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Intrinsic Hall effect and separation of Rashba and Dresselhaus spin splittings in semiconductor quantum wells*

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We have proposed a method to separate Rashba and Dresselhaus spin splittings in semiconductor quantum wells by using the intrinsic Hall effect. It is shown that the interference between Rashba and Dresselhaus terms can deflect the electrons in opposite transverse directions with a change of sign in the macroscopic Hall current, thus providing an alternative way to determine the different contributions to the spin–orbit coupling.

Keywords: Hall effect, spin–orbit coupling, spintronics, spin transport

PACC: 7170E, 7210B, 7280E

1. Introduction

Spintronics (spin transport electronics) has become a very active and promising field, [1−4] which combines the basic quantum mechanics of coherent spin dynamics and technological applications in information processing and storage devices.[5−7] In particular, how to systematically manipulate spins in traditional semiconductors via electronic approach comes to be an important issue, which can be revealed by recent extensive studies on spin generation,[8,9] spin dephasing,[10] spin accumulation,[11] and spin Hall effect [12,13] in two-dimensional and bulk semiconductors. Among these spin-related phenomena, spin–orbit (SO) coupling mechanisms in semiconductors provide a basis for extensive device applications, and a source of interesting physics in a wide range. Therefore, how to measure the strength of spin–orbit interactions has been of much concern for a long time.

There exist two different kinds of SO interactions in semiconductor quantum well structures. One is the Rashba SO coupling,[14] whose strength is described by the Rashba coefficient α, and can be tuned by gate voltages. This coupling stems from the structure inversion asymmetry (SIA) of the confining potential of two-dimensional electron (or hole) systems. The other is the Dresselhaus SO term β,[15] which is due to the bulk inversion asymmetry (BIA) [16,17] and the interface inversion asymmetry (IIA).[18,19] Both Rashba and Dresselhaus SO interactions can result in spin splitting of the bands and give rise to a variety of spin-dependent phenomena that allow one to manipulate spins in a controllable way. However, it is difficult to separate the relative contributions of Rashba and Dresselhaus terms to the SO coupling. To obtain the Rashba coefficient α, the Dresselhaus contribution is usually neglected. At the same time, Dresselhaus and Rashba terms can compensate each other, resulting in such interesting effects as the vanishing of the spin splitting in certain k-space directions,[20] the absence of spin relaxation in specific crystallographic directions,[21] the lack of SdH beating,[22] and the buildup of a nonballistic spin-field-effect transistor.[23] Recently, it has been experimentally demonstrated[24] that angular dependent measurements of the spin-galvanic photocurrent [25] allow one to separate contributions due to Dresselhaus and Rashba terms.

In this paper we propose a method to extract the relative strengths of Rashba and Dresselhaus spin–orbit interactions by using the anomalous Hall effect (AHE). The AHE is characterized by an additional term in the expression of the transverse Hall resistivity. This term is proportional to the magnetization M of the sample. The total Hall resistivity becomes $\rho_{xy} = R_0 B + R_H M$, where B is the magnetic field,

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$R_0 = 1/ne$ is the ordinary Hall coefficient, and R_H is the anomalous Hall coefficient. Although the AHE has been known for half a century, its origin has been keenly debated for decades. They have emerged anew because of fresh theoretical insights and strong interest in spin currents for spin-based electronics. Recently, systematic works on metallic ferromagnets[26] and semiconductor ferromagnets[27] have attributed the AHE to the real-space or momentum-space Berry-phases. This intrinsic version of Luttinger's theory is receiving strong experimental support,[28] and therefore will be a starting point in this paper. Our results show that by tuning the ratio of the Rashba interaction to the Dresselhaus interaction, the Hall current may flow along different transverse directions, thus providing an alternative way to extract the relative strengths of Rashba and Dresselhaus terms in semiconductor quantum well structures.

2. Theoretical model: Rashba Hamiltonian

Consider the conduction electrons in semiconductor quantum wells. In this two-dimensional electron gas (2DEG) system, the Rashba SO interaction arises from the quantum well asymmetry in the growth (z) direction, and the Dresselhaus term is due to the BIA. Thus the total SO interaction is given by

$$V_{\rm so} = \alpha(\tau_x k_y - \tau_y k_x) + \beta(\tau_x k_x - \tau_y k_y), \qquad (1)$$

where τ_i ($i = x, y, z$) are the usual Pauli matrices. Taking into account the kinetic energy, the Zeeman magnetic field normal to the heterostructure, and the uniform electric field, $\boldsymbol{E}$, we can write the $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian as

$$\begin{aligned} H &= \mu k^2 + V_{\rm so} + h_0\sigma_z + e\boldsymbol{E}\cdot\boldsymbol{r} \\ &\equiv H_0 + e\boldsymbol{E}\cdot\boldsymbol{r}, \end{aligned} \qquad (2)$$

where $\mu = \hbar^2/2m^*$ with m^* being the band effective mass, $h_0 = g^* B\mu_{\rm B}/2$ denotes the Zeeman splitting due to the external weak magnetic field or the internal exchange field, which will occur in the presence of magnetic doping.

The eigenvalues of the unperturbed Hamiltonian H_0 are $\varepsilon_{1(2)k} = \gamma k^2 \mp \Delta_k$ with $\Delta_k = \sqrt{h_0^2 + (\alpha^2+\beta^2)k^2 + 4\alpha\beta k_x k_y}$. The corresponding Bloch eigenstates are given by

$$\Phi_1(\boldsymbol{r}) = \begin{pmatrix} \sin\dfrac{\theta_k}{2}\mathrm{e}^{\mathrm{i}\chi_k} \\ -\cos\dfrac{\theta_k}{2} \end{pmatrix} \mathrm{e}^{\mathrm{i}k\cdot r} \equiv u_{1k}\mathrm{e}^{\mathrm{i}k\cdot r}, \qquad (3a)$$

$$\Phi_2(\boldsymbol{r}) = \begin{pmatrix} \cos\dfrac{\theta_k}{2}\mathrm{e}^{\mathrm{i}\chi_k} \\ \sin\dfrac{\theta_k}{2} \end{pmatrix} \mathrm{e}^{\mathrm{i}k\cdot r} \equiv u_{2k}\mathrm{e}^{\mathrm{i}k\cdot r}, \qquad (3b)$$

where we have introduced u_{nk} ($n = 1, 2$) to denote momentum-dependent spinors. The factors θ_k and χ_k in Eqs.(3) are defined by $\cos\theta_k = h_0/\Delta_k$ and $\tan\chi_k = (\alpha k_x + \beta k_y)/(\alpha k_y + \beta k_x)$, respectively. Note that although the SO coupling behaves like an effective magnetic field in k-space, it does not destroy time-reversal symmetry, which is broken by the real Zeeman splitting h_0. The interplay of SO coupling and Zeeman splitting will, as will be shown below, induce an intrinsic Hall effect for the electric current.

3. Adiabatic approximation and intrinsic Hall conductivity: general derivation and model-specific analysis

In the presence of a uniform electric field, by introducing a phase transformation $\bar{\Phi}(t, \boldsymbol{r}) = \exp[\mathrm{i}e\boldsymbol{E}\cdot\boldsymbol{r}t]\,\Phi(\boldsymbol{r})$, the term $e\boldsymbol{E}\cdot\boldsymbol{r}$ in Eq.(2) is eliminated and the $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian is transformed as $\bar{H}(t) = H(\boldsymbol{p} - f(t))$ with $f(t) = e\boldsymbol{E}t$. Since $f(t)$ changes very slowly, it can be treated as an adiabatic parameter. The time-dependent Schrödinger equation for $\bar{\Phi}(t, \boldsymbol{r})$ is expressed as

$$\mathrm{i}\frac{\partial}{\partial t}\bar{\Phi}(t, \boldsymbol{r}) = \bar{H}(t)\,\bar{\Phi}(t, \boldsymbol{r}). \qquad (4)$$

Within the adiabatic approximation, the resultant instantaneous eigenvalue equation at time t is given by $\bar{H}(t)\,\bar{\Phi}_{nk}(t) = \varepsilon_{nk}(t)\,\bar{\Phi}_{nk}(t)$, where $\varepsilon_{nk}(t)$ are the instantaneous eigenenergies. Since the electric field is assumed to be homogeneous in 2DEG, the instantaneous eigenstate, like the static eigenstate $\Phi_{nk}(\boldsymbol{r})$, can be written in the Bloch form, $\bar{\Phi}_{nk}(t, \boldsymbol{r}) = \mathrm{e}^{\mathrm{i}k\cdot r}u_{nk}(t)$, where the function $u_{nk}(t)$ is obtained from the static spinor u_{nk} via a shift in k-space

$$u_{nk}(t) = u_{n,k-f(t)}. \qquad (5)$$

The solution of Eq.(4) in the adiabatic approximation is given by

$$\bar{\Psi}_{nk}(t,\boldsymbol{r}) = \exp\left(\mathrm{i}\int_0^t \mathrm{d}t'[\varepsilon_{nk}(t') + \gamma_{nk}(t')]\right)\bar{\Phi}_{nk}(t,\boldsymbol{r}), \tag{6}$$

where $\gamma_{nk}(t)$ is a consequence of the geometric effect and is given by

$$\begin{aligned}\gamma_{nk}(t) = & \ \mathrm{i}\int_0^t \mathrm{d}t'\left\langle \bar{\Phi}_{nk}(t')\left|\frac{\partial}{\partial t'}\right|\bar{\Phi}_{nk}(t')\right\rangle \\ = & \ \mathrm{i}\int_0^t \mathrm{d}t' u^*_{nk}(t')\frac{\partial}{\partial t'}u_{nk}(t').\end{aligned} \tag{7}$$

When the vector $f(t)$ is parallel to the reciprocal-lattice vector $\boldsymbol{G}$, we have a period T for a closed loop in the momentum space.[29] The Berry phase is defined as $\int_0^T \mathrm{d}t\gamma_k(t)$. The Berry connection defined in the parameter space is given by $\boldsymbol{a}_n(\boldsymbol{k}) = \langle u_{nk}|\nabla_k|u_{nk}\rangle$. By using Eq.(5), the Berry phase for $f(t)//\boldsymbol{G}$ can be written in a general form, $\Gamma_{n,f}(k) = \mathrm{i}\oint_{C(f)} \mathrm{d}\boldsymbol{k}\cdot\boldsymbol{a}_n(\boldsymbol{k})$, where $C(f)$ is a closed loop on which $f(t)$ moves. In general, the Berry phase depends on the details of $C(f)$. [30]

The above adiabatic description illustrates how the Berry phase and Berry connection arise from the perturbation of the electric field. However, we must go further to take into account the mixing between the adiabatic eigenstates. This is essential for the Berry curvature to occur in the transport equations. Given the zero-order adiabatic spinors $u_{nk}(t)$, the first-order wavefunctions are given within time-dependent (non-degenerate) perturbation theory by

$$\left|\tilde{u}^{(1)}_{nk}(t)\right\rangle = |u_{nk}(t)\rangle + \sum_{n'\neq n}\frac{|u_{n'k}(t)\rangle\left\langle u_{n'k}(t)|\mathrm{i}\frac{\partial u_{nk}(t)}{\partial t}\right\rangle}{\varepsilon_{n'k}(t)-\varepsilon_{nk}(t)}, \tag{8}$$

which are the starting quantum states for further treatment.

The macroscopic currents are obtained from the quantum-mechanical and thermodynamical average over the current operator. Using expression (8) for the first-order wavefunctions as a response to the electric field $\boldsymbol{E}$, the electric current $\boldsymbol{J}$ is given by

$$\boldsymbol{J} = -e\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\left\langle\tilde{u}_{nk}(t)\left|\boldsymbol{v}\right|\tilde{u}_{nk}(t)\right\rangle = \boldsymbol{J}_N + \boldsymbol{J}_{\mathrm{H}}, \tag{9}$$

where $\boldsymbol{v} = \partial H_0/\partial\boldsymbol{k}$ is the velocity operator in the Bloch states and f_n denotes the Fermi–Dirac distribution function for the nth band under an electric field.

$$\boldsymbol{J}_N = -e\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\frac{\partial\varepsilon_{nk}}{\partial k} \tag{10}$$

is the normal current along the direction of the electric field, and

$$\boldsymbol{J}_{\mathrm{H}} = -\mathrm{e}\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\left\{\sum_{n'\neq n}\frac{\langle u_{nk}(t)\left|\boldsymbol{v}\right|u_{n'k}(t)\rangle\left\langle u_{n'k}(t)|\mathrm{i}\frac{\partial u_{nk}(t)}{\partial t}\right\rangle}{\varepsilon_{n'k}(t)-\varepsilon_{nk}(t)} + \mathrm{c.c.}\right\} \tag{11}$$

is the additional current contribution due to the mixing of the adiabatic wavefunctions. Its Hall properties of the flow perpendicular to the electric field can be seen as follows. To further simplify the expression for $\boldsymbol{J}_{\mathrm{H}}$, notice that in Eq.(11) $\langle u_{nk}\left|\boldsymbol{v}\right|u_{n'k}\rangle/(\varepsilon_{n'k}-\varepsilon_{nk}) = \left\langle\frac{\partial u_{n\boldsymbol{k}}}{\partial k}|u_{n'k}\right\rangle$ and $\left|\frac{\partial u_{nk}}{\partial t}\right\rangle = -\frac{eE}{\hbar}\cdot|\frac{\partial u_{nk}}{\partial\boldsymbol{k}}\rangle$. By substituting these two equalities into Eq.(11), we obtain

$$\begin{aligned}J_{\mathrm{H}} = & \ -e\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\left\{\left\langle\frac{\partial u_{nk}(t)}{\partial\boldsymbol{k}}|\mathrm{i}\frac{\partial u_{nk}(t)}{\partial t}\right\rangle + \mathrm{c.c.}\right\} \\ = & \ \frac{e^2}{\hbar}\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\left\{\left\langle\frac{\partial u_{nk}(t)}{\partial\boldsymbol{k}}\right|\boldsymbol{E}\cdot\left|\mathrm{i}\frac{\partial u_{nk}(t)}{\partial\boldsymbol{k}}\right\rangle + \mathrm{c.c.}\right\} \\ = & \ \frac{e^2}{\hbar}\boldsymbol{E}\times\boldsymbol{z}\sum_n\int_{BZ}\frac{\mathrm{d}^2k}{(2\pi)^2}f_n\Omega_{n,z},\end{aligned} \tag{12}$$

where

$$\Omega_n = \mathrm{i}\nabla_k \times \boldsymbol{a}_n(\boldsymbol{k}) = \mathrm{i}\left\langle \frac{\partial u_{nk}(t)}{\partial \boldsymbol{k}} \right| \times \left| \frac{\partial u_{nk}(t)}{\partial \boldsymbol{k}} \right\rangle \tag{13}$$

is the usual Berry curvature[31,32] which is related to the Berry phase Γ_n by $\Gamma_n = \int_A \mathrm{d}S \cdot \Omega_n$, with A being the area enclosed by the adiabatic loop $C(f)$ in the wave-vector space. One can see from Eq.(12) that the current $\boldsymbol{J}_{\mathrm{H}}$ is perpendicular to the external electric field, thus it is Hall-like. Equation (12) can also be obtained from the semiclassical equation of motion for the Bloch electrons.[33] The distribution function in Eq.(12) is formally obtained by a systematic solution of the Boltzmann equation. In the linear response approximation, $f_n = f_n^{(0)} + \delta f_n$, where $f_n^{(0)}$ is the equilibrium part and δf_n is proportional to the electric field. Then the equation of the total current has a form (to the first order in $\boldsymbol{E}$)

$$\boldsymbol{J} = -e\sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2} \delta f_n \frac{\partial \varepsilon_{nk}}{\partial \boldsymbol{k}} + \frac{e^2}{\hbar}\boldsymbol{E} \times \boldsymbol{z} \sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2} f_n^{(0)} \boldsymbol{\Omega}_{n,z}. \tag{14}$$

One can see that the first normal term in Eq.(14) arises from the drift of the distribution function and thus is along the direction of the electric field, whereas the intrinsic (second) term of $\boldsymbol{J}$ is due to the first-order change of the electron wavefunctions and behaves as being deflected by an effective magnetic field. Remarkably, the eigenstate mixing in Eq.(8) disappears in the transport equation, and the Hall current looks like an adiabatic response to the electric field. This is also the reason why the single-band semiclassical approximation[33] gives the same expression as that in Eq.(12).

The subsequent Hall conductivity is given by

$$\sigma_{\mathrm{H}} = J_{\mathrm{H},y}/E_x = -\frac{e^2}{\hbar}\sum_n \int \frac{\mathrm{d}^2k}{(2\pi)^2} f_n^{(0)} \Omega_{n,z}. \tag{15}$$

For the present Hamiltonian (2), after a straightforward derivation, we obtain the expressions for Berry curvatures as follows:

$$\boldsymbol{\Omega}_{1,z} = \frac{1}{2}\frac{h_0(\alpha^2-\beta^2)}{\Delta_k^{3/2}}, \quad \Omega_{2,z} = -\Omega_{1,z}. \tag{16}$$

When $h_0 = 0$, the time-reversal symmetry is recovered and the Berry curvature for both bands vanishes. Interestingly, when the Rashba parameter α and Dresselhaus parameter β are tuned to $\alpha = \beta$, $\Omega_{n,z}$ vanishes again. At zero temperature, suppose only the bottom band is occupied, then the Hall conductivity is given by

$$\sigma_{\mathrm{H}} = -\frac{e^2}{4\pi h}\int_0^{2\pi} \mathrm{d}\theta \int_0^{k_f^{(+)}} k\mathrm{d}k \frac{h_0(\alpha^2-\beta^2)}{\Delta_k^{3/2}}, \tag{17}$$

where we have transformed the parameters to those in polar coordinates in k space, and $k_f^{(+)}$ is the Fermi wavevector of the bottom band. To obtain an analytic expression for Eq.(17), we take $k_f^{(+)}$ to be infinity. Then the expression (17) reduces to

$$\begin{aligned}\sigma_{\mathrm{H}} = & -\frac{e^2}{4\pi h}\int_0^{2\pi} \mathrm{d}\theta \frac{\alpha^2-\beta^2}{[h_0^2 + (\alpha^2+\beta^2) + 2\alpha\beta\sin(2\theta)]^{1/2}} \\ = & \frac{\beta^2-\alpha^2}{|\alpha^2-\beta^2|}\frac{e^2}{2h},\end{aligned} \tag{18}$$

which is a product of the Berry phase $\Gamma_1 = (\alpha^2 - \beta^2)\pi/|\alpha^2-\beta^2|$ (in the zero-h_0 limit) and the quantum conductance $e^2/\hbar$. One can see from Eq.(18) that by tuning the Rashba parameter α around a fixed value of β, the Berry phase and, subsequently the intrinsic charge-Hall conductivity, may change their signs, thus providing an interesting electric method to detect these two kinds of spin–orbit interactions.

Equation (18) corresponds to the ideal case of single-band occupation and infinite Fermi wavevector. For realistic consideration, we need to take into account the usual case that both bands are occupied. The effect of finite value of Fermi wavevector should also be considered. The Fermi wavevectors $k_f^{(\pm)}$ for

the two bands are obtained from the Fermi energy ε_f by the relation

$$\varepsilon_f = \mu k_f^2 \pm \sqrt{h_0^2 + \left(k_f^{\pm}\right)^2 c(\theta_f)}, \tag{19}$$

where $c(\theta_f) = \alpha^2 + \beta^2 + 2\alpha\beta\sin(2\theta_f)$ with the Fermi anisotropic angle θ_f determined by $\cos\theta_f = k_{f,x}/k_f$. On the other hand, the Fermi energy ε_f is pinned by the electron density $N_{\rm e}$. In the absence of Zeeman splitting ($h_0 = 0$), the two bands are obviously occupied with the same weight. Then we have the relation between the electron density and the Fermi energy as follows:

$$\begin{aligned} N_{\rm e} = & \frac{1}{(2\pi)^2}\sum_n \int_0^{2\pi} \frac{1}{2} k_f^2 {\rm d}\theta \\ = & \frac{1}{2\pi}\left(\frac{\varepsilon_f}{\mu} + \frac{\alpha^2+\beta^2}{2\mu^2}\right). \end{aligned} \tag{20}$$

In this case, the Fermi wavevectors $k_f^{\pm}$ for the two bands are

$$k_f^{\pm} = \frac{2\mu\varepsilon_f + c(\theta_f) \mp \sqrt{c^2(\theta) + 4\mu c(\theta_f)\varepsilon_f}}{2\mu^2}. \tag{21}$$

The presence of Zeeman field shifts the degeneracy of the two bands at point $\boldsymbol{k} = 0$. If h_0 is weak enough to satisfy the inequality $N_{\rm e} > \dfrac{\alpha^2+\beta^2}{4\pi\mu^2} + \dfrac{h_0}{2\pi\mu}$, then the Fermi energy and wavevectors are still determined by Eqs.(20) and (21). On the other hand, in the strong Zeeman-splitting region, $N_{\rm e} < \dfrac{\alpha^2+\beta^2}{4\pi\mu^2} + \dfrac{h_0}{2\pi\mu}$, only the bottom band is occupied. Then the Fermi energy will be dependent on the amplitude of h_0. In this single-band occupation region, we still need to consider two different cases for k-integral in Eq.(15): (i) In the regime $-h_0 < \varepsilon_f < h_0$, then the lower limit of k in the integrand in Eq.(15) is zero, and the upper limit is $k_f^{(+)}$ in Eq.(21), whereas the Fermi energy ε_f is obtained via the equation

$$N_{\rm e} = \frac{1}{(2\pi)^2}\int_0^{2\pi}\frac{1}{2}k_f^2{\rm d}\theta = \frac{1}{4\pi}\left(\frac{\varepsilon_f}{\mu} + \frac{\alpha^2+\beta^2}{4\mu^2} + \frac{1}{2\pi}\int_0^{2\pi}{\rm d}\theta\frac{\sqrt{c^2(\theta)+4\mu c(\theta)\varepsilon_f + 4\mu^2 h_0^2}}{2\mu^2}\right). \tag{22}$$

(ii) When the Fermi energy is low enough to satisfy $\varepsilon_f < -h_0$, then the lower and upper limits of k in Eq.(15) are $k_f^{(-)}$ and $k_f^{(+)}$, respectively. The Fermi energy is obtained via

$$N_{\rm e} = \frac{1}{(2\pi)^2}\int_0^{2\pi}{\rm d}\theta\int_{k_f^{(+)}}^{k_f^{(+)}} k{\rm d}k = \frac{1}{4\pi^2}\int_0^{2\pi}\frac{\sqrt{c^2(\theta)+4\mu c(\theta)\varepsilon_f+4\mu^2h_0^2}}{4\mu^2}{\rm d}\theta. \tag{23}$$

Therefore we have obtained the Fermi energy and Fermi wavevectors in different parameter regimes.

4. Numerical results and discussion

As an example of application of Eqs.(15)–(23) to real systems, we consider the GaAs 2DEG system with parameters $\mu = \hbar^2/2m^* = 568\,{\rm meV\ nm^2}$, $N_{\rm e} = 0.05\,nm^{-2}$. The Dresselhaus parameter β can be obtained by the thickness of the 2DEG as $\beta \approx \gamma(\pi/w)^2$, with material-specific constant γ, which is 27.5 meV nm^3 for GaAs.[34] Then we have $\beta \sim 10\,{\rm meV}$ for a typical value 5 nm of w. The Rashba parameter α can be varied in a wide range by adjusting the external potential applied perpendicular to the plane. Given the value of the effective Zeeman factor, $g^* = 0.44$ for bulk GaAs, the Zeeman splitting h_0 can be varied in a range of 0–0.06 meV, which is relatively small compared with other energy scales. However, the situation is totally different if the Zeeman splitting is tuned by the magnetic impurity doping in the paramagnetic semiconductor hosts. Recent advances in diluted magnetic semiconductors[35] has made it possible to manipulate the kinetic exchange interaction of the free carriers in a controllable way. In the mean-field approximation, the kinetic exchange interaction splitting can be expressed as $h_0 = -N_0\beta x S B_S(\xi)$. Here, $N_0\beta$ is the exchange integral, x is the magnetic ion concentration, $B_S(\xi)$ is the standard Brillouin function [36] and $\xi = g_S\mu_{\rm B}SB/k_{\rm B}T$ with S and g_S being the total spin quantum number and the Landé factor of the mag-

netic ions, respectively. For Mn doping in GaAs host, it has been acknowledged[37] that the Mn ions in $3d^5$ configuration will substitute the cations, thus leading to $S = 5/2$ and $g_S = 2$. The p-d exchange integral in (Ga,Mn)As has been measured[38] to be $N_0\beta \approx -1\,\mathrm{eV}$. The exchange interaction h_0 can be changed in a wide range of 4–16 meV under a weak external magnetic field of the order of 10–100 mT at a typical low temperature $T = 1\,\mathrm{K}$ and Mn concentration $x = 0.05$.

Figure 1 summarizes the behaviours of the Hall conductivity which is numerically obtained from Eq.(15) at zero temperature. In Fig.1(a) is shown a contour plot of this quantity in the parameter plane of Rashba and Dresselhaus SO coefficients. The electron density has been fixed at $N_\mathrm{e} = 0.05\,\mathrm{nm}^{-2}$. One can see that on switching the interaction α to the Dresselhaus interaction, the Hall conductivity will change its sign, which means that when α is tuned to be larger than β, the Hall current will flow in the inverse transverse direction compared with the case of $\alpha < \beta$. Figure 1(b) shows the Hall conductivity for different values of Zeeman splitting. It reveals that the Hall conductivity can be increased by increasing the Zeeman splitting. The reason is that a large Zeeman splitting will prominently imbalance the electronic population of the two bands, which in turn, due to the opposite signs of Berry curvatures (see Eq.(26)), enlarges the Hall effect as shown in Fig.1(b). It should be addressed that although the temperature dependence of this intrinsic Hall effect is only reflected by the Fermi distribution function in Eq.(3), it does not mean that this Hall conductivity is insensitive to thermal excitation. To ensure that the Hall effect is not completely overshadowed by thermal excitation, the Zeeman splitting h_0 and the spin–orbit interaction must exceed the energy broadening by impurity scattering.

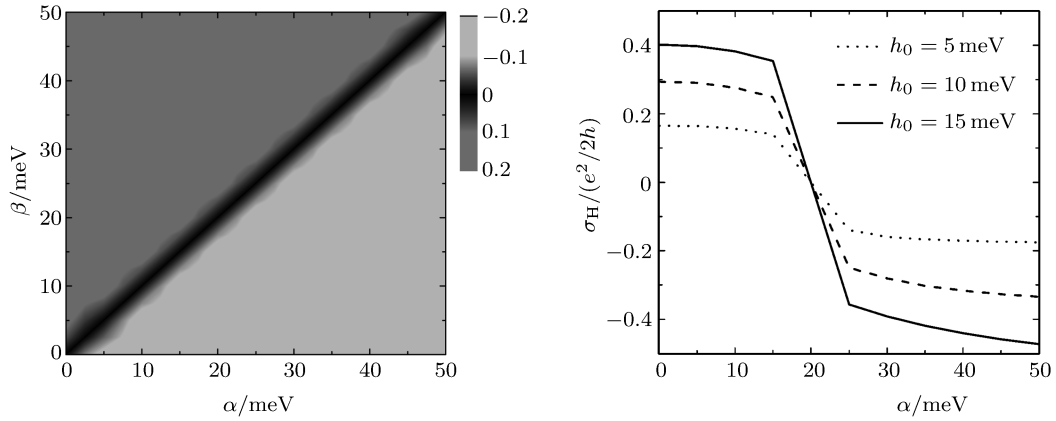


Fig.1. Hall conductivity (in units of $e^2/2h$) in a GaAs quantum well system depended on (a) the Rashba spin–orbit interaction α and the Dresselhaus interaction β. (b) Hall conductivity as a function of α (β=20 meV) for different values of Zeeman splitting.

5. Conclusion

In summary, we have analysed the interplay of two kinds of spin–orbit interactions in influencing the intrinsic Hall conductivity. It has been shown that by tuning the ration of the Rashba SO interaction to the Dresselhaus interaction in different parameter regimes, the flow of Hall current may be along different transverse directions, thus providing a feasible way to determine the relative strength of Rashba and Dresselhaus terms in spin–orbit coupled semiconductor quantum well structures.

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