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Entangling two single-mode Gaussian states by use of a beam splitter*

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Considering two light beams which are in general single-mode Gaussian states and incident on input ports of an ideal beam splitter, respectively, this paper investigates how separability and entanglement of the output lights depend on degrees of nonclassicality and purities of the input states. The minimum and maximum amounts of attainable entanglement in the output state are found.

Keywords: Gaussian states, entanglement, purity, nonclassicality
PACC: 4250

1. Introduction

The beam splitter as a linear and passive device of generating entangled fields has been extensively studied.[1−4] When each of input modes on a beam splitter is in a single-mode Gaussian state, Kim *et al*[2] showed that for creating an entangled Gaussian state it is necessary that one of the input fields is nonclassical. Separability conditions of the states of the output fields have also been established when each of the input modes is in a squeezed thermal state, or, one input mode is in a squeezed thermal state and another is in a vacuum state, or, one input mode is in a squeezed vacuum state and another is in a thermal state.[2] Wang[3] generalized Kim's conclusion and showed that for any kinds of input states the entangled output state from a beam splitter always requires nonclassicality in the input states. An explicit formula for von Neumann entropy of the reduced density matrix of one of the output modes has also been found when each of the input modes is in the single-mode squeezed vacuum state.[3] Gaussian states belong to one of the important types of quantum states in continuous-variables quantum information processes. On the one hand, Gaussian states are easily prepared in laboratory. On the other hand, a necessary and sufficient condition for separability, and various entanglement measures for Gaussian states have been established.[5] We notice that nonclassicality of one-mode Gaussian states can be quantified[6] and all parameters of a one-mode Gaussian state can be expressed in terms of two physical quantities: nonclassicality degree and purity. In this paper, we consider that two single-mode lights in general Gaussian states are incident on an ideal beam splitter and investigate how separability and entanglement of the output state depend on nonclassicality degrees and purities of the input states. We take the logarithmic negativity[7] as a measure of entanglement. The influence of nonclassicality degrees and purities of the input states, and parameters of the beam splitter on the entanglement of the output state is investigated in detail. The minimum and maximum amounts of attainable entanglement in the output state are found.

2. Nonclassicality of Gaussian states

A s-mode Gaussian state is defined by a Wigner characteristic function of the form

$$\Gamma(\boldsymbol{x}) = \exp\Big(-\frac{1}{2}\boldsymbol{x}^{+}\boldsymbol{\Gamma}\boldsymbol{x}\Big), \tag{1}$$

where

$$\boldsymbol{x}^{+} = [x_1^*, x_1, x_2^*, x_2, \cdots, x_s^*, x_s]$$

with complex amplitudes $x_i (i = 1, 2, ..., s)$, and $\boldsymbol{\Gamma}$ is a covariance matrix. The Glauber and Sudarshan P representation of the state is related to the characteristic function by the integral[8]

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$$P(\boldsymbol{z})=(\pi)^{-2s}\int \mathrm{d}^2\boldsymbol{x}\exp\Big\{\sum_{i=1}^{s}\Big(\frac{|x_i|^2}{2}-x_iz_i^*+x_i^*z_i\Big)\Big\}\Gamma(\boldsymbol{x})=(\pi)^{-2s}\int \mathrm{d}^2\boldsymbol{x}\exp\Big\{-\frac{1}{2}\boldsymbol{x}^+\Big(\boldsymbol{\Gamma}-\frac{1}{2}\Big)\boldsymbol{x}+\boldsymbol{x}^+\boldsymbol{z}\Big\}. \quad (2)$$

If a quantum state is nonclassical, the P representation is either singular or negative definite.[9] This means that for a nonclassical Gaussian state the integral (2) cannot be worked out. For this case, a smooth and positive definite function that becomes acceptable as a classical distribution function may be introduced by the convolution transformation of the P representation[9]

$$R(\tau,\boldsymbol{y})=(\pi\tau)^{-s}\int \mathrm{d}^2\boldsymbol{z} \times\exp\Big(-\frac{1}{\tau}\sum_{i=1}^{s}|z_i-y_i|^2\Big)P(\boldsymbol{z}), \quad (3)$$

where τ is a positive continuous parameter. Substituting Eq.(2) into Eq.(3), we find that the positive definite behaviour of $R(\tau,\boldsymbol{y})$ requires the condition

$$\boldsymbol{\Gamma}+\Big(\tau-\frac{1}{2}\Big)\boldsymbol{I}>0. \quad (4)$$

Let $\lambda_i(\lambda_1\leq\lambda_2\leq\cdots\leq\lambda_{2s})$ be eigenvalues of $\boldsymbol{\Gamma}$. The condition Eq.(4) is equivalent to $\lambda_1+\tau-\frac{1}{2}\geq 0$. The degree of nonclassicality of a Gaussian state is defined as[9]

$$\tau=\max\Big\{0,\frac{1}{2}-\lambda_1\Big\}. \quad (5)$$

Obviously, the degree of nonclassicality of a Gaussian state is in the range $0\leq\tau\leq\frac{1}{2}$.

For a single-mode Gaussian state with the covariance matrix

$$\boldsymbol{\Gamma}=\begin{bmatrix} a & b\\ b^* & a\end{bmatrix}, \quad (6)$$

from Eq.(5), we find that the degree of nonclassicality is given by

$$\tau=\max\Big\{0,-a+|b|+\frac{1}{2}\Big\}. \quad (7)$$

The same result can also be obtained by calculating the Bures distance between the state and the set of all classical one-mode Gaussian ones.[6]

For a one-mode Gaussian state that is described by a density matrix operator $\hat{\rho}$ with a covariance matrix of the form Eq.(6), its purity $u=\mathrm{tr}(\hat{\rho}^2)$ is equal to $1/(2\sqrt{\det\Gamma})$. In terms of the nonclassicality degree τ and the purity u, we can write the elements of the covariance matrix Eq.(6) as

$$a=\frac{1}{2}+\frac{\tau^2+1/(2u)^2-1/4}{1-2\tau}, \quad (8)$$

$$|b|=\frac{\tau-\tau^2+1/(2u)^2-1/4}{1-2\tau}. \quad (9)$$

3. Entanglement generated via a beam splitter

Consider a lossless beam splitter whose input ports are respectively fed by two single-mode fields with complex amplitudes α_1 and α_2. The complex amplitudes of the fields emerging from the output ports are given by [10]

$$\begin{bmatrix}\alpha_1'\\ \alpha_2'\end{bmatrix}=\boldsymbol{M}_{\mathrm{B}}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}, \quad (10)$$

where

$$\boldsymbol{M}_{\mathrm{B}}=\begin{bmatrix}\cos\theta & \sin\theta\mathrm{e}^{\mathrm{i}\varphi}\\ -\sin\theta\mathrm{e}^{-\mathrm{i}\varphi} & \cos\theta\end{bmatrix}. \quad (11)$$

The phase difference φ between the reflected and the transmitted fields is induced by the beam splitter.

Suppose that the input modes are in single-mode Gaussian states with the covariance matrices

$$\boldsymbol{M}=\begin{bmatrix}a_1 & b_1\\ b_1^* & a_1\end{bmatrix}$$

and

$$\boldsymbol{N}=\begin{bmatrix}a_2 & b_2\\ b_2^* & a_2\end{bmatrix}$$

respectively, where a_j are real parameters and $b_j=|b_j|\mathrm{e}^{\mathrm{i}\phi_j}(j=1,2)$. Thus, the characteristic function of the input field is

$$\Gamma(\alpha_1,\alpha_2)=\exp\Big(-\frac{1}{2}\boldsymbol{y}^+\boldsymbol{V}_i\boldsymbol{y}\Big), \quad (12)$$

where $\boldsymbol{y}^+=[\alpha_1^*,\alpha_1,\alpha_2^*,\alpha_2]$ is a row vector of the complex amplitudes of the input modes, and the covariance matrix $\boldsymbol{V}_i$ is given by

$$\boldsymbol{V}_i=\begin{bmatrix}\boldsymbol{M} & 0\\ 0 & \boldsymbol{N}\end{bmatrix}. \quad (13)$$

From the unitary transformation Eq.(10), we see that the output field is in a two-mode Gaussian state with the covariance matrix

$$\boldsymbol{V}_{\rm o} = \mathbf{L}_{\rm B}^{+}\boldsymbol{V}_i\boldsymbol{L}_{\rm B} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{C} \\ \boldsymbol{C}^{+} & \boldsymbol{B} \end{bmatrix}, \tag{14}$$

where

$$\boldsymbol{L}_{\rm B} = \boldsymbol{R}^{-1}\begin{bmatrix} M_{\rm B} & 0 \\ 0 & M_{\rm B}^{*} \end{bmatrix}\boldsymbol{R}, \tag{15}$$

and

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

In Eq.(14), 2×2 matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\mathbf{C}$ are given by

$$\begin{aligned} \boldsymbol{A} &= \begin{bmatrix} a_1\cos^2\theta + a_2\sin^2\theta & b_1\cos^2\theta + b_2\sin^2\theta \mathrm{e}^{2\mathrm{i}\varphi} \\ b_1^{*}\cos^2\theta + b_2^{*}\sin^2\theta\mathrm{e}^{-2\mathrm{i}\varphi} & a_1\cos^2\theta + a_2\sin^2\theta \end{bmatrix}, \\ \boldsymbol{B} &= \begin{bmatrix} a_1\sin^2\theta + a_2\cos^2\theta & b_2\cos^2\theta + b_1\sin^2\theta\mathrm{e}^{-2\mathrm{i}\varphi} \\ b_2^{*}\cos^2\theta + b_1^{*}\sin^2\theta\mathrm{e}^{2\mathrm{i}\varphi} & a_1\sin^2\theta + a_2\cos^2\theta \end{bmatrix}, \\ \boldsymbol{C} &= \cos\theta\sin\theta\begin{bmatrix} (a_1-a_2)\mathrm{e}^{\mathrm{i}\varphi} & b_1\mathrm{e}^{-\mathrm{i}\varphi} - b_2\mathrm{e}^{\mathrm{i}\varphi} \\ b_1^{*}\mathrm{e}^{\mathrm{i}\varphi} - b_2^{*}\mathrm{e}^{-\mathrm{i}\varphi} & (a_1-a_2)\mathrm{e}^{-\mathrm{i}\varphi} \end{bmatrix}. \end{aligned} \tag{17}$$

Any physical state must satisfy Heisenberg uncertainty relation. For a two-mode Gaussian state with a covariance matrix $\boldsymbol{V}$, the Heisenberg uncertainty relation has the compact form[11,12]

$$\boldsymbol{V} + \frac{1}{2}\boldsymbol{E} \geq 0, \tag{18}$$

where

$$\boldsymbol{E} = \boldsymbol{E}_1 \oplus \boldsymbol{E}_1$$

with

$$\boldsymbol{E}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Peres[13] showed that if a quantum state is separable the partial transpose of the state is still positive. It means that the partial transposed state must obey the Heisenberg uncertainty relation. This requirement is equivalent to the condition [11,12]

$$\boldsymbol{V} + \frac{1}{2}\boldsymbol{T}_1\boldsymbol{E}\boldsymbol{T}_1 \geq 0, \tag{19}$$

where $\boldsymbol{T}_1$ is the partial transposition operator with the matrix form

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Simon showed that Eq.(19) is a necessary and sufficient criterion for separability of a two-mode Gaussian state.[11,14] Substituting Eq.(14) into Eq.(19), we obtain the separability condition of the output state

$$\begin{aligned} &2(|b_1|^2 + |b_2|^2)(1+\cos 4\theta) - 2(a_1-a_2)^2\cos 4\theta \\ &\quad + 4|b_1||b_2|(1-\cos 4\theta)\cos(\phi_1 - \phi_2 - 2\varphi) \\ &\geq 2(a_1+a_2)^2 - 16(a_1^2 - |b_1|^2)(a_2^2 - |b_2|^2) - 1. \end{aligned} \tag{20}$$

From Eq.(20), we see that the separability condition depends only on the difference between the phase difference of the input states and the beam splitter phase. In the present discussion, we consider the cases with two different choices of the beam splitter phase

$$\text{case 1}: \quad \varphi = (\phi_1 - \phi_2)/2, \tag{21}$$

$$\text{case 2}: \quad \varphi = (\phi_1 - \phi_2)/2 - \pi/2. \tag{22}$$

In the case 1(2), the third term of the left side of the inequality Eq.(20) takes the maximal (minimal) value for given input fields and a fixed incident angle θ. Therefore, the case 2(1) may mean the maximal (minimal) violation of the inequality Eq.(20) for the given conditions. We shall show that the case 1 and the case 2 provide the lowest and highest bounds of attainable entanglement in the output state respectively.

For the case 1, upon substitution of a_i and b_i with u_i and τ_i according to Eqs.(8) and (9), the separability condition (20) can be written as

$$c_{1s} = 1 - (u_1^2 + u_2^2) + u_1^2u_2^2 - \mu_1 - \mu_2\cos 4\theta \geq 0, \tag{23}$$

where

$$\mu_1 = 2(1+u_1^2u_2^2)(\tau_1+\tau_2) + 2\tau_1\tau_2(u_1^2+u_2^2-2) + 2(u_1\tau_1-u_2\tau_2)^2 - u_1^2(3\tau_1+\tau_2) - u_2^2(\tau_1+3\tau_2), \quad (24)$$

$$\mu_2 = (u_1^2-u_2^2)(\tau_1-\tau_2) + 2\tau_1\tau_2(u_1^2+u_2^2) - 2(u_1^2\tau_1^2+u_2^2\tau_2^2). \quad (25)$$

When $\tau_1 = \tau_2$, from Eqs.(24) and (25), we have $\mu_1 = 4\tau(1-\tau)(1-u_1^2)(1-u_2^2)$ and $\mu_2 = 0$. Thus,

$$c_{1s} = (1-u_1^2)(1-u_2^2)[1-4\tau(1-\tau)] \geq 0.$$

Therefore, in case 1, the output state is separable if the input states have the same nonclassicality degree. This conclusion holds as long as the input modes are in single-mode Gaussian states which may be mixed or pure. As a special case of the present situation, Kim *et al.*[2] showed that the output state can not become entangled as the input fields are in the pure squeezed states with the same real squeezing parameter and the beam splitter phase $\varphi = 0$. When $\mu_2 \geq 0$, the left side of the inequality Eq.(23) takes the minimal value

$$c_{1s} = (1-u_1^2)(1-u_2^2)(1-2\tau_1)(1-2\tau_2) \geq 0, \quad (26)$$

at the phase $\theta = 0$. Therefore, the output state must be separable when the input states have the property $\mu_2 \geq 0$. When $\tau_1 \neq \tau_2$ but $u_1 = u_2 = u$, we have $\mu_2 = -2u^2(\tau_1-\tau_2)^2 < 0$. In this case, the inequality Eq.(23) becomes

$$c_{1s} = (1-u^2)^2[1-2(\tau_1+\tau_2)] + 4\tau_1\tau_2(1-u^2)^2 - 2u^2(\tau_1-\tau_2)^2(1-\cos 4\theta) \geq 0. \quad (27)$$

If both of the input states are pure, i.e. $u = 1$, this inequality is always violated whenever $\theta \neq 0, \theta \neq \pi/2$, and $\tau_1 \neq \tau_2 \neq 0$. Therefore, in this case, the output state must be entangled if the input states are pure. It is noticed that the third term of the left side of the inequality Eq.(27) has the maximal value at $\theta = \pi/4$. This means that the inequality is maximally violated at $\theta = \pi/4$. For various values of u, the left side of the inequality Eq.(27) at $\theta = \pi/4$ is plotted as a function of $\Delta = \tau_1 - \tau_2$ in Fig.1. We observe that except with $u = 1$ the output state becomes entangled only when Δ is beyond a certain value. The smaller the purity is the larger the required Δ for the output state to be entangled is. From Fig.1, we also observe that for a given Δ the output state may not be entangled if the purity u is too small.

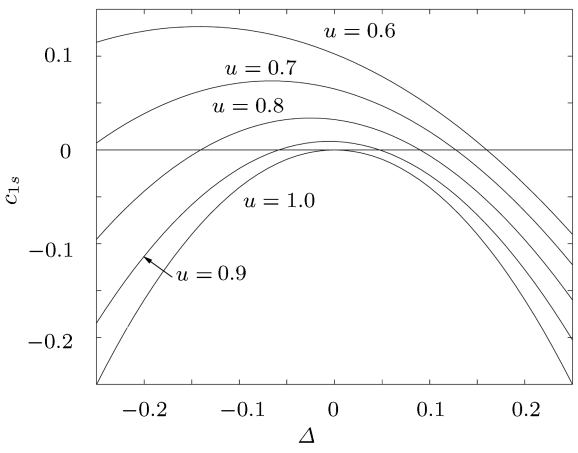


Fig.1. c_{1s} versus $\Delta = \tau_1 - \tau_2$ with $\tau_2 = 0.25$ and $u_1 = u_2 = u = 0.6, 0.7, 0.8, 0.9, 1.0$.

For the case 2, on substitution of a_i and b_i with u_i and τ_i according to Eqs.(8) and (9), we can write the separability condition Eq.(20) as

$$c_{2s} = \lambda_1 + \lambda_2 \cos 4\theta \geq 0, \quad (28)$$

where

$$\lambda_1 = 1 - 4(\tau_1+\tau_2+2\tau_1\tau_2) - (1-2\tau_1)(1-2\tau_2) \times [u_1^2+u_2^2-u_1^2u_2^2(1+2\tau_1+2\tau_2-4\tau_1\tau_2)],$$
$$\lambda_2 = [1-u_1^2(1-2\tau_1)(1-2\tau_2)] \times [1-u_2^2(1-2\tau_1)(1-2\tau_2)]. \quad (29)$$

It is obvious that $\lambda_2 \geq 0$. Thus, the separability condition Eq.(28) may be maximally violated at $\theta = \pi/4$. In this case, we have the separability condition

$$c_{2s}\left(\theta = \frac{\pi}{4}\right) = -4[\tau_1(1-\tau_1)+\tau_2(1-\tau_2)] \times [1-u_1^2u_2^2(1-2\tau_1)(1-2\tau_2)] \geq 0. \quad (30)$$

Therefore, it is always violated provided one of the input states is nonclassical. It is noticed that the violation does not depend on the purities of the input states.

We now turn to discuss entanglement measure of the output state. For a two-mode Gaussian state with a covariance matrix $\boldsymbol{V}$ of the form Eq.(14), the logarithmic negativity is introduced to qualitatively measure entanglement,[7] which is defined as

$$E_N = \max\{0, -\log_2 2c_-\}, \quad (31)$$

where

$$2c_-^2 = \Sigma(\boldsymbol{V}) - \sqrt{[\Sigma(\boldsymbol{V})]^2 - 4\det \boldsymbol{V}}, \quad (32)$$

and $\Sigma(\boldsymbol{V}) = \det \boldsymbol{A} + \det \boldsymbol{B} - 2\det \boldsymbol{C}$. From Eqs.(13) and (14), we find

$$\begin{aligned}\det \boldsymbol{V}_{\rm o} =& \det \boldsymbol{V}_i = (a_1^2 - |b_1|^2)(a_2^2 - |b_2|^2) \\ =& \frac{1}{16u_1^2u_2^2} \end{aligned} \tag{33}$$

Therefore, $\det \boldsymbol{V}_{\rm o}$ is determined only by the purities of the input states and the determinant of the covariance matrix of a Gaussian state is invariant under the unitary transformation Eq.(11). Fixing the $\det \boldsymbol{V}$, from Eq.(32), we have

$$\frac{\partial c_-^2}{\partial \Sigma(\boldsymbol{V})} = \frac{1}{2}\Big[1 - \frac{\Sigma(\boldsymbol{V})}{\sqrt{[\Sigma(\boldsymbol{V})]^2 - 4\det \boldsymbol{V}}}\Big] < 0. \tag{34}$$

Therefore, the entanglement measure Eq.(32) is a monotonically increasing function of $\Sigma(\boldsymbol{V})$.

From Eq.(14), we find

$$\begin{aligned}\Sigma(\boldsymbol{V}_{\rm o}) =& \frac{1}{2}(a_1^2 + a_2^2 - |b_1|^2 - |b_2|^2)(1 + \cos 4\theta) \\ &+ a_1a_2(1 - \cos 4\theta) - |b_1||b_2|(1 - \cos 4\theta) \\ &\times \cos(\phi_1 - \phi_2 - 2\varphi). \end{aligned} \tag{35}$$

It is seen that $\Sigma(\boldsymbol{V}_{\rm o})$ has the minimal(maximal) value in the case 1(2) when the angle θ is fixed. Therefore, for a given value of θ, the entanglement of the output state is maximized in the case 2 and minimized in the case 1.

For the case 1, from Eq.(35), we have

$$\Sigma(\boldsymbol{V}_{\rm o}) = \frac{\nu + \mu_2 \cos 4\theta}{4u_1^2u_2^2(1 - 2\tau_1)(1 - 2\tau_2)}, \tag{36}$$

where

$$\begin{aligned}\nu =& 2(u_1^2\tau_1^2 + u_2^2\tau_2^2) + (2\tau_1\tau_2 + 1)(u_1^2 + u_2^2) \\ &- u_1^2\tau_2 - u_2^2\tau_1 - 3u_1^2\tau_1 - 3u_2^2\tau_2. \end{aligned} \tag{37}$$

According to the above discussion about separability of the output state, the entanglement must be zero if $\mu_2 \geq 0$. When $\mu_2 < 0$, Equation (36) has the maximal value at $\theta = \pi/4$. Therefore, the entanglement is maximized at $\theta = \pi/4$. Figure 2 shows the entanglement as a function of u and τ_1 with $\tau_2 = 0.25$ and $\theta = \pi/4$. The entanglement is plotted as a function of the purities with $\tau_1 = 0.2$, $\tau_2 = 0.45$ and $\theta = \pi/4$ in Fig.3.

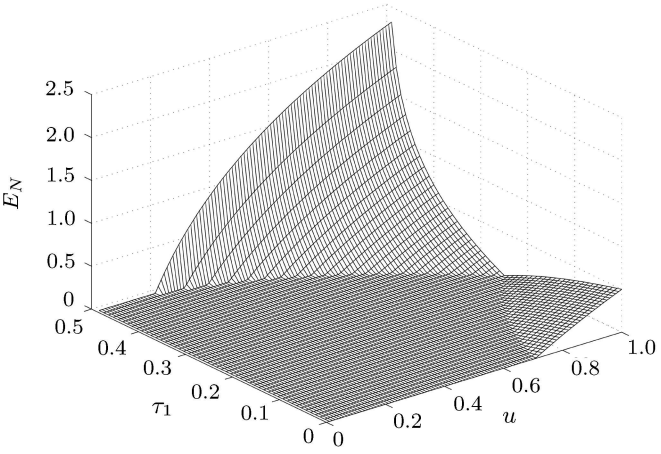


Fig.2. The entanglement in the case 1 as a function of u and τ_1 with $\tau_2 = 0.25$.

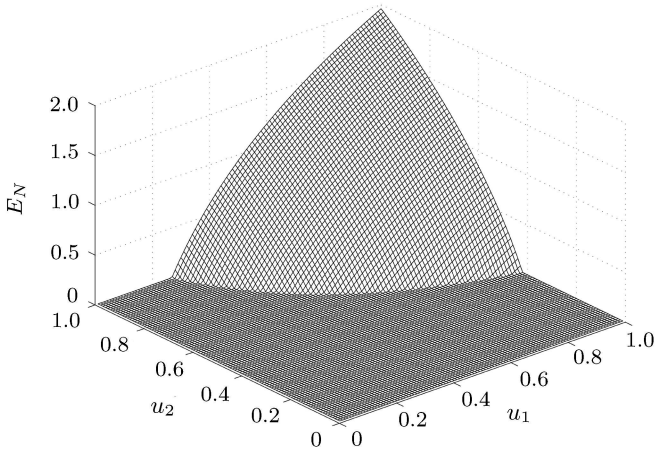


Fig.3. The entanglement in the case 1 as a function of u_1 and u_2 with $\tau_1 = 0.2$, $\tau_2 = 0.45$.

From these figures, we observe that the difference between degrees of nonclassicality of the input states is required for nonzero entanglement in the output state and the purities must be beyond a certain value for the nonzero entanglement when the difference between degrees of nonclassicality of the input states is given.

For the case 2, from Eq.(36), we have

$$\Sigma(\boldsymbol{V}_{\rm o}) = \frac{\kappa - \lambda_2 \cos 4\theta}{8u_1^2u_2^2(1 - 2\tau_1)(1 - 2\tau_2)}, \tag{38}$$

where

$$\kappa = [1 + u_1^2(1 - 2\tau_1)(1 - 2\tau_2)]$$

$$\times [1 + u_2^2(1 - 2\tau_1)(1 - 2\tau_2)]. \quad (39)$$

From Eq.(29), it is easily seen that λ_2 is always larger than zero. Therefore, Eq.(38) is maximized at $\theta = \pi/4$. From Eq.(31), we find that for given input states the maximal entanglement of the output state is

$$E_N = -\frac{1}{2}\log_2(1 - 2\tau_1)(1 - 2\tau_2). \quad (40)$$

Thus, the entanglement is only determined by the nonclassicality of the input states.

4. Summary

We investigate the influence of nonclassicality degrees and purities of the input states on separability and entanglement of the output state, when two single-mode lights (in general Gaussian states) are incident on an ideal beam splitter. The necessary and sufficient condition for separability of the output state has been established, which is expressed in terms of nonclassicality degrees and purities of the input states, and parameters of the beam splitter. As a measure of entanglement of the output state, the logarithmic negativity is calculated. We find that the entanglement is maximized(minimized) for a 50 : 50 beam splitter when the phase difference of the input states differs from the beam splitter induced phase by $\pi/2(0)$. An explicit formula for the maximum entanglement has been found, which depends on the nonclassicality degrees of the input states but are irrelative to purities of the input states. We also find that in contrast to the maximum amount of the attainable entanglement the minimum one depends on the purities of the input states and the difference between degrees of nonclassicality of the input states. For a fixed value of the nonclassicality difference, the minimum attainable entanglement becomes nonzero only when the purities are larger than a certain value.

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