Vol 15 No 12, December 2006
1009-1963/2006/15(12)/2953-06

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Entanglement concentration for multi-atom GHZ class state via cavity QED*

Jiang Chun-Lei(姜春蕾), Fang Mao-Fa(方卯发)†, and Zheng Xiao-Juan(郑小娟)

College of Physics and Information Science,

Hunan Normal University, Changsha 410081, *China*

(Received 28 March 2006; revised manuscript received 10 July 2006)

In this paper, we propose a physical scheme to concentrate non-maximally entangled atomic pure states by using atomic collision in a far-off-resonant cavity. The most distinctive advantage of our scheme is that the non-maximally entangled atoms may be far from or near each other and their degree of entanglement can be maximally amplified. The photon-number-dependent parts in the effective Hamiltonian are cancelled with the assistance of a strong classical field, thus the scheme is insensitive to both the cavity decay and the thermal field.

Keywords: entangled atomic states, GHZ state, concentration, cavity quantum electrodynamics

PACC: 4250, 0365

1. Introduction

Entanglement is recognized nowadays as a key ingredient for a fundamental test of quantum mechanics and as a basic resource of quantum information processing, such as quantum cloning,[1] quantum dense coding and quantum teleportation.[2−5] To fulfil perfect quantum information processing, the quantum channel must be maximally entangled usually. But no one operation is perfect, and no one transmission channel is free of noise. In the real processing of storage and transmission of quantum entangled states, the maximally entangled states usually collapse into non-maximally entangled ones or even mixed states because of the unavoidable noise. Then, to realize the faithful quantum information processing, we must convert the non-maximally entangled states into maximally entangled ones. There are many theoretical and experimental schemes that can achieve this conversion, several entanglement purification schemes have been proposed.[6−14]

For the non-maximally entangled pure states, the conversion process is usually termed as entanglement concentration or distillation,[7] and the process dealing with the mixed states is usually named as entanglement purification.[6] Entanglement distillation (concentration) is to concentrate a smaller number of maximally entangled states from a large number of non-maximally entangled states by local operation and classical communication (LOCC),[7] while the entanglement purification can only increase the entanglement of the mixed states by LOCC.[6] For the whole entangled system, the entanglement has not increased.[15]

Bennett *et al* have presented the first entanglement concentration scheme where the entanglement of the whole system is transferred to the smaller number of entangled pairs.[7] Bose *et al* proposed another physical scheme to concentrate maximally entangled polarization states of photons using entanglement swapping.[14] Experimental entanglement distillation schemes have also been proposed for the non-maximally entangled polarization states of photons. These are mostly the distillation schemes for entangled polarization states of photons,[10,13] and we find few concentration scheme for atomic states. In experiments, atoms are the optimal carriers of quantum information for the quantum computer, so the study on atomic states is of practical significance. Cao *et al* proposed a scheme to concentrate the non-maximally entangled atomic pure states via cavity quantum electrodynamics(QED).[16] They used an effective scheme proposed by Zheng,[17] which is insensitive to the thermal field just when the cavity is initially in vacuum state.

In this paper, we present a feasible entanglement

*Project supported by the National Natural Science Foundation of China (Grant No 10374025).

†Corresponding author. E-mail: mffang@hunnu.edu.cn

distillation scheme to concentrate maximally entangled states from non-maximally entangled pure states via cavity QED. In this case the entangled atoms need not to be distributed, thus the scheme can easily amplify the entanglement maximally. So it may be used as an entanglement relay station. In the case of quantum communication, the entangled atoms are distributed among distant users, then our scheme can concentrate their entanglements by using an auxiliary atom. The scheme is insensitive to both cavity decay and thermal field, which is important experimentally.

We use this scheme to concentrate multi-atom Greenberger–Horne–Zeilinger (GHZ) class states, which play an important role in quantum communication, so our discussion concerning these states is of practical significance.

2. The model and the effective Hamiltonian

Consider N identical two-level atoms interacting with a single-mode cavity field and driven by a classical field. In the rotating-wave approximation, the Hamiltonian is($\hbar = 1$):[18,19]

$$H = \omega_0 \sum_{j=1}^{N} S_{z,j} + w_a a^+ a + \sum_{j=1}^{N} \left[g(a^+ S_j^- + a S_j^+) + \Omega \left(S_j^+ \mathrm{e}^{-\mathrm{i}\omega t} + S_j^- \mathrm{e}^{\mathrm{i}\omega t} \right) \right], \tag{1}$$

where $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, with $|e_j\rangle$ and $|g_j\rangle (j = 1, 2, \ldots, N)$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity mode respectively; g is the atom–cavity coupling strength, and Ω is the Rabi frequency of the classical field, ω_0 is the atomic transition frequency, ω_a is the cavity frequency and ω is the frequency of the classical field. Assume that $\omega_0 = \omega$, then the interaction Hamiltonian, in the interaction picture, is

$$H_i = \sum_{j=1}^{N} \left[g(\mathrm{e}^{-\mathrm{i}\delta t} a^+ S_j^- + \mathrm{e}^{\mathrm{i}\delta t} a S_j^+) + \Omega(S_j^+ + S_j^-) \right], \tag{2}$$

where $\delta = \omega_0 - \omega_a$. With a large detuning $\delta \gg g/2$ and a strong driving field ($2\Omega \gg \delta$,g) limit, the effective Hamiltonian can be described as follows:[20]

$$H_{\mathrm{eff}} = \lambda \left[\frac{1}{2} \sum_{j=1}^{N} (|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \sum_{j,k=1}^{N} (S_j^+ S_k^+ + S_j^+ S_k^- + \mathrm{H.c}) \right] \quad j \neq k, \tag{3}$$

where $\lambda = g^2/2\delta$. The distinct feature of this effective Hamiltonian is that it is independent of the photon number of the cavity field. Without the strong classical field, the Stark shift terms are proportional to the photon number, and the terms $S_j^+ S_k^+ + H.c$ do not exist. According to Ref.[21], $H_0 = \Omega \sum_{j=1}^{N} (S_+ + S_-)$, and it is easy to prove $[H_0 \;\; H_{\mathrm{eff}}] = 0$, so the evolution operator of the system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}H_0 t} \mathrm{e}^{-\mathrm{i}H_{\mathrm{eff}} t}. \tag{4}$$

We note that the atomic state evolution operator $U(t)$ is independent of the cavity field state, so the latter can be in a thermal state.

3. The concentration of GHZ class state

We first consider the two-atom concentration case. By solving Schrödinger equation, we can obtain the evolution of different initial states during the interaction:

$$
\begin{aligned}
|g_1\rangle|g_2\rangle \rightarrow\ & \mathrm{e}^{-\mathrm{i}\lambda t}\Big[\cos(\lambda t)\Big[\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle\Big]\times\Big[\cos\Omega t|g_2\rangle-\mathrm{i}\sin\Omega t|e_2\rangle\Big]\\
&-\mathrm{i}\sin(\lambda t)\Big[\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle\Big]\times\Big[\cos\Omega t|e_2\rangle-\mathrm{i}\sin\Omega t|g_2\rangle\Big]\Big],\\
|e_1\rangle|g_2\rangle \rightarrow\ & \mathrm{e}^{-\mathrm{i}\lambda t}\Big[\cos(\lambda t)\Big[\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle\Big]\times\Big[\cos\Omega t|g_2\rangle-\mathrm{i}\sin\Omega t|e_2\rangle\Big]\\
&-\mathrm{i}\sin(\lambda t)\Big[\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle\Big]\times\Big[\cos\Omega t|e_2\rangle-\mathrm{i}\sin\Omega t|g_2\rangle\Big]\Big],\\
|g_1\rangle|e_2\rangle \rightarrow\ & \mathrm{e}^{-\mathrm{i}\lambda t}\Big[\cos(\lambda t)\Big[\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle\Big]\times\Big[\cos\Omega t|e_2\rangle-\mathrm{i}\sin\Omega t|g_2\rangle\Big]\\
&-\mathrm{i}\sin(\lambda t)\Big[\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle\Big]\times\Big[\cos\Omega t|g_2\rangle-\mathrm{i}\sin\Omega t|e_2\rangle\Big]\Big],\\
|e_1\rangle|e_2\rangle \rightarrow\ & \mathrm{e}^{-\mathrm{i}\lambda t}\Big[\cos(\lambda t)\Big[\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle\Big]\times\Big[\cos\Omega t|e_2\rangle-\mathrm{i}\sin\Omega t|g_2\rangle\Big]\\
&-\mathrm{i}\sin(\lambda t)\Big[\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle\Big]\times\Big[\cos\Omega t|g_2\rangle-\mathrm{i}\sin\Omega t|e_2\rangle\Big]\Big]. \qquad (5)
\end{aligned}
$$

Next, we will consider the detailed concentration procedures. Suppose that the non-maximally entangled state is in the form

$$|\psi\rangle_{12}=a|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2, \qquad (6)$$

where $|a|^2+|b|^2=1$. If the atoms in this entangled state are stored together, to obtain the maximally entangled state of these atoms, we just need to send the two atoms simultaneously into the cavity. After an interaction time $\lambda t=\lambda t_1=\pi/4$ and modulating the classical field $\Omega t_1=k\pi$, with k being an integer, the state of the system can be written as

$$
\begin{aligned}
a|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2 \rightarrow\ & \frac{a}{\sqrt{2}}(|e_1\rangle|e_2\rangle-\mathrm{i}|g_1\rangle|g_2\rangle)+\frac{b}{\sqrt{2}}(|g_1\rangle|g_2\rangle-\mathrm{i}|e_1\rangle|e_2\rangle)\\
=\ & \frac{1}{\sqrt{2}}\Big[(a-\mathrm{i}b)|e_1\rangle|e_2\rangle+(b-\mathrm{i}a)|g_1\rangle|g_2\rangle\Big]\\
=\ & \frac{1}{\sqrt{2}}(|e_1\rangle|e_2\rangle+\mathrm{e}^{\mathrm{i}\theta}|g_1\rangle|g_2\rangle), \qquad (7)
\end{aligned}
$$

where $\theta=\mathrm{arctg}\dfrac{|b|}{|a|}-\mathrm{arctg}\dfrac{|a|}{|b|}$. Then, if atom 1 belongs to Alice, atom 2 belongs to Bob, to obtain the maximally entangled state of the atoms in the state described by Eqs.(6), an auxiliary atom initially in ground state $|g\rangle_a$ and the cavity, as expressed before, have to be introduced into the location of Alice or Bob. Without loss of generality, here we assume that they are all in Alice's location, and the auxiliary atom is identical with the two entangled atoms.

Alice will send atom 1 and the auxiliary atom simultaneously to the cavity in different directions. After an interaction time t, the evolution of the system is given by

$$
\begin{aligned}
(a|e\rangle_1|e\rangle_2+b|g\rangle_1|g\rangle_2)|g_a\rangle \rightarrow\ & \mathrm{e}^{-i\lambda t}\Big\{a|e_2\rangle\Big[\cos\lambda t(\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle)(\cos\Omega t|g_a\rangle-\mathrm{i}\sin\Omega t|e_a\rangle)\\
&-\mathrm{i}\sin\lambda t(\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle)(\cos\Omega t|e_a\rangle-\mathrm{i}\sin\Omega t|g_a\rangle)\Big]\\
&+b|g_2\rangle\Big[\cos\lambda t(\cos\Omega t|g_1\rangle-\mathrm{i}\sin\Omega t|e_1\rangle)(\cos\Omega t|g_a\rangle-\mathrm{i}\sin\Omega t|e_a\rangle)\\
&-\mathrm{i}\sin\lambda t(\cos\Omega t|e_1\rangle-\mathrm{i}\sin\Omega t|g_1\rangle)(\cos\Omega t|e_a\rangle-\mathrm{i}\sin\Omega t|g_a\rangle)\Big]\Big\}. \qquad (8)
\end{aligned}
$$

Choosing $\lambda t = \lambda t_2 = \text{arctg}\dfrac{|b|}{|a|}$ and $\Omega t_2 = k\pi$, after the two atoms escape from the cavity, Alice can detect atom 1, if it is in ground state, while atom 2 and the auxiliary atom will be in a maximally entangled state:

$$\frac{1}{\sqrt{2}}(|e\rangle_2|e\rangle_a + \mathrm{i}|g\rangle_2|g\rangle_a), \tag{9}$$

where we have discarded the common phase factor. If atom 1 is in an exited state, the process fails. The successful probability is $P_{\text{succ}} = \dfrac{1}{2}$. After a rotation of the auxiliary atom, the state described above will become:

$$\frac{1}{\sqrt{2}}(|e\rangle_2|e\rangle_a + |g\rangle_2|g\rangle_a). \tag{10}$$

In the following, we will discuss the concentration of multi-atom GHZ class states:

$$|\psi\rangle_{12\ldots N} = a|e_1e_2\ldots e_N\rangle + b|g_1g_2\ldots g_N\rangle, \tag{11}$$

where $|a|^2 + |b|^2 = 1$. The effective Hamiltonian H_{eff} can also be rewritten as

$$H_{\text{eff}} = 2\lambda S_x^2, \tag{12}$$

where $S_x = \dfrac{1}{2}\sum_{j=1}^{N}(S_j^+ + S_j^-)$. The evolution operator of the system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}H_{\text{eff}}t} = \mathrm{e}^{-2\mathrm{i}(\Omega S_x + \lambda S_x^2)t}. \tag{13}$$

Using the representation of the operator S_z, the atomic states $|e_1e_2\ldots e_N\rangle$ and $|g_1g_2\ldots g_N\rangle$ can be expressed as $|N/2, N/2\rangle$ and $|N/2, -N/2\rangle$, respectively. On the other hand, such states can be expanded in terms of the eigenstates of S_x:[21,22]

$$\begin{aligned}|N/2, N/2\rangle &= \sum_{M=-N/2}^{N/2} C_M(-1)^{N/2-M}|N/2, M\rangle_x,\\ |N/2, -N/2\rangle &= \sum_{M=-N/2}^{N/2} C_M|N/2, M\rangle_x.\end{aligned} \tag{14}$$

First, if the entangled atoms in the state described by Eqs.(11) are stored or used in the same location, to obtain the maximally entangled state of these atoms, we just need to simultaneously send these atoms into the cavity. If N is even, M is any integer, by choosing $\lambda t = \lambda t_3 = \pi/4$ and modulating the classical field as $\Omega t_3 = k\pi$, the state of the system will become

$$\begin{aligned}&\frac{a}{\sqrt{2}}\sum_{M=-N/2}^{N/2} C_M(-1)^{N/2-M}\left[\mathrm{e}^{-\mathrm{i}\pi/4} + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^M\right]|N/2, M\rangle_x\\ &+\frac{b}{\sqrt{2}}\sum_{M=-N/2}^{N/2} C_M\left[\mathrm{e}^{-\mathrm{i}\pi/4} + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^M\right]|N/2, M\rangle_x\\ =\ &\frac{a}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\pi/4}|e_1e_2\ldots e_N\rangle + \frac{a}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\pi/4}(-1)^{N/2}|g_1g_2\ldots g_N\rangle\\ &+\frac{b}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\pi/4}|g_1g_2\ldots g_N\rangle + \frac{b}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\pi/4}(-1)^{-N/2}|e_1e_2\ldots e_N\rangle\\ =\ &\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\pi/4}\left[\left[a + \mathrm{i}b(-1)^{-N/2}\right]|e_1e_2\ldots e_N\rangle + \left[\mathrm{i}a(-1)^{N/2} + b\right]|g_1g_2\ldots g_N\rangle\right]\\ =\ &\frac{1}{\sqrt{2}}|e_1e_2\ldots e_N\rangle + \mathrm{e}^{\mathrm{i}\phi}|g_1g_2\ldots g_N\rangle,\end{aligned} \tag{15}$$

where ϕ is a phase factor. On the other hand, in the case when N is odd, with $\lambda t = \lambda t_4 = \pi/4$, and $\Omega t_4 = (2k + 3/4)\pi$, we also can obtain the maximally entangled state of the atoms in the state described by Eqs.(11) for different ϕ. Finally, when these N atoms are distributed among distant users including Alice, without loss of generality, suppose that Alice has accessed atom 1 and an auxiliary atom prepared in ground state. After Alice has sent her two atoms to the cavity, the state of the system will become

$$(a|e_1e_2\dots e_N\rangle + b|g_1g_2\dots g_N\rangle)|g_a\rangle \rightarrow \ \mathrm{e}^{-\mathrm{i}\lambda t}\Big[a\Big[|e_2\dots e_N\rangle\cos\lambda t\Big[\cos\Omega t|e_1\rangle - \mathrm{i}\sin\Omega t|g_1\rangle\Big] \times\Big[\cos\Omega t|g_a\rangle - \mathrm{i}\sin\Omega t|e_a\rangle\Big] - \mathrm{i}\sin\lambda t\Big[\cos\Omega t|g_1\rangle - \mathrm{i}\sin\Omega t|e_1\rangle\Big] \times\Big[\cos\Omega t|e_a\rangle - \mathrm{i}\sin\Omega t|g_a\rangle\Big]\Big] + b\Big[|g_2\dots g_N\rangle\Big[\cos\lambda t \times\Big[\cos\Omega t|g_1\rangle - \mathrm{i}\sin\Omega t|e_1\rangle\Big]\Big[\cos\Omega t|g_a\rangle - \mathrm{i}\sin\Omega t|e_a\rangle\Big] - \mathrm{i}\sin\lambda t \times\Big[\cos\Omega t|e_1\rangle - \mathrm{i}\sin\Omega t|g_1\rangle\Big]\Big[\cos\Omega t|e_a\rangle - \mathrm{i}\sin\Omega t|g_a\rangle\Big]\Big]\Big]. \qquad (16)$$

With $\lambda t = \lambda t_5 = \mathrm{arctg}\dfrac{|b|}{|a|}$ and $\Omega t_5 = k\pi$, after the two atoms escape from the cavity, Alice can detect atom 1, if the atom 1 is in ground state, the atoms $2, 3, \dots, N$ and the auxiliary atom will be in a maximally entangled state:

$$\frac{1}{\sqrt{2}}(|e_ae_2\dots e_N\rangle + \mathrm{i}|g_ag_2\dots g_N\rangle), \qquad (17)$$

where we have discarded the common phase factor. If atom 1 is in an exited state, the process fails. The successful probability is also $P_{\mathrm{succ}} = \dfrac{1}{2}$. The whole process is shown in Fig.1.

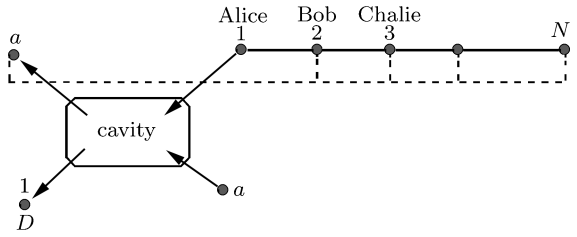


Fig.1. Schematic diagram of the concentration process for N-atom non-maximally entangled states. The atoms located in the solid line denotes the initial entangled ones, and the atoms located in the dashed line denotes the concentrated ones. D denotes a detector.

After a rotation of the auxiliary atom, the state will become

$$\frac{1}{\sqrt{2}}(|e_ae_2\dots e_N\rangle + |g_ag_2\dots g_N\rangle). \qquad (18)$$

We find that we only need to carry out local operation at one location, and need only one auxiliary atom and a cavity as an auxiliary system for the N-atom case, since the auxiliary atom and the atom held by Alice who does the whole concentration process are identical. So in the end, atom 1 in the originally non-maximally entangled state is replaced by the auxiliary atom, and this is feasible. The auxiliary atom and the atoms $2, 3, \dots, N$ will be maximally entangled, but they never interact with each other in the whole concentration process. The scheme we used is a simple one.

4. Conclusions

We have proposed a scheme to concentrate multi-atom non-maximally GHZ class states. Different from the previous ones, first, the current scheme uses the far-off-resonant interaction model, with the assistance of a strong classical field, so that the photon-number-dependent parts in the effective Hamiltonian are cancelled. This allows the cavity in a thermal state, therefore it is more feasible in experiment. Second, in the process of storing the entangled atomic states or quantum dense coding or quantum logic gate, etc., the entangled atoms need not be distributed, thus we can easily amplify the entanglement maximally. In the case of quantum communication, in which the entangled atoms are distributed among distant users, our scheme can concentrate them by using an auxiliary atom. Third, we obtain the maximally entangled states not at the cost of other entanglement sources.

References

[1] Gu Y J, Zheng Y Z, Chen L B and Guo G C 2002 *Chin. Phys. Lett.* **19** 752
[2] Wang X W, Liu X and Fang M F 2006 *Chin. Phys.* **15** 676
[3] Zheng S B 2005 *Chin. Phys.* **14** 1825
[4] Lin X and Li H C 2005 *Chin. Phys.* **14** 1724

[5] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfuter H and Zeilinge A 1997 *Nature* **390** 575

[6] Bennett C H, Brassard G, Popescu S, Schumacher B, Smolin J A and Wootters W K 1996 *Phys. Rev. Lett.* **76** 722

[7] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev.* A **53** 2046

[8] Duan L M, Giedke G, Cirac J I and Zoller P 2000 *Phys. Rev. Lett.* **84** 4002

[9] Pan J W, Gasparoni S, Ursin R, Weihs G and Zeilinger A 2003 *Nature* **423** 417

[10] Romero J L, Roa L, Retamal J C and Saavedra C 2002 *Phys. Rev.* A **65** 052319

[11] Pan J W, Simon C, Brukner C and Zeilinger A 2001 *Nature* **410** 1067

[12] Kwiat P G, Barraza L S, Stefanov A and Gisin N 2001 *Nature* **409** 1014

[13] Murao M, Plenio M B, Popescu S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 4075

[14] Bose S, Vedral V and Knight P L 1999 *Phys. Rev.* A **60** 194

[15] Bennettt C H, DiVincenzo D P, Smolin J A and Wootters W K 1996 *Phys. Rev.* A **54** 3824

[16] Cao Z L and Yang M 2004 *Physica* A **341** 251

[17] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[18] Koashi M, Buzek V and Imoto N 2000 *Phys. Rev.* A **62** 050302

[19] Bollinger J J, Itano W M, Wineland D J and Heinzen D J 1996 *Phys. Rev.* A **54** 4649

[20] Rauschenbeutel A, Nogues G and Osnaghi S 2000 *Science* **288** 2024

[21] Zheng S B 2002 *Phys. Rev.* A **66** 060303

[22] Brink D M and Satchler G R 1975 *Angular Momentum*(Oxford: Clarendon)