle|i\rangle|c[i]\rangle|0\rangle$ in this paper.

2.1. Sub-procedure 1: the quantum VQ for $x \in S$

Algorithm: Sub-procedure 1

Step1 Let $\delta = \frac{\delta_0}{2}$.

Step2 Generate the initial state $|\Psi_0\rangle \geq \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} (|\delta\rangle|x\rangle|i\rangle|0\rangle|0\rangle)$.

Step3 Perform operation $t = \left[\frac{\pi}{4}\sqrt{N}\right]$ times: $|\Psi_t\rangle \geq (G_{\rm inequality})^t|\psi_0\rangle$.

Step4 Observe the third register: let i_0 be the outcome. Compute $d_0 = d(x, c[i_0])$ classically. If $d_0 < \delta$, the result is i_0, otherwise go to Sub-procedure 2.

If there exists a vector $c[i_0] \in C$ and $d(x, c[i_0]) < \frac{\delta_0}{2}$, $c[i_0]$ is the closest codevector of $x \in X$ and it is the unique closest codevector.[1] And the iteration $G_{\rm inequality}$ acts only on N – dimensional subspace.[9,11] Therefore, by Grover's algorithm, sub-procedure 1 performs the operation $G_{\rm inequality}$ $\left[\frac{\pi}{4}\sqrt{N}\right]$ times approximately. That is, sub-procedure 1 has time complexity $\left[\frac{\pi}{4}\sqrt{N}\right]$.

2.2. Sub-procedure 2: the quantum VQ for $x \in (T - S)$ or $x \in (I - T)$

Algorithm: Sub-procedure 2

Step1 Initialize $m = 1$ and $\lambda = \frac{6}{5}$.

Step2 We choose an experiential value $\hat{\delta}$ such that the set T can cover the whole encoding space I approximately.

Step3 Choose j uniformly at random among the nonnegative integers not bigger than m.

Step4 Generate the initial state $|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} (|\delta\rangle|x\rangle|i\rangle|0\rangle|0\rangle)$.

Step5 Apply j iterations of the iteration $G_{\rm inequality}$ starting from the state

$$|\Psi_0\rangle : \ \Psi_j \geq (G_{\text{inequality}})^j |\psi_0\rangle.$$

Step6 Observe the third register: let h be the outcome.

Step7 Perform classical computing $y_0 = d(x, c[h])$. If $y_0 < \hat{\delta}$, classically fully search the corresponding neighbourhood $\Omega(c[h])$ to obtain the closest codevector $c[i_0]$; The result is i_0 and exit.

Step8 Otherwise, set m to $\min\{\lambda m, \sqrt{N}\}$ and go to step 3.

Step9 Encoding for $I - T$: If $y_0 \geq \hat{\delta}$, or i_0 does not exist, apply classical method to encode.

We have conclusion that the closest codevector $c[i_0]$ of vector $x \in T - S$ is included in the neighbourhood $\Omega(c[h])$ of step 7 (see Fig.1). The reason is that $d(x, c[j]) \geq d(c[j], c[h]) - d(c[h], x) > \hat{\delta}$ for an arbitrary $c[j] \notin \Omega(c[h])$.

The hybrid idea of sub-procedure 2 is that applying the iteration $G_{\text{inequality}}$ repeatedly to find a neighbourhood $\Omega(c[h])$ which comprises very few elements, then fully searching the neighbourhood classically to find the solution.

In addition, the set $I - T$ comprises very few vectors statistically and classical method is applied to encode vector $x \in I - T$.

Figure 1 illustrates the hybrid idea of sub-procedure 2.

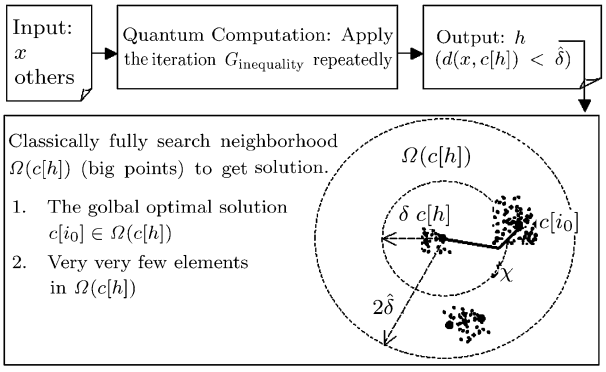


Fig.1. The illustration of hybrid idea of sub-procedure 2: 1. Embed the iteration $G_{\text{inequality}}$ in BBHT algorithm and apply the iteration repeatedly. This quantum method will find a neighbourhood $\Omega(c[h])$. 2. The global optimal solution $c[i_0] \in \Omega(c[h])$, and there are very few elements in $\Omega(c[h])$. Therefore, we can classically search $\Omega(c[h])$ to obtain solution.

The iteration $G_{\text{inequality}}$ is embedded in the BBHT algorithm in sub-procedure 2. The case of no solution is handled by BBHT algorithm.[8] The iteration $G_{\text{inequality}}$ acts only on $N-$ dimensional subspace.[9,11] Thus, the number of performing the iteration $G_{\text{inequality}}$ in sub-procedure 2 is less than $\frac{9}{4}\sqrt{\frac{N}{\inf_\Omega}}$ when $\inf_\Omega \ll N$.

The four statistical properties of image for VQ are listed below (see Fig.2):

1. Almost feature vectors x concentrate on themselves centroids generally. That is, almost vectors $x \in I$ are included in set S. The solution is unique for $x \in S$.

2. The set $T - S$ comprises the points of near edges of image in practice. The solution is included in a small neighbourhood $\Omega(c[h])$ for input vector $x \in T - S$.

3. The set $I - T$ comprises special points such as very detail points or points at edges maybe.

4. Let $|S| : |T - S| : |I - T| = a : b : c$. Statistically, we have (see Fig.2) $80\% \lesssim a \lesssim 99\%$, $1\% \lesssim b \lesssim 19\%$, $c \lesssim 1\%$.

Therefore, the time complexity of the whole quantum encoding algorithm is less than $\sqrt{N}$ statistically.

The phenomenon should be noticed, that the above properties are not powerful for classical methods to solve the open problem of the fast encoding of VQ, by contrast, it is powerful for quantum methods.

Figure 2 shows these statistical properties of image.

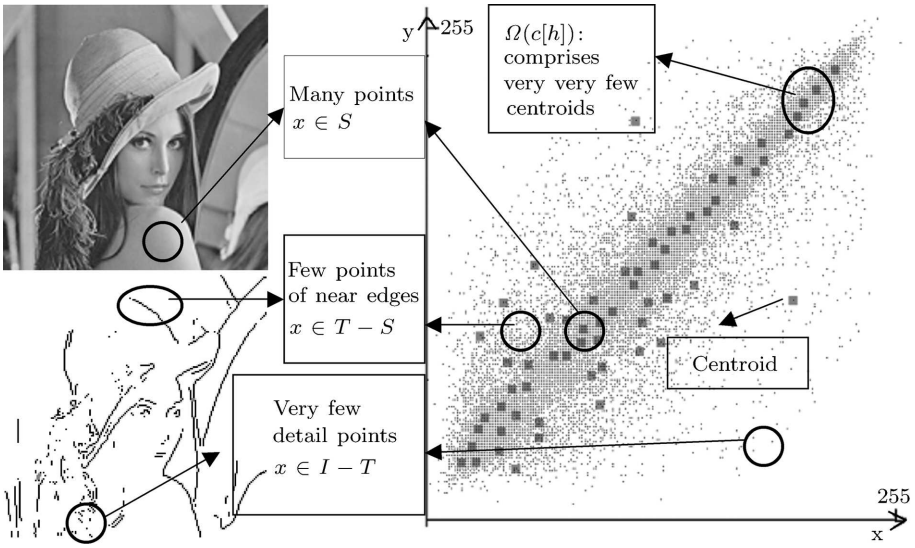


Fig.2. The illustration of the properties of image that is applied to design the hybrid algorithm: (Image Lena with size 256×256 is divided into 2×1 image blocks to form vectors.[1]) 1. Almost feature vectors x concentrate on themselves centroids generally. And the solution is unique for $x \in S$. 2. The set $T - S$ comprises the points of near edges of image and it includes few points in practice. And the solution is included in a small set $\Omega(c[h])$ for $x \in T - S$. 3. The set $I - T$ comprises special points such as very detail points or points at edges maybe. 4. Statistically, $|S| \gg |T - S| \gg |I - T|$. The above four statistical properties of image for VQ are applied to accelerate the quantum algorithm: 1. Sub-procedure 1 acts on the set S with time complexity $\left\lceil \frac{\pi}{4}\sqrt{N} \right\rceil$ approximately. 2. Sub-procedure 2 acts on the set $T - S$ with time complexity less than $\frac{9}{4}\sqrt{\frac{N}{\inf_{\Omega}}}$ approximately. 3. Classical full search algorithm acts on $I - T$. The phenomenon should be noticed, that these properties are not powerful for classical methods to solve the open problem of the fast encoding of VQ, by contrast, it is powerful for quantum methods.

The hybrid algorithm is not only an image compression method, but also it can be applied to image recognition.[12]

3. The cost physical resource for accessorial data structure

The main accessorial resource is cost to save the classical data structure of all neighbourhoods $\Omega(c[i])$. And the data structure can be constructed by using adjacency list technique[13] before encoding. Adjacency list technique is a basic technique, and it is applied on image compression usually.[1] The adjacency list to save all $\Omega(c[i])$ can be constructed by the following method:[13]

First, allocate a single list to save all neighbours of each codevector $(c[i])$, where $(c[i]) \in C$ (i.e., centroid in Fig.2) and $i = 0, 1, \ldots, (N-1)$.

Second, all of the addresses of all single lists are saved in an array.

The structure of the adjacency list is diagrammatized in Fig.3.

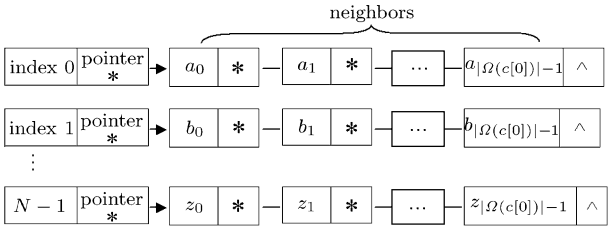


Fig.3. The illustration of the adjacency list to save all neighbourhoods $\Omega(c[i])$. Each single list saves all neighbours of $(c[i])$. The space complexity of the adjacency list is $O(N \log_2 N)$ and it is very good. It is very easy to realize the adjacency list with a modern electronic computer. Therefore, it is a better tradeoff that costs $O(N \log_2 N)$ classical bits to decrease the running time by a factor of 45 of the pure quantum algorithm in Ref.[11].

It is easy to estimate the space complexity of the adjacency list.[13] The total number of the classical bits required to save the adjacency list is approximately $\sum_{i=0}^{N-1} (|\Omega(c[i])| + 1)(\log_2 N + 4 \times 8)$ (bits).

In general, $|\Omega(c[i])|$ is very small (see Fig.2).

Therefore, the space complexity is $O(N \log_2 N)$ bits approximately and it is very good.

It is very easy to realize the adjacency list with a modern electronic computer.[13] Therefore, it is a better tradeoff that costs $O(N \log_2 N)$ classical bits

to permute the running time by a factor of 45 of the pure quantum algorithm presented in Ref.[11].

4. Conclusion

VQ has high compression ratio and simple structure. However, the performance comes at the cost of increased computational complexity. Fast encoding of VQ is an open problem and is one of the two key techniques of VQ. Many classical encoding algorithms of VQ that can obtain global optimal solution have computational complexity $O(N)$. In this paper, a hybrid quantum VQ encoding algorithm between classical method and quantum algorithm is presented. The number of its operations is less than $\sqrt{N}$ for most images. It is a good tradeoff that costs $O(N \log_2 N)$ classical bits to decrease the running time by a factor of 45 of the pure quantum algorithm in Ref.[11].

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