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Open-loop frequency response for a chaotic masking system*

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In this paper, a new numerical simulation approach is proposed for the study of open-loop frequency response of a chaotic masking system. Using Chua's circuit and the Lorenz system as illustrative examples, we have shown that one can employ chaos synchronization to separate the feedback network from a chaotic masking system, and then use numerical simulation to obtain the open-loop synchronization response, the phase response, and the amplitude response of a chaotic masking system. Based on the analysis of the frequency response, we have also proved that changing the amplitude of the exciting (input) signal within normal working domain does not influence the frequency response of the chaotic masking system. The new numerical simulation method developed in this paper can be extended to consider the open-loop frequency response of other systems described by differential or difference equations.

Keywords: open-loop, frequency response, chaotic masking system
PACC: 0545, 0547, 4752

1. Introduction

Due to the potential application of chaos synchronization in secure communications,[1−4] various methods to realize chaos synchronization have been proposed and studied in the past decade.[5−10] However, practical applications such as real time, audio communication system using chaos synchronization have not been applicable because of lacking understanding of some fundamental characteristic of chaotic systems. The white Gaussian noise channel is the most basic communication channel. In order to efficiently reduce the noisy chaotic signal[11,12] and perform the noise-induced synchronization[13,14] when a chaotic signal is transmitted via a white Gaussian noise channel, it is necessary to study the frequency response of chaotic systems. Although some methods[15−17] have been developed to explore that the frequency response of a chaotic masking system has impact on masking signal, the methods[15,16] did not fully reveal the fundamental properties of frequency response of chaotic systems. In Ref.[17] Chen and Pei obtained an analytic solution using the error equations and used it to study the characteristic of frequency response of synchronized Chua's circuit. However, it is difficulty to generalize this approach to investigate the frequency response of other chaotic systems. In this paper, we shall use the basic idea of chaos synchronization and separate the feedback network from a chaotic system to study the open-loop frequency response of a chaotic masking system via numerical simulations. In particular, we shall focus on Chua's circuit and the Lorenz system.

Consider the following general differential equation:

$$\dot{X} = f(X), \tag{1}$$

where $X \in R^n$. The solution to Eq.(1) can be obtained using an integral condition, and the evolving sequence of the state vector can be described by the iterative process $X, f(X), f(f(X)), \ldots$. This suggests that a differential equation may be considered as a system with an internal feedback. It is easy to compute the frequency response of the closed-loop system, but how to obtain its open-loop frequency response? The open-loop frequency response is the frequency response of the system without feedback, or that obtained by disconnecting the feedback (i.e., the closed-

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loop is broken). The dynamical system (1) inherently contains a 'feedback' which is a part of the system, how do we cut off the 'feedback'? Chaotic synchronization provides a method to solve the problem, since two dynamical systems become synchronized if and only if the states of the two systems possess the same evolution form. Therefore, we may use chaos synchronization to find the open-loop frequency response of a chaotic masking system. In the following, we use Chua's circuit to illustrate the idea.

2. The open-loop frequency response of chaotic masking system based on Chua's circuit

The block diagram showing the open-loop frequency response of chaotic masking system by Chua's circuit is depicted in Fig.1, where the driving system (transmitter) is denoted by the subscript d, while the response (driven) system (receiver) by the subscript r. The driving system is given by

$$\begin{aligned} \dot{x}_{\rm d} &= \alpha[y_{\rm d} - g(x_{\rm d})], \\ \dot{y}_{\rm d} &= x_{\rm d} - y_{\rm d} + z_{d}, \\ \dot{z}_{\rm d} &= -\beta z_{\rm d}, \end{aligned} \tag{2}$$

where α,β, a and b are parameters, and g is Chua's diode, defined by

$$g(x_{\rm d}) = bx_{\rm d} + \frac{1}{2}(a-b)(|x_{\rm d}+1| - |x_{\rm d}-1|). \tag{3}$$

When $\alpha = 9$, $\beta = 14.2$, $a = -1/7$, $b = 2/7$, the system exhibits chaotic behaviour. The response system is described by

$$\begin{aligned} \dot{x}_{\rm r} &= \alpha[y_{\rm r} - g(x_{\rm r})], \\ \dot{y}_{\rm r} &= s(t) - y_{\rm r} + z_{\rm r}, \\ \dot{z}_{\rm r} &= -\beta z_{\rm r}, \end{aligned} \tag{4}$$

where $s(t) = x_{\rm d}(t) + m_{\rm i}(t)$, with $m_{\rm i}(t)$ the (external) exciting signal, is sent out from the transmitter to drive the response system. In fact, the above driving-response system is the same as the chaotic masking system designed by Cuomo *et al.*[5] It is seen from Fig.1 that the synchronization (driving) signal $x_{\rm d}$ can be considered as a bias signal of the response chaotic system just as the transistor's bias voltage. A bias signal $x_{\rm d}$ combined with an exciting signal, $m_{\rm i}(t)$, is used as an input to the response system. Because the output signal, $x_{\rm r}(t)$, of the response chaotic system is no longer fed back to the input, we may regard the response system as an open-loop system of a chaotic masking system and the frequency response of the response system as an open-loop frequency response.

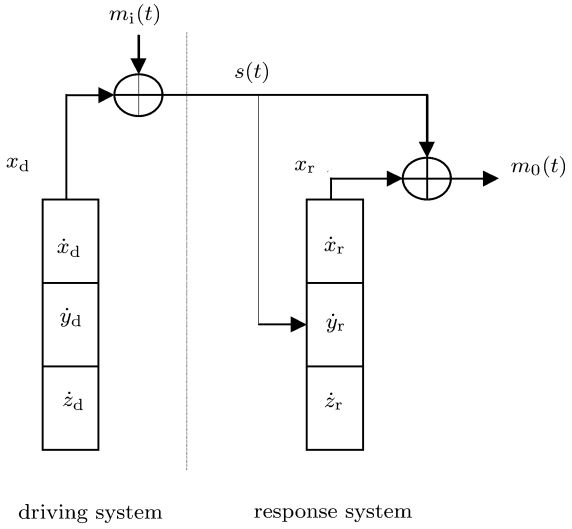


Fig.1. The block diagram showing the open-loop frequency response of Chua's circuit.

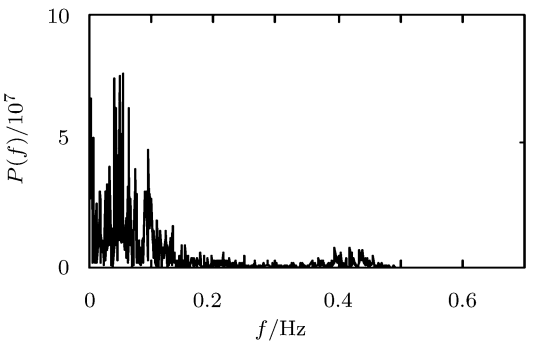


Fig.2. The power spectrum of Chua's circuit.

A basic idea can be clearly seen from the above discussion: we consider the driving and response systems as a whole system. When the two subsystems are synchronized, the whole system behaves like a single system. Since the output signal from the response system is not fed back to the input of the driving system, the response system can then be considered as an open-loop system. Next, we use a numerical integration scheme to investigate the open-loop frequency response of Chua's circuit. The numerical approach is based on a fourth-order Runge–Kutta method, with integration step size $h = 0.02$ (the sampling time step size of the exciting signal is also taken as $h = 0.02$).

We adopt the identical initial values for the driving system and the response system so that it does not have transient behaviour for the response system. The total number of integration steps, N, is taken to be 20,000. The power spectrum of Chua's circuit has been obtained, as shown in Fig.2. It is seen that the frequencies are mainly located in the interval [0, 0.5]. For simplicity in the following analysis, without loss of generality, we assume that the external exciting signal of the response system is a simple sinusoidal wave with amplitude $A_{\rm i}$, frequency f, and phase $\varphi_{\rm i}(t) = 0$. Therefore, $m_{\rm i}(t)$ can be written as

$$m_{\rm i}(t) = A_{\rm i}\sin[2\pi ft + \varphi_{\rm i}(t)]A_{\rm i}\sin(2\pi ft),$$
$$0 \le f < \infty. \tag{5}$$

This exciting signal is then added to the synchronization signal $x_{\rm d}(t)$, and thus the input signal to the response system is

$$s(t) = x_{\rm d}(t) + m_{\rm i}(t), \tag{6}$$

where $x_{\rm d}(t)$ is the chaos output signal of the driving system, which may be represented by

$$x_{\rm d}(t) = \sum_{f=0}^{\infty} A_{\rm i}(f)\sin[2\pi ft + \varphi(f)],$$
$$f_{\rm L} \le f \le f_{\rm H}. \tag{7}$$

It is seen from Fig.2 that $f_{\rm L} = 0$ and $f_{\rm H} = 0.5$. The output signal, $m_0(t)$, of the response system can be obtained by subtracting the output signal, $x_{\rm r}(t)$, of the response chaotic system from the input signal, $s(t)$, i.e.

$$m_0(t) = s(t) - x_{\rm r}(t)\,. \tag{8}$$

If the response chaotic system has very good damping property for the exciting signal, $m_{\rm i}(t)$, in frequency domain considered, then $m_{\rm i}(t)$ in the bias signal cannot be transferred to the output, but the bias signal can. Roughly speaking, the 'damping property' of a chaotic masking system means that it has the good property to 'filter' the signal considered in a certain frequency domain (this will be illustrated in Fig.8.). This result in the output signal of the response chaotic system, $x_{\rm r}(t) = x_{\rm d}(t) + m_{\rm i}(t)$, approx $x_{\rm d}(t)$, and the output signal of the open-loop frequency response system, $m_0(t)$, approx. $m_{\rm i}(t)$ obtained from Eqs.(6) and (8) (shown in Fig.1). This is the basic principle of chaos masking system introduced by Cuomo *et al.*[5]

Now let us further investigate the output signal, $m_0(t)$, which is considered as the response to the external exciting signal, $m_{\rm i}(t)$. It is well known that the output of a linear system to a sinusoidal input is a sinusoidal wave with the same frequency, though having different magnitude and phase. However, for a (nonlinear) chaotic system, this is no longer true. It can be seen from Fig.2 that the frequency spectrum of a chaotic masking system illustrates the nonlinear characteristic of the chaotic system. We will consider the synchronization signal $x_{\rm d}(t)$ as a bias signal to the response chaotic system , which is added by the exciting signal. Thus the exciting signal, $m_{\rm i}(t)$, will disturb the synchronization response of the response system for the drive system. To a sinusoidal input, the output of the response chaotic system is no longer a single tone wave, but almost continuous frequency wave. Furthermore, it has been observed using various testing external sinusoidal signals with different frequencies, that the amplitude and phase of the chaotic frequency response are functions of the external frequency. To analyse the frequency characteristic of a chaotic masking system, Eq.(8) can be written as

$$m_0(t) = A_0(f)\sin[2\pi ft + \varphi_0(f)], \quad 0 \le f < \infty. \tag{9}$$

The corresponding transfer function can be expressed as

$$H(\mathrm{j}f) = m_0(\mathrm{j}f)/m_{\rm i}(\mathrm{j}f) = H(f)\mathrm{e}^{\mathrm{j}\varphi(f)},$$
$$0 \le f < \infty, \tag{10}$$

where $\mathrm{j}f$ is the complex form of the frequency. Because the exciting signal is a single frequency wave, its amplitude $A_{\rm i}$ is a constant and thus the module of the transfer function can be expressed as $H(f) = A_0(f)/A_{\rm i}$, which indicates that the transfer function $H(f)$ and the amplitude $A_0(f)$ of the response signal $m_0(f)$ are in proportion with a constant ratio.

Similarly, the phase response can be written as $\varphi(f) = \varphi_0(f)$ due to $\varphi_{\rm i}(t) = 0$, which is actually the phase of $m_0(f)$. Besides the usual amplitude and phase responses, there exists another important response in the study of chaotic systems, which is called "synchronization response". This is an important measurement for the distortion of the response signal with respect to the exciting signal, which is a function of frequency, given by

$$\eta(f) = I_{\max}(f)/N, \tag{11}$$

where $N = 20,000$ is the total number of sampling data of the time series. $I_{\max}(f)$ is the total number of synchronous points between the response signal and the exciting signal for a given frequency f under a synchronous threshold value d which is adjusted to reach

an optimal synchronous response. The total number of the synchronous points is

$$I_{\max}(f) = \sum_{i=1}^{N} W(i), \tag{12}$$

where

$$W(i) = \begin{cases} 1 & \text{if } \|m_0(i) - m_{\rm i}(i)\| \le d \\ 0 & \text{otherwise,} \end{cases} \quad (1 \le i \le N). \tag{13}$$

One must adjust the amplitude and phase of the output signal of the response system in order to obtain the best synchronization response for a given exciting signal with frequency f and amplitude $A_{\rm i}$. The amplitude response, $H(f)$, and the phase response, $\varphi(f)$, can be found once the synchronous response $\eta(f)$ is measured.

In this study, the frequency f of the exciting signal is chosen from the interval $[0, 10\text{Hz}]$, which covers the frequency spectrum of Chua's chaotic circuit. In all numerical simulations, the exciting signal of the response system is a single-tone wave with amplitude $A_{\rm i} = 0.3$ (modulation ratio $= -16.6885$ dB), and the phase $\varphi_{\rm i}(t) = 0$, i.e.

$$m_{\rm i}(t) = 0.3\sin(2\pi f t). \tag{14}$$

In order to understand the numerical results, we plot the synchronization response curve, $\eta(f)$, the phase response curve, $\varphi(f)$, and the amplitude response curve, $H(f)$, in Figs.4–6, respectively. Now, we describe the process of numerical simulation to show how to obtain the best threshold value d. For a given frequency, $f = f_0$, of the exiting signal determined by Eq.(14), assume a threshold value $d = d_0$. We then move the sampling window of the output signal, $m_0(t)$, right or left to find the phase deviation to obtain the largest number of synchronous points, denoted by $I_{\max}(f_0)$ between the input and the output signals. The phase response curve, $\varphi(f_0)$ can be calculated based on the total number of sampling of the exciting signal, $1/hf_0$, within one period. Further, we add an adjusting factor, α to the output signal $m_0(t)$, to define a new output signal $m_0'(t) = \alpha m_0(t)$ and then change α such that the number of synchronous points reaches the maximum, $I_{\max}(f_0)$, from which we obtain the amplitude response, $H(f_0)$. Finally, we obtain the synchronization response $\eta(f_0)$ using Eqs.(11)–(13). We adjust the threshold value d, based on the observation of synchronization response by repeating a number of numerical simulations. The numerical simulation result is shown in Fig.3. It can be seen that the synchronization response is mainly located in the interval $[0.04A_{\rm i}, 0.07A_{\rm i}]$. The synchronization response reduces rapidly as d decreases for $d < 0.04A_{\rm i}$, and slowly increases as d increases for $d > 0.07A_{\rm i}$. We then arbitrarily choose a value of d in the interval $[0.04A_{\rm i}, 0.07A_{\rm i}]$, say $d = 0.0667A_{\rm i}$, and compute $\varphi(f_0)$, $H(f_0)$ and $\eta(f_0)$ accordingly. For example, suppose $f = 0.4$ Hz, move the output signal, $m_0(t)$, to right by 18 points. When the number of the synchronous points reaches its maximum, we calculate $\varphi(f = 0.4\text{ Hz}) = -2\pi \times 18 \times h \times f = -2\pi \times 18/125 = -0.288\pi$. Tuning the factor $\alpha = 1.96$ in $m_0'(t) = \alpha m_0(t)$, we find that the number of the synchronous points reach the maximum $I_{\max}(f = 0.4\text{ Hz}) = 8,644$, and thus we have the amplitude response $H(f = 0.4\text{ Hz}) = 1/1.96 = 0.5102$. Finally, we have the synchronization response $\eta(f = 0.4Hz) = I_{\max}(f = 0.4\text{Hz})/N = -8,6400/20,000 = -0.4322$. It is seen from Fig.4 that the worst synchronization response appears at $f = 0.5$ Hz, and the synchronization response $\eta(f)$ monotonically increases as the difference, $|f - 0.5\text{Hz}|$, increases. The response $\eta(f)$ tends to unity as f approaches 0 or 3.

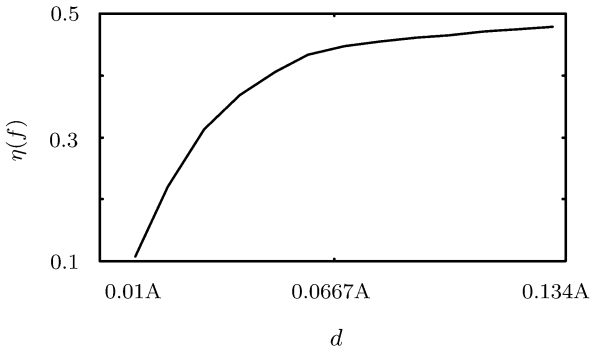


Fig.3. The synchronization response vs threshold value in Chua's circuit.

Figure 5 depicts the phase response of Chua's chaotic system, which shows a jump in the interval $f \in [0.5\text{Hz}, 0.55\text{Hz}]$. It is observed that the phase of the response signal lags behind the phase of the exciting signal for $f \le 0.5$ Hz, and the phase difference is measured as negative (indicating the phase lag). The negative phase difference reaches its peak at $f = 0.5$ Hz, and $|\varphi(f)|$ decreases continuously as the f decreases from 0.5 Hz to 0, and reaches its minimum, 0, at $f = 0$. The phase $\varphi(f)$ has a positive peak at $f = 0.55$ Hz. For $f \ge 0.55$ Hz as f increases, $\varphi(f)$ decreases continuously, and reaches 0 around $f = 1.0$ Hz. Figure 6 shows the amplitude response curve. It is

seen that the trend is almost the same as that of the phase response (see Fig.5), except for the scaling on the vertical axis. The amplifying coefficient, $H(f)$, equals 1 at $f = 0$, and then decreases continuously for $f \in [0.0\text{Hz}, 0.5\text{Hz}]$, and finally reaches its minimum value at $f = 0.5$ Hz. It has a jump in the range of $f \in [0.5\text{Hz}, 0.55\text{Hz}]$. When $f > 0.55$ Hz, $H(f)$ decreases continuously and equals 0.97 around $f = 3$ Hz.

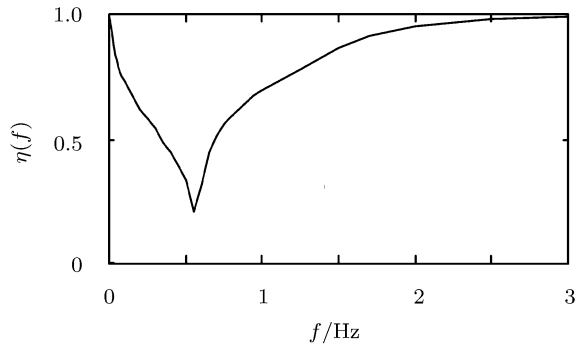


Fig.4. The synchronization response of Chua's circuit.

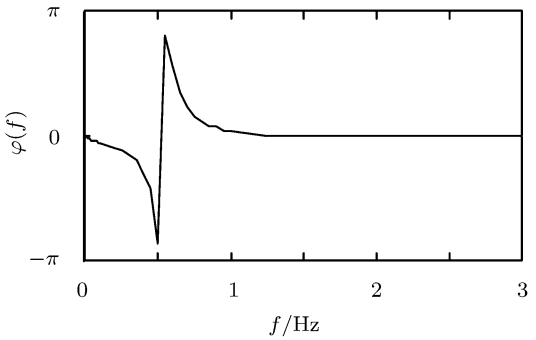


Fig.5. The phase response of Chua's circuit.

To have a better view on the frequency response characteristic near $f = 0$ and $f = 3$ Hz (which are the two marginal areas of the responses shown in Figs.5 and 6), we show some numerical results in Figs.7 and 8. Figure 7 shows the exciting signal, $m_{\text{i}}(t)$, and the response signal, $m_0(t)$, where the frequency and amplitude of $m_0(t)$ are taken to be $f = 0.1$ Hz and $A_{\text{i}} = 0.3$, respectively. Note that for a clear view, the response curve, $m_0(t)$, has been shifted down by 1 unit. It is seen that the output signal of the response system is similar to the exciting signal but with some small amplitude, noise-type disturbances, which can be completely removed using an appropriate filter, and thus fully recover the exciting signal, $m_{\text{i}}(t)$. Figure 8, on the other hand, depicts $m_{\text{i}}(t)$ and $m_0(t)$ of the response system at $f = 3$ Hz with $A_{\text{i}} = 0.3$. Again, the response signal $m_0(t)$ is shifted down by 0.5 unit for a clear view. It shows clearly that the response signal is almost identical to the exciting signal. The amplitude and phase of the response signal, $m_0(t)$, do not change, as compared with that of the exciting signal, $m_{\text{i}}(t)$. Figure 8 also shows that high-frequency 'noise' have been filtered, for which the frequency of the exciting signal is beyond the frequency domain of the response chaotic system. This implies that the response chaotic system has very good damping property for high-frequency signals. But the chaotic system is not good for relatively low high-frequency signals in the frequency domain of the system. The input signal will change the frequency response if it is in the frequency domain of the response chaotic system.

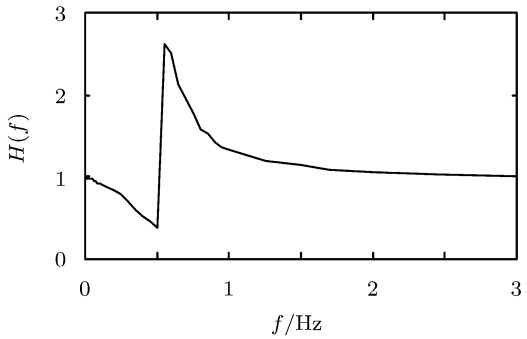


Fig.6. The amplitude response of Chua's circuit.

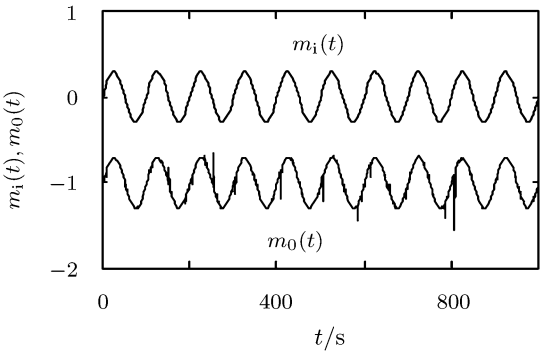


Fig.7. The time sequences of the exciting signal with $A_{\text{i}} = 0.3$, $f_{\text{i}} = 0.01$ Hz, and the output signal of the response system for Chua's circuit.

Next, we consider the relation between the synchronization response and the amplitude of the exciting signal. The frequency of the exciting signal is fixed at $f = f_0$, but the amplitude, A_{i}, is varied from 10^{-13} to 3.0, which yields the modulation ratio from -266.23 to 3.3115 dB. Here, the synchronous threshold value is still chosen to be $d = 0.0667A_{\text{i}}$. Then, for a given input frequency $f_0 = 0.05$ Hz, $\varphi(A_{\text{i}})$ and $H(A_{\text{i}})$

are adjusted to obtain an optimal synchronization response $\eta(A_{\rm i})$. Numerical simulation results indicate that $\eta(A_{\rm i})$ has very little changes with respect to $A_{\rm i}$ in the given interval. For example, it has been found that $0.8216 < \eta(A_{\rm i}) < 0.8228$ for $10^{-13} < A_{\rm i} < 0.1$. However, the response system loses synchronization at $A_{\rm i} = 10^{-16}$.

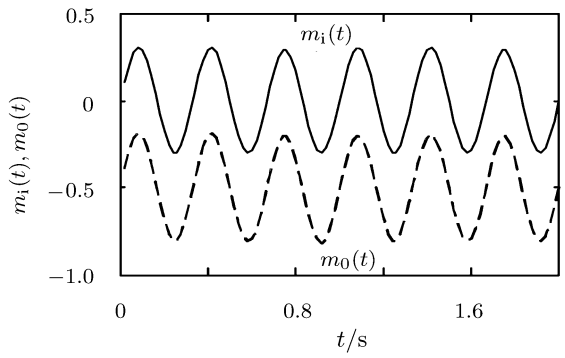


Fig.8. The time sequences of the exciting signal with $A_{\rm i} = 0.3$, $f_{\rm i} = 3$ Hz, and the output signal of the response system for Chua's circuit.

3. The open-loop frequency response of chaotic masking system based on Lorenz system

To illustrate the usefulness of the method described above, we use another well-known chaotic system - the Lorenz system to study the open-loop frequency response of chaotic masking system. The driving Lorenz system is given by

$$\begin{aligned} \dot{x}_{\rm d} &= \sigma(y_{\rm d} - x_{\rm d}), \\ \dot{y}_{\rm d} &= \dot{x}_{\rm d} - y_{\rm d} - x_{\rm d} z_{\rm d}, \\ \dot{z}_{\rm d} &= x_{\rm d} y_{\rm d} - \gamma z_{\rm d}, \end{aligned} \tag{15}$$

where the parameters σ, ρ and γ are taken as $\sigma = 16$, $\rho = 45.6$, $\gamma = 4$, for which the system exhibits chaotic behaviour. The response chaotic system can be written as

$$\begin{aligned} \dot{x}_{\rm r} &= \sigma(y_{\rm r} - x_{\rm r}), \\ \dot{y}_{\rm r} &= \rho s(t) - y_{\rm r} - s(t) z_{\rm r}, \\ \dot{z}_{\rm r} &= s(t) y_{\rm r} - \gamma z_{\rm r}, \end{aligned} \tag{16}$$

where

$$s(t) = x_{\rm d}(t) + m_{\rm i}(t)$$

is the driving signal. Based on Eqs.(15) and (16), we can also draw the block diagram of the open-loop frequency response for the Lorenz system which is actually the chaos masking scheme designed by Cuomo *et al.*[5] In computer simulation, we choose the integration step size $h = 0.01$, which is the same as the sampling step size of the exciting signal. The total number for the integration is $N = 30,000$. The simulation results are shown in Figs.9–14.

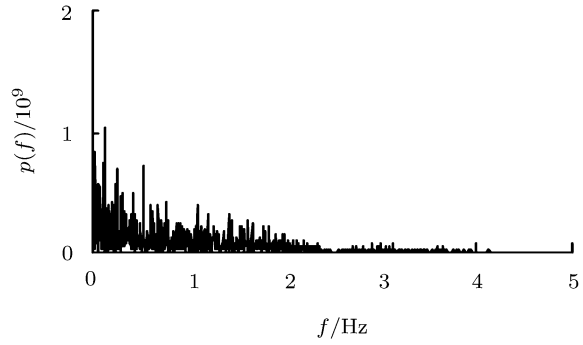


Fig.9. The power spectrum of the Lorenz system.

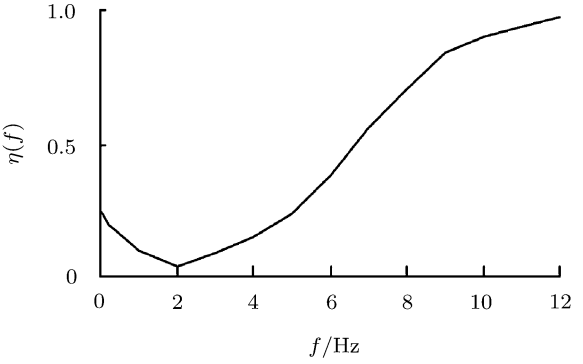


Fig.10. The synchronization response of the Lorenz system.

Figure 9 shows the power spectrum of a chaotic masking system, while Figs.10–12 depict, respectively, the synchronization response curve, $\eta(f)$, the phase response curve, $\varphi(f)$, and the amplitude response curve, $H(f)$, for $A_{\rm i} = 0.05$ and $d = 0.07A_{\rm i}$. Figures 13 and 14, on the other hand, show the time history of the output signal of the response system and the exciting signal for $f = 0.05$ and 12 Hz, respectively. In order to give a better comparison, the curve $m_{\rm i}(t)$ has been shifted down by 0.5 unit in Fig.13, and 0.075 unit in Fig.14.

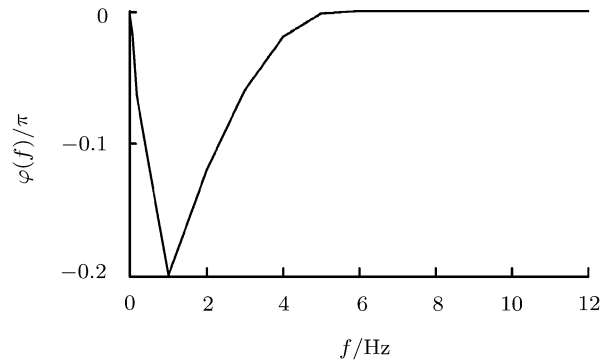


Fig.11. The phase response of the Lorenz system.

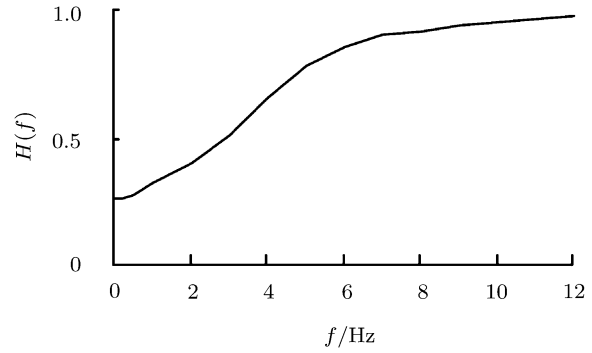


Fig.12. The amplitude response of the Lorenz system.

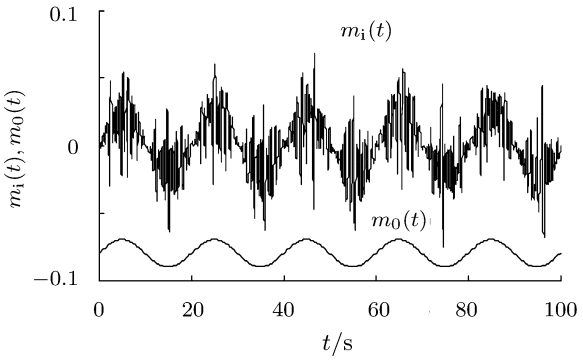


Fig.13. The time sequences of the exciting signal with $A_{\rm i} = 0.05, f = 0.05$ Hz, and the output signal of the response system for the Lorenz system.

Similarly, we have studied the relation between the synchronization response and the amplitude of the exciting signal. This is achieved by measuring the synchronization response, $\eta(A_{\rm i})$, for a fixed frequency of the exciting signal, but varying its amplitude, $A_{\rm i}$, as well as a fixed synchronous threshold value, $d = 0.07A_{\rm i}$. It is seen from the numerical simulation results that when the frequency $f = 0.05$ Hz and amplitude $A_{\rm i} \in [0.001, 5]$ (corresponding to MR$-66.242-7.734$ dB), the synchronization response $\eta(A_{\rm i}) \in [0.225, 0.228]$. When $f = 12$ Hz and $A_{\rm i} \in [0.001, 5]$, we have $\eta(A_{\rm i}) \in [0.991, 0.995]$.

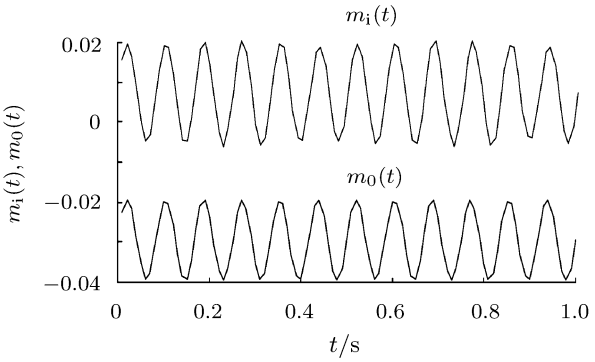


Fig.14. The time sequences of the exciting signal with $A_{\rm i} = 0.05$, $f = 12$ Hz, and the output signal of the response system for the Lorenz system.

4. Conclusion

In conclusion, we have studied the open-loop frequency response of a chaotic masking system. For Chua's circuit, the change of the frequency response is mainly located in the domain where the frequency is three times as large as the highest frequency of a chaotic masking system, i.e., $0 \le f \le 1.5$ Hz. However, for the Lorenz system, the variations of the frequency response is mainly within [0Hz, 9Hz]. This indicates that different chaotic systems may have different relations between the domains of the variations of frequency response and the frequency band of the system. If the frequency f of the exciting signal is in the domain of the frequency response, the output signal of the response system is distorted, and the exciting signal cannot be correctly recovered. If f exceeds the highest frequency of the domain of the frequency response, the exciting signal can be fully recovered. If f is close to the lowest frequency of the domain of the frequency response, then the output signal of the response system is almost the same as the exciting signal with small disturbing noises, and thus the exciting signal can be also recovered using an appropriate filter. This suggests that for such an exciting (message) signal whose frequency exceeds the domain of the frequency response, chaotic masking fails to be used in secure communications.

We have shown that the amplitude of the exciting signal does not have impact on the frequency response of a chaotic masking system. However, it should be

pointed out that the amplitude of the exciting signal must be restricted to a certain region such that the chaotic system is working properly, because, for example, a too strong exciting signal can drive the response chaotic system into periodic oscillations. Although the theoretical results given in Ref.[17] agree, in some aspect, with the results obtained in this paper using method, the method in Ref.[17] cannot be extended to deal with other differential equations or iterative systems, but our method used this paper can.

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