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Perfect synchronization of chaotic systems: a controllability perspective*

Sun Ming-Xuan(孙明轩)†, He Xiong-Xiong(何熊熊), and Yu Li(俞 立)

College of Information Engineering, Zhejiang University of Technology, Hangzhou 310014, *China*

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This paper presents a synchronization method, motivated from the constructive controllability analysis, for two identical chaotic systems. This technique is applied to achieve perfect synchronization for Lorenz systems and coupled dynamo systems. It turns out that states of the drive system and the response system are synchronized within finite time, and the reaching time is independent of initial conditions, which can be specified in advance. In addition to the simultaneous synchronization, the response system is synchronized un-simultaneously to the drive system with different reaching time for each state. The performance of the resulting system is analytically quantified in the face of initial condition error, and with numerical experiments the proposed method is demonstrated to perform well.

Keywords: controllability, finite time convergence, synchronization, chaos
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1. Introduction

Chaotic behaviour is sensitive to initial conditions and different trajectories could be resulted due to shifts in initial conditions. Recent years have witnessed great progress in chaos synchronization and many schemes have been developed with the use of the drive-response configuration, introduced in Ref.[1]. The method can make two identical or non-identical chaotic systems synchronizing whatever the initial conditions are.

So far many works have been devoted to use the method of active control, suggested in Ref.[2], for chaotic synchronization. The idea behind it is the fully-controlled-synchronization manner, which applies control elements to each state variable to be synchronized. This method has been applied successfully to nonlinear Bloch equations,[3] Chua circuits,[4] and Lorenz systems.[2] Designs have been looking for new ways from the Lyapunov approach.[5] Following its success in exactly modelled chaotic systems, extensive works attempt to cope with parameter uncertainties: see for instance, adaptive active control.[6−8]. In the published literature, there has been considerable interest in the Lyapunov syntheses.[9−11] However, the method of active control deserves further study. In Ref.[12], this method was used as a tool for sequential synchronization, assuring that control elements could apply with specified activating instances. This is probably useful for the security enhancement in secure communication.

The mainstream of research remains in asymptotic synchronization. Our recent work offers an alternative, in Ref.[13], where the synchronizing error between two chaotic systems was guaranteed to go to zero exactly at a pre-specified instant and remains at zero after the instant; we call it perfect synchronization. The finite time convergence is due to the introduction of initial rectified attractors (IRA). The reaching time by the IRA-based method is independent of initial conditions and is able to be given in advance.

In this paper, the problem of perfect synchronization is solved based on the constructive controllability analysis. As will be shown, this work gives actually an IRA-based design. From this observation, the proposed method in this paper can be considered as an extension of Ref.[13]. We shall verify that the chaotic systems under consideration can be simultaneously or un-simultaneously synchronized with ease, by setting the reaching time instants to be the same or not. Both Lorenz and coupled dynamo systems are taken for case studies. In the face of initial condition error, we shall also show that the proposed method can still perform

*Project supported by the National Natural Science Foundation of China (Grant No 60474005).
†E-mail: mxsun@zjut.edu.cn

well.

2. Matched nonlinearity and asymptotic synchronization

The synchronization problem is solved, in this section, on the basis of feedback linearization and pole assignment techniques. Consider an uncontrolled chaotic system described by

$$\dot{y} = Ay + F(y), \tag{1}$$

where $y \in R^n$ is the state vector. $A \in R^{n\times n}$ and F indicates the nonlinearity, which are assumed to be known. Let us introduce control matrix B and define $B_{\rm s} = {\rm span}\{B\}$. F is said to satisfy the matching conditions, if $F \in B_{\rm s}$. For the matched nonlinearity, there exists $f \in R^r$ such that $F = Bf$, and the controlled chaotic system can be written as

$$\dot{y} = Ay + B(f(y) + u). \tag{2}$$

We regard Eq.(2) as the response system, while the drive system is in the same form of Eq.(1), expressed by

$$\dot{x} = Ax + Bf(x), \tag{3}$$

Let us define the synchronizing error as $e = x - y$, and, from Eqs.(2) and (3), the evolution of which is governed by

$$\dot{e} = Ae + B(f(x) - f(y) - u). \tag{4}$$

Consequently, we can choose

$$u = f(x) - f(y) - v \tag{5}$$

to obtain the error system as

$$\dot{e} = Ae + Bv. \tag{6}$$

Assumption 1 System (6) is completely controllable.

An algebraic test for controllability of system (6), which can be applied, is given as follows.[14] System (6) is completely controllable if and only if

$$\text{rank } \Gamma_{\rm c}[A, B] = n. \tag{7}$$

where $\Gamma_{\rm c}[A, B] = [B, AB, A^2B, \cdots, A^{n-1}B]$ is the controllability matrix.

For the synchronization purpose, we appeal to the pole assignment technique for choosing the feedback gain K in the linear feedback control

$$v = -Ke\,. \tag{8}$$

Let us define $A_K = A - BK$. Applying Eq.(6), we attain the following error system:

$$\dot{e} = A_K e. \tag{9}$$

The poles of the closed-loop system can be placed at any desired locations and the asymptotic synchronization can be achieved. It should be noted that, to achieve the asymptotic synchronization, we need not apply full control elements if $B \neq I$. We are fortunate as the controllability of typical chaotic systems we are interested is ensured and $B \neq I$. The design is feasible. However, there are cases where the matrix B has to be set as a unity matrix, for example, the coupled dynamos system. In the case of $B = I$, the controllability-based method becomes the active control method.[2]

One may wonder about the controllability of system (6) by utilizing the linear feedback. It is interesting to note that if system $\{A, B\}$ is not controllable, then the resulting system $\{A - BK, B\}$ is not controllable.

3. Perfect synchronization

In this section, from a controllability perspective, we shall consider the problem of perfect synchronization of two identical chaotic systems.

3.1. Simultaneous synchronization

We now present our design to the solution of the perfect synchronization problem with the same reaching time for all state variables. Our design consists of two steps.

Step I Linear Feedback Design. For system (6), the control component v in Eq.(5) is chosen as

$$v = -Ke + r, \tag{10}$$

where r is the action to cause the finite-time convergence to be achieved. Applying Eq.(10) and defining $A_K = A - BK$, we attain the following linear system:

$$\dot{e} = A_K e + Br. \tag{11}$$

Step II Initial Rectifying Action Design. From linear control theory, system (11) is completely controllable too under Assumption 1, and

$$\text{rank } \Gamma_{\rm c}[A_K, B] = n. \tag{12}$$

Therefore, following Ref.[15], it is possible to achieve perfect synchronization by choosing the action profile of r as follows:

$$r(t)=\begin{cases}-B^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-t)}W_{\mathrm{c}}^{-1}(t_1,t_0)\mathrm{e}^{A_K^{\mathrm{T}}(t_1-t_0)}e(t_0), & t_0\le t\le t_1,\\ 0, & t>t_1,\end{cases}\tag{13}$$

where $W_{\mathrm{c}}(t_1,t_0)$ is the controllability Gramian

$$W_{\mathrm{c}}(t_1,t_0)=\int_{t_0}^{t_1}\mathrm{e}^{A_K(t_1-\tau)}BB^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-\tau)}\mathrm{d}\tau,\tag{14}$$

which is nonsingular as the system is controllable, and can be calculated off-line as both A_K and B are known.

Initial rectifying action $r(t)$ given by Eq.(13) satisfies

$$\int_{t_0}^{t}\mathrm{e}^{-A_K\tau}Br(\tau)d\tau=-\mathrm{e}^{-A_Kt_0}e(t_0).$$

By the definition,[13] the system (11) has an initial rectified attractor. As is shown, this design gives actually an IRA-based one.

Theorem 1 For the drive system (3) and the response system (2), let the controller (5), together with the linear feedback (10) and the initial rectifying action (13), be applied. Then

i) the response system (2) can perfectly synchronize the drive system (3) in the absence of initial condition error, in particular,

$$e(t)=\begin{cases}\mathrm{e}^{A_K(t-t_0)}\Big[I-\mathrm{e}^{A_K(t_0-t_1)}\displaystyle\int_{t_0}^{t}\mathrm{e}^{A_K(t_1-\tau)}BB^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-\tau)}\mathrm{d}\tau W_{\mathrm{c}}^{-1}(t_1,t_0)\mathrm{e}^{A_K(t_1-t_0)}\Big]e(t_0), & t\in[t_0,t_1],\\ 0, & t>t_1;\end{cases}\tag{15}$$

ii) in the presence of bounded initial condition error, the designed controller guarantees the synchronizing error to be bounded, and converge to zero as time approaches infinity.

Proof i) It follows immediately by applying Eq.(13) to Eq.(11).

ii) For the case where initial condition error exists, the initial rectifying action given in Eq.(13) can be expressed as

$$r(t)=\begin{cases}-B^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-t)}W_{\mathrm{c}}^{-1}(t_1,t_0)\mathrm{e}^{A_K(t_1-t_0)}(e(t_0)+\xi(t_0)), & t_0\le t\le t_1,\\ 0, & t>t_1,\end{cases}\tag{16}$$

where $\xi(t_0)$ indicates the initial condition error. Applying Eq.(16) we have, for $t_0\le t\le t_1$,

$$e(t)=\mathrm{e}^{A_K(t-t_0)}\Big[I-\mathrm{e}^{A_K(t_0-t_1)}\int_{t_0}^{t}\mathrm{e}^{A_K(t_1-\tau)}BB^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-\tau)}\mathrm{d}\tau W_{\mathrm{c}}^{-1}(t_1,t_0)\mathrm{e}^{A_K(t_1-t_0)}\Big](e(t_0)+\xi(t_0)).$$

Since A_K is designed to be stable, there exist constants $M>0$ and $\alpha>0$ such that $\|\mathrm{e}^{A_Kt}\|\le M\mathrm{e}^{-\alpha t}$ and $\|\mathrm{e}^{A_K^{\mathrm{T}}t}\|\le M\mathrm{e}^{-\alpha t}$. We have

$$\Big\|\int_{t_0}^{t}\mathrm{e}^{A_K(t_1-\tau)}BB^{\mathrm{T}}\mathrm{e}^{A_K^{\mathrm{T}}(t_1-\tau)}\mathrm{d}\tau\Big\|\le\frac{1}{2\alpha}M^2\|BB^{\mathrm{T}}\|(1-\mathrm{e}^{-2\alpha(t_1-t_0)}),\quad t_0\le t\le t_1.$$

Therefore, there exists constant $M_1>0$ such that

$$\|e(t)\|\le M_1M\mathrm{e}^{-\alpha(t-t_0)}(\|e(t_0)\|+\|\xi(t_0)\|),\quad t_0\le t\le t_1.\tag{17}$$

Since $r(t)=0$, $t>t_1$, then $e(t)=\mathrm{e}^{A_k(t-t_1)}e(t_1)$. Furthermore,

$$\|e(t)\|\le M\mathrm{e}^{-\alpha(t-t_1)}\|e(t_1)\|),\quad t>t_1.\tag{18}$$

The boundedness and asymptotic convergence of $e(t)$ can be concluded by combining Eqs.(17) and (18).

Uncertainties in the dynamics Eq.(3) could mean that we fail to achieve the perfect synchronization. In

this case, our work has to conduct a re-design to incorporate adaptive and robust controls for dealing with structured and unstructured uncertainties. In this paper, we focus on design issues based on the constructive controllability analysis. However, for simplicity, the re-design is not carried out.

One may wish that the time profile of the synchronizing error is smooth. Namely, its derivative is continuous for $t = t_0$ and $t = t_1$. The continuity is obviously guaranteed for $t_0 < t < t_1$. Unfortunately, for the current design, the smooth merging for both time instants $t = t_0$ and $t = t_1$ cannot be ensured and there is no room to improve it. In particular, smooth merging requires that

$$\dot{e}(t_0) = A_K e(t_0) + B(f(x(t_0)) - f(y(t_0))), \tag{19}$$

$$\dot{e}(t_1) = 0. \tag{20}$$

Condition (19) implies that $u(t_0) = 0$. However, the proposed design leads to

$$u(t_0) = f(x(t_0)) - f(y(t_0)) - r(t_0),$$
$$r(t_1) = -B^{\mathrm{T}} W_{\mathrm{c}}^{-1}(t_1, t_0) \mathrm{e}^{A_K(t_1 - t_0)} e(t_0).$$

The smooth merge for both instants $t = t_0$ and $t = t_1$ cannot be ensured since it is impossible for the present design to have

$$r(t_0) = f(x(t_0)) - f(y(t_0)),$$
$$e(t_0) = 0,$$

as $x(t_0) \neq y(t_0)$.

Although the chaotic systems under consideration are sensitive to initial conditions, from Theorem 1, it is demonstrated that the synchronizing error converges to zero asymptotically as time goes in the presence of initial condition error. Moreover, due to the introduction of initial rectifying action, performance improvement is attained in comparison with the conventional asymptotic synchronization. We shall present numerical results for illustration, by taking the Lorenz system as an example. However, synchronization for the unified chaotic system,[16,17] follows similar control design. An active control design can be found in Ref.[18]. Let us consider two identical Lorenz systems with

$$A = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{bmatrix}, \tag{21}$$

and $\sigma, r, b > 0$ are three system parameters taken as in the original study,[19] i.e. $\sigma = 10, r = 28$ and $b = 8/3$. It is easy to check that rank $[B, AB, A^2B] = 3$. The controllability of the system $\{A, B\}$ makes it possible for us to conduct the proposed design. The eigenvalues of matrix A are $\{\lambda_i, i = 1, 2, 3\} = \{-22.8277, 11.8277, -2.6667\}$ and matrix A is unstable due to the existence of one positive eigenvalue. We fail to apply the initial rectifying action directly to Eq.(6), by setting $v = r$. The proposed two-step design is then applied and the resulting closed-loop poles are $\{\lambda_{K,i}, i = 1, 2, 3\} = \{-1.5, -1, -2\}$, Appealing to the pole-placement technique we attain the feedback gain

$$K = \begin{bmatrix} 35.2 & -8 & 0 \\ 0 & 0 & -1.1667 \end{bmatrix}.$$

The simulation for the synchronization without using initial rectifying action is first carried out for comparison. The control input is calculated according to Eqs.(5) and (10) where we set $r = 0$. The initial time $t_0 = 3$ is chosen. Initial conditions of the drive system and the response system are set to $x(0) = [6, 1, 7]^{\mathrm{T}}$ and $y(0) = [8, 5, 9]^{\mathrm{T}}$ respectively. The simulation result is shown in Fig.1, from which the error states converge to zero as time increases and the asymptotic synchronization is achieved.

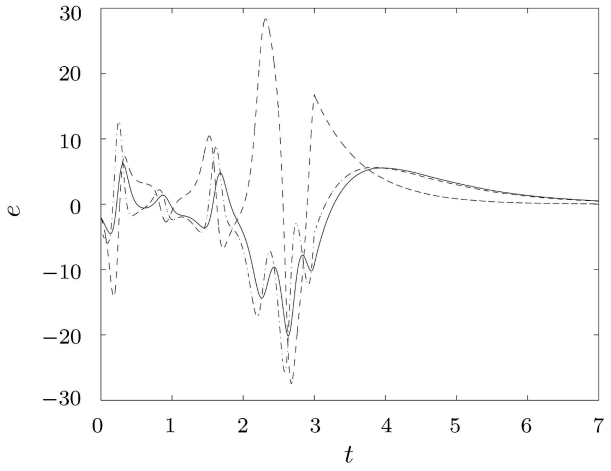


Fig.1. Simultaneous synchronization without IRA: e_1 (solid line), e_2 (dot-dash line), and e_3 (dash line). The control u is activated at $t \geq 3$.

With the selection of the rectifying duration $[t_0, t_1] = [3, 4]$, the rectifying action given in Eq.(13)

is then introduced in order to achieve the perfect synchronization. The initial conditions of the drive system and the response system keep the same. The numerical simulation is delineated in Fig.2. It is verified from the figure that the error states are simultaneously forced to be zero at $t_1 = 4$ and remain zero after the instant.

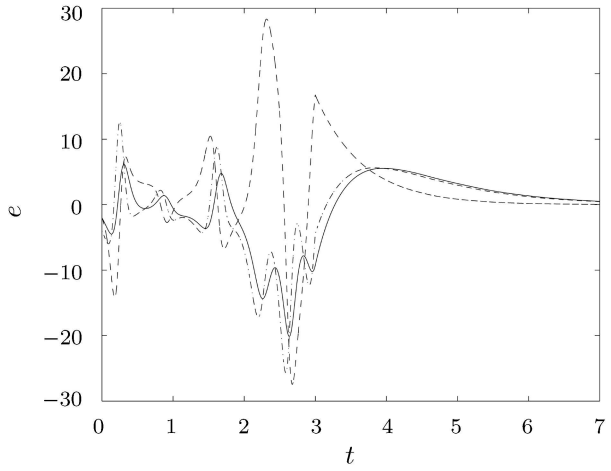


Fig.2. Simultaneous synchronization with IRA: e_1 (solid line), e_2 (dot-dashed line), and e_3 (dashed line). The initial condition of the response system is set as $y(0) = [8, 5, 9]^{\mathrm{T}}$. The rectifying action r is applied over the interval.[3,4]

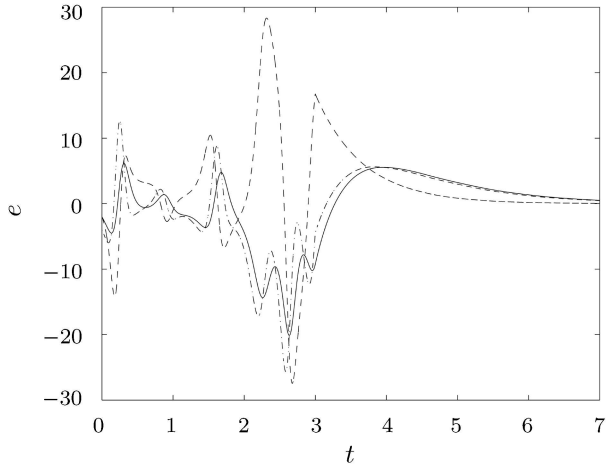


Fig.3. Synchronization performance in the presence of initial condition error: e_1 (solid line), e_2 (dot-dashed line), and e_3 (dashed line). Initial condition error $e(t_0)$ is assumed to be $\xi = [0.5, 0.5, 0.5]^{\mathrm{T}}$. The rectifying action r is applied over the interval.[3,4]

To examine the robustness with respect to initial condition error, let the initial condition of the drive system remain the same and the initial condition of the response system be reset to $[8, 5, 9]^{\mathrm{T}} + \xi$, with $\xi = [0.5, 0.5, 0.5]^{\mathrm{T}}$ indicating the initial condition error. Figure 3 shows the synchronizing result, which clearly depicts better performance achieved in comparison with Fig.1. We stress that the pole assignment by state feedback in our design plays a crucial role in enhancing the robustness to initial condition error.

3.2. Un-simultaneous synchronization

The synchronization problem is solved for the case $B = I$. What we shall characterize for this case is the synchronizing performance achieved unsimultaneously. The uncontrolled coupled dynamos system[20] is taken as the drive system, which is described by

$$\dot{x} = Ax + f(x), \tag{22}$$

where $x = [x_1, x_2, x_3]^{\mathrm{T}}$ is the state of the drive system, and

$$A = \begin{bmatrix} -\mu & \alpha & 0 \\ -\alpha & -\mu & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} x_2 x_3 \\ x_1 x_3 \\ 1 - x_1 x_2 \end{bmatrix}, \tag{23}$$

where $\alpha, \mu > 0$ are two system parameters. The system exhibits chaotic behaviour as $\alpha = 2$ and $\mu = 1$. The response system is the coupled dynamo system with the added control input,

$$\dot{y} = Ay + f(y) + u, \tag{24}$$

where $y = [y_1, y_2, y_3]^{\mathrm{T}}$ is the state of the response system and $u = [u_1, u_2, u_3]^{\mathrm{T}}$ is the control input.

For easy presentation of our control design, let us define error states $e_i = x_i - y_i, i = 1, 2, 3$ and the error system be particularly rewritten as

$$\begin{aligned} \dot{e}_1 &= -\mu e_1 + \alpha e_2 + x_2 x_3 - y_2 y_3 - u_1, \\ \dot{e}_2 &= -\alpha e_1 - \mu e_2 + x_1 x_3 - y_1 y_3 - u_2, \\ \dot{e}_3 &= -x_1 x_2 + y_1 y_2 - u_3. \end{aligned} \tag{25}$$

Based on feedback linearization, we adopt the input u in the form of

$$\begin{aligned} u_1 &= x_2 x_3 - y_2 y_3 - v_1, \\ u_2 &= x_1 x_3 - y_1 y_3 - v_2, \\ u_3 &= -x_1 x_2 + y_1 y_2 - v_3, \end{aligned} \tag{26}$$

and use the linear feedback control with initial rectifying action given by

$$\begin{aligned} v_1 &= (\lambda_1 + \mu) e_1 - \alpha e_2 + r_1, \\ v_2 &= \alpha e_1 + (\lambda_2 + \mu) e_2 + r_2, \\ v_3 &= \lambda_3 e_3 + r_3. \end{aligned} \tag{27}$$

It follows by substituting Eqs.(26) and (27) into Eq.(25) that, for $i = 1, 2, 3$,

$$\dot{e}_i = \lambda_i e_i + r_i. \tag{28}$$

We would like the reaching time to be respectively adjustable for each error state and the convergence rate to be able to be improved by simply setting shorter reaching time. To this end, the rectifying actions r_i, $i = 1, 2, 3$, are chosen as

$$r_i(t) = \frac{2\lambda_i \mathrm{e}^{\lambda_i(t_{1,i}-t)}}{\mathrm{e}^{\lambda_i(t_{0,i}-t_{1,i})} - \mathrm{e}^{\lambda_i(t_{1,i}-t_{0,i})}} e_i(t_{0,i}),$$

$$i = 1, 2, 3, \tag{29}$$

where the duration $\Delta_i = t_{1,i} - t_{0,i}$ is the interval of time during which the rectifying action acts, and $t_{1,i}$ is the reaching time of ith error state.

Theorem 2 For the coupled dynamo system (22) and the response system (24), let the controller (26), together with (27) and (29), be applied. If $\lambda_i, i = 1, 2, 3$, are chosen to be negative, then system (24) can perfectly synchronize system (22) unsimultaneously, and for $i = 1, 2, 3$,

$$e_i(t) = \begin{cases} \left[1 - \dfrac{\mathrm{e}^{\lambda_i(t_{0,i}-t)} - \mathrm{e}^{\lambda_i(t-t_{0,i})}}{\mathrm{e}^{\lambda_i(t_{0,i}-t_{1,i})} - \mathrm{e}^{\lambda_i(t_{1,i}-t_{0,i})}}\right] \mathrm{e}^{\lambda_i(t-t_{0,i})} e_i(t_{0,i}), & t_{0,i} \le t \le t_{1,i}, \\ 0, & t > t_{1,i}. \end{cases} \tag{30}$$

Proof It follows immediately by applying the initial rectifying action Eq.(29) to the error system Eq.(28).

When introducing $b_i r_i$ as the second term on the right-hand side of Eq.(28), we have $\dot{e}_i = \lambda_i e_i + b_i r_i$. If the initial rectifying action is chosen as

$$r_i(t) = \frac{2\lambda_i}{b_i} \frac{\mathrm{e}^{\lambda_i(t_{1,i}-t)}}{\mathrm{e}^{\lambda_i(t_{0,i}-t_{1,i})} - \mathrm{e}^{\lambda_i(t_{1,i}-t_{0,i})}} e_i(t_{0,i}),$$

then the resultant trajectory will be the same as that when we choose $b_i = 1$. Also show that it is not necessary to introduce $b_i (\neq 1)$ in Eq.(28).

Applying Eqs.(26), (27) and (29), we choose the following values in the numerical test: $x(0) = [1, -1, 3]^{\mathrm{T}}$, $y(0) = [3, 1, 4]^{\mathrm{T}}$, $\lambda_i = -0.5$, $i = 1, 2, 3$. And the following setting for rectifying durations are adopted: $\{t_{0,1}, t_{1,1}\} = \{10, 11\}$; $\{t_{0,2}, t_{1,2}\} = \{11.5, 12.5\}$; $\{t_{0,3}, t_{1,3}\} = \{13, 14\}$. The simulation result is shown in Fig.4, where un-simultaneous synchronization is observed within the specified durations.

Like the IRA-based approach,[13] an impulsive action might be involved in the initial rectifying action given by Eq.(29). It is easy to see that the following ζ-function:

$$\zeta(t) = \begin{cases} 0, & t < 0, \\ \dfrac{2\lambda \mathrm{e}^{\lambda(\Delta - t)}}{\mathrm{e}^{\lambda\Delta} - \mathrm{e}^{-\lambda\Delta}}, & 0 \le t \le \Delta, \\ 0, & t > \Delta, \end{cases} \tag{31}$$

will approach Dirac delta function as Δ tends to zero. From Eq.(29), such an impulsive action will be included as t_1 tends to t_0. The impulsive action is not realizable from a practical point of view. Thus, we do not suggest to use very short duration.

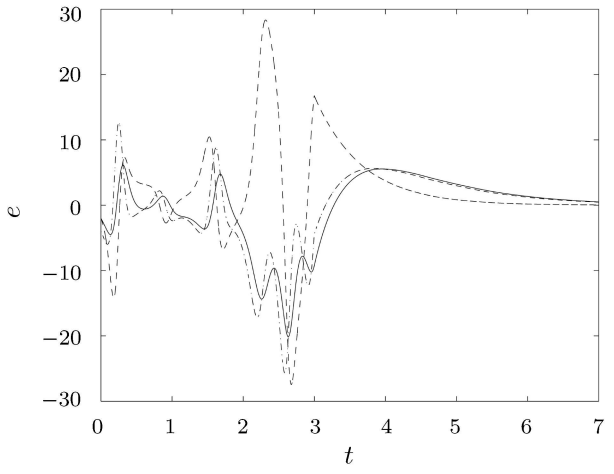


Fig.4. Un-simultaneous synchronization with IRA: e_1 (solid line), e_2 (dot-dash line), and e_3 (dash line). The rectifying durations are respectively set as $\{t_{0,1}, t_{1,1}\} = \{10, 11\}; \{t_{0,2}, t_{1,2}\} = \{11.5, 12.5\}; \{t_{0,3}, t_{1,2}\} = \{13, 14\}$.

4. Conclusion

In this paper, we have addressed the problem of perfect synchronization of chaotic systems, and our approach to the solution of this problem employs the constructive controllability analysis. It has been shown that states of the drive system and the response

system are able to be synchronized within finite time. Major features of the proposed method lie in that the reaching time is independent of the initial conditions, and can be pre-specified. The perfect synchronization of the undertaken chaotic systems can be guaranteed in simultaneous or un-simultaneous ways, which has been testified via computer simulations for Lorenz systems and coupled dynamos systems, and the resulting systems in the face of initial condition error could perform well also.

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