Vol 15 No 12, December 2006
1009-1963/2006/15(12)/2878-05
Chinese Physics
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and IOP Publishing Ltd

A new multi-scroll chaotic system

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(Received 9 April 2006; revised manuscript received 5 June 2006)

This paper proposes a new simple autonomous chaotic system which can generate multi-scroll chaotic attractors. The characteristic of this new multi-scroll chaotic system is that the $4n+2m+4$-scroll chaotic attractors are generated easily with n and m varying under $n \leq m$. Various number of scroll chaotic attractors are illustrated not only by computer simulation but also by the realization of an electronic circuit experiment on EWB (Electronics Workbench).

Keywords: multi-scroll chaotic system, circuit, scroll
PACC: 0545

1. Introduction

Multi-scroll chaotic generators have been studied with increasing interest in the last few years due to their great potential in many chaos-based technologies and information systems. So, the generation, control, synchronization and application of multi-scroll chaotic attractors have recently become a focal topic for research.[1−15] During the past ten years, a large number of multi-scroll chaotic systems have been proposed by several researchers. For example, Suykens and Vandewalle proposed the quasi-linear function approach to generate a family of n-double scroll chaotic attractors;[1−3] Yalcin *et al* proposed a method which adds some additional breakpoints into piece-wise linear function of nonlinear resistor in Chua's circuit in 2001, and also proposed a method which uses the step circuit to generate a family of scroll-grid chaotic attractors, including 1D n-scroll, 2D $n \times m$-grid scroll and 3D $n \times m \times l$-grid scroll chaotic attractors in 2002;[4,5] Lü *et al* proposed a saturated circuit series approach for generating 1D n-scroll, 2D $n \times m$-grid scroll and 3D $n \times m \times l$-grid scroll chaotic attractors in 2004;[6] very recently, Yu *et al* introduced a novel nonlinear modulating function approach for generating n-scroll chaotic attractors based on a general jerk circuit,[7] and so on. All the above works develop this research direction greatly. In this paper, a new simple multi-scroll chaotic system that can easily generate $4n + 2m + 4$-scroll chaotic attractors with n and m varying under $n \leq m$ is studied. Various attractors are illustrated not only by computer simulation but also by the experiment of an electronic circuit on Electronics Workbench (EWB).

The rest of this paper is organized as follows. In Section 2, a new multi-scroll chaotic system is presented and analysed. Section 3 is devoted to numerical simulations for illustration and verification of our result. In Section 4, an electronic circuit is designed and the results of experiment on EWB are given. Finally, some concluding remarks and comments are given in Section 5.

2. A new multi-scroll chaotic system

The autonomy differential equations of this new multi-scroll chaotic system can be described as

$$\begin{aligned} \dot{x} &= ay - bf(y+z), \\ \dot{y} &= -ax + y - cf(y+z) + bg(x+z), \\ \dot{z} &= -az + bg(x+z), \end{aligned} \tag{1}$$

where

$$f(y+z) = \sum_{i=-n}^{n} \operatorname{sgn}(y+z+2i), \tag{2}$$

$$g(x+z) = \sum_{j=-m}^{m} \operatorname{sgn}(x+z+2j), \tag{3}$$

a, b are constants and x, y, z are variables of the system (1). Taking $a = 10$, $b = 5$, $c = 0.5$, $n = 0$, $m = 0$ and let the right hand side of the system (1) equal to zero, we can obtain the following equations:

$$ay - b\operatorname{sgn}(y+z) = 0,$$

$$-ax + y - c\,\mathrm{sgn}(y+z) + b\,\mathrm{sgn}(x+z) = 0,$$
$$-az + b\,\mathrm{sgn}(x+z) = 0. \quad (4)$$

The equilibrium points of Eq.(1) can be solved from Eq.(4): $((b-ac+ab)/a^2,\ b/a,\ b/a)$, $(0,\ 0,\ 0)$, $(-(b-ac+ab)/a^2,\ -b/a,\ -b/a)$. Linearizing Eq.(1) at each equilibrium point, one can observe that the Jacobian of them are the same as

$$J = \begin{bmatrix} 0 & a & 0 \\ -a & 1 & 0 \\ 0 & 0 & -a \end{bmatrix}. \quad (5)$$

Each equilibrium point is unstable because its characteristic roots from the Jacobian are the same as -10, $0.5 + 9.98$ i, $0.5 - 9.98$ i. So, the trajectories of the system will not stay in any equilibrium point. In fact, for a given initial value, as time elapses, the trajectory of Eq.(1) spirally diverges around its equilibrium point or changes around to another. After a long time, the trajectories will be overflowed the whole chaotic regime and 4-scroll chaotic attractors can be formed, which is shown in Fig.1. Likewise, the number of equilibrium points is varied with the natural number n and m of Eq.(1) and each equilibrium point is also unstable.

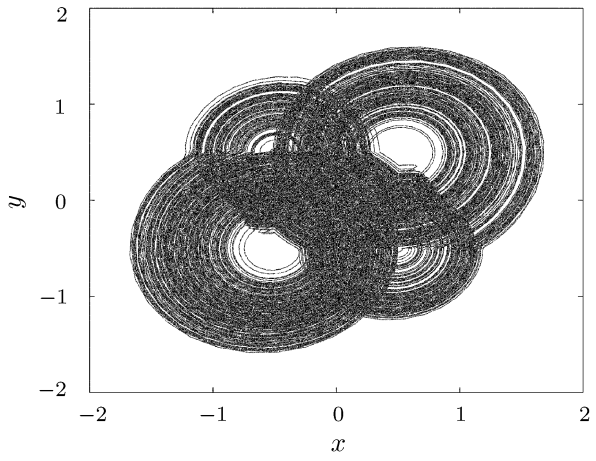


Fig.1. The 4-scroll chaotic attractors ($n = 0$, $m = 0$).

3. Simulation results

In this section, the fourth-order Runge–Kutta method is used to solve the system of differential equations (1), with step size equal to 0.001 in numerical simulation. Choose the value of initial conditions of Eq.(1): $x(0) = 0.001$, $y(0) = 0.001$, $z(0) = 0.001$ and system parameters of the system (1): $a = 10$, $b = 5$, $c = 0.5$. When we choose the different value of the natural number n and m, the $4n + 2m + 4$-scroll chaotic attractors can be obtained. For example, $n = 0$, $m = 5$; $n = 1$, $m = 5$; $n = 2$, $m = 5$; $n = 4$, $m = 5$; the 14-scroll, 18-scroll, 22-scroll and 30-scroll chaotic attractors are obtained as shown in Figs.2, 3, 4, 5 respectively. When $n = 2$, $m = 3$, the 18-scroll chaotic attractors can be also generated as shown in Fig.6. By comparing with Fig.3, we know that the number of scrolls is the same, but the arrangement of them is different. When we want to generate other even number of N-scrolls, the same thing will also happen if the value of the natural number n and m satisfies the following conditions:

$$4n + 2m + 4 = N, \quad n \le m. \quad (6)$$

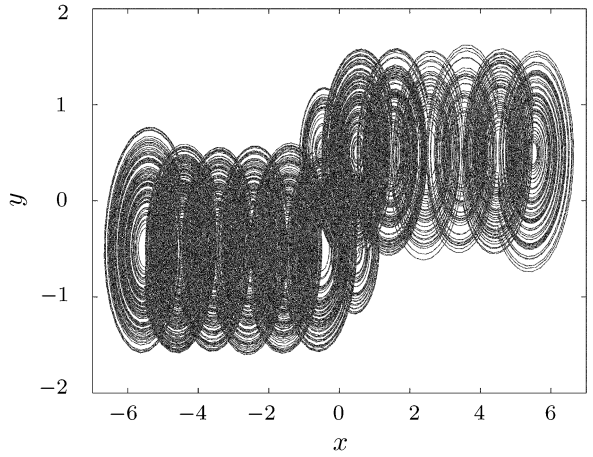


Fig.2. The 14-scroll chaotic attractors ($n = 0$, $m = 5$.)

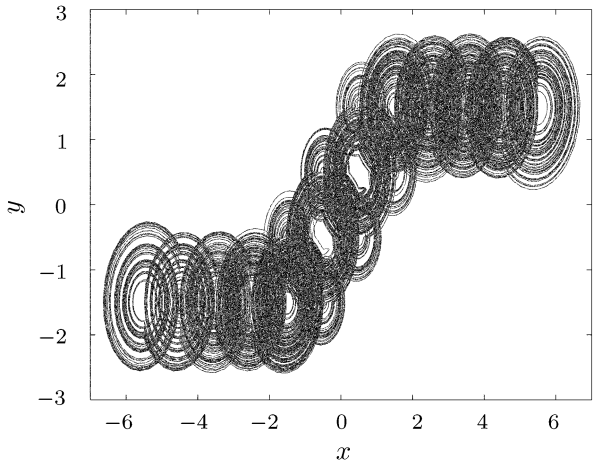


Fig.3. The 18-scroll chaotic attractors ($n = 1$, $m = 5$).

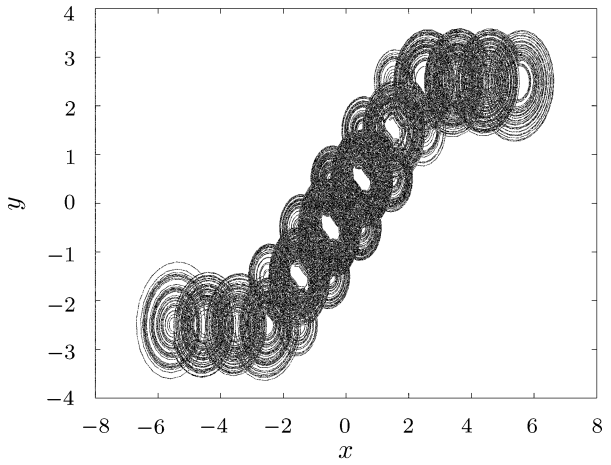


Fig.4. The 22-scroll chaotic attractors ($n = 2$, $m = 5$).

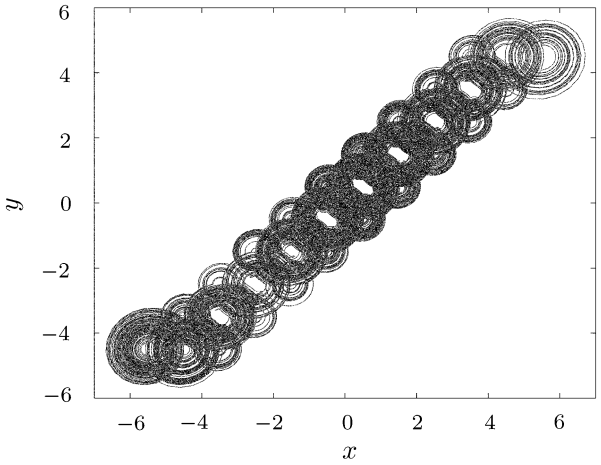


Fig.5. The 30-scroll chaotic attractors ($n = 4$, $m = 5$.)

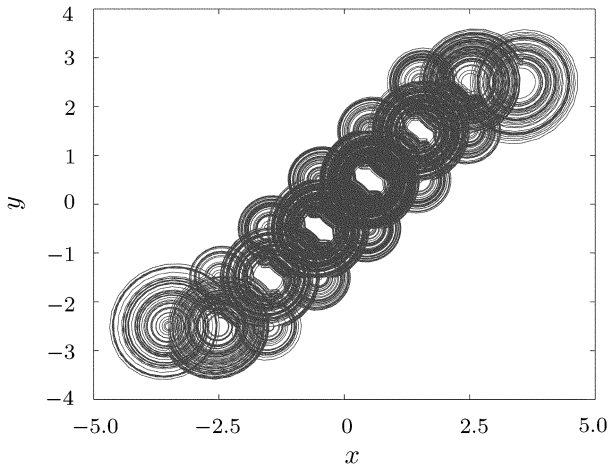


Fig.6. The 18-scroll chaotic attractors ($n = 2$, $m = 3$.)

4. Circuit experiment on EWB

In this section, an electronic circuit is designed to realize system (1) and experiment on EWB. The LF347 is used to realize addition and subtraction of the signal and nonlinear function. The circuit is designed and shown in Fig.7. The power supply of LF347 is ±15 V. According to the theory of circuit and the characteristic of each circuit component, the following equations can be obtained:

$$\begin{aligned}
\dot{x} &= \frac{1}{R_0C_0}y - \frac{15}{R_2C_0}\sum_{i=-n}^{n}\operatorname{sgn}(y+z+2i),\\
\dot{y} &= -\frac{1}{R_0C_0}x + \frac{1}{R_1C_0}y + \frac{15}{R_2C_0}\\
&\quad\times\sum_{j=-m}^{m}\operatorname{sgn}(x+z+2j)\\
&\quad-\frac{15}{R_3C_0}\sum_{i=-n}^{n}\operatorname{sgn}(y+z+2i),\\
\dot{z} &= -\frac{1}{R_0C_0}z + \frac{15}{R_2C_0}\sum_{j=-m}^{m}\operatorname{sgn}(x+z+2j). \qquad (7)
\end{aligned}$$

So

$$\begin{aligned}
&1/R_1C_0 = 1, \quad a = 1/R_0C_0,\\
&b = 15/R_2C_0, \quad c = 15/R_3C_0.
\end{aligned}$$

Taking

$$\begin{aligned}
&R_0 = 10\ \text{k}\Omega, \quad R_1 = 100\ \text{k}\Omega, \quad R_2 = 300\ \text{k}\Omega,\\
&R_3 = 3\ \text{M}\Omega, \qquad C_0 = 10\ \mu\text{F}
\end{aligned}$$

into the system (7), we find that it will be equal to system (1). When S_0, S_1, S_2, …, S_n and K_0, K_1, K_2, …, K_m ($n \le m$) are closed, the $4n + 2m + 4$-scroll chaotic attractors can be obtained. For example, $n = 0$, $m = 5$, namely S_0 and K_0, K_1, K_2, K_3, K_4, K_5 are closed, the 14-scroll chaotic attractors can be generated, which is shown in Fig.8, and $n = 1$, $m = 5$, namely, S_0, S_1 and K_0, K_1, K_2, K_3, K_4, K_5 are closed, the 18-scroll chaotic attractors can be generated, which is shown in Fig.9. When $n = 2$,m=3, namely S_0, S_1, S_2 and K_0, K_1, K_2, K_3 are closed, the 18-scroll chaotic attractors can also be generated, which is shown in Fig.10. By comparing Fig.9, one can observe that the number of scrolls is the same, but arrangement is different. At the same time, one can also observe that there is a good qualitative agreement between the numerical simulation and the experimental measurement on EWB.

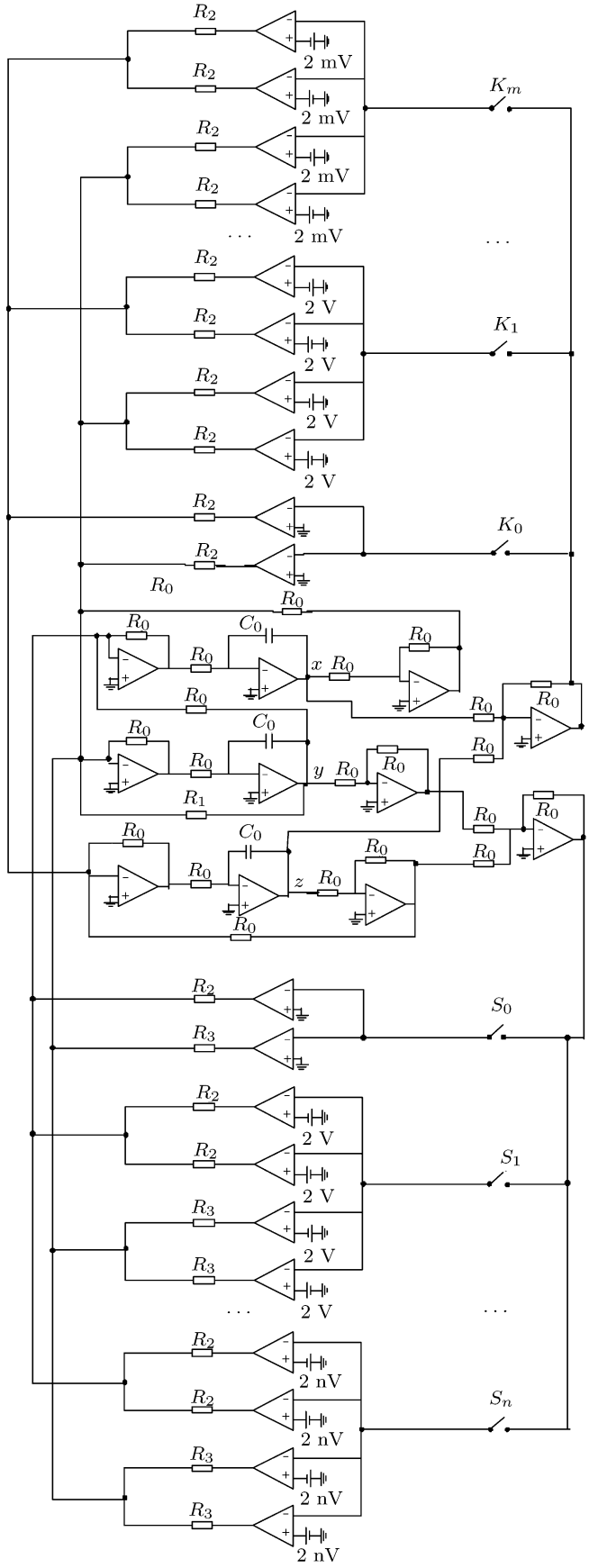


Fig.7. The circuit of realizing the system (1).

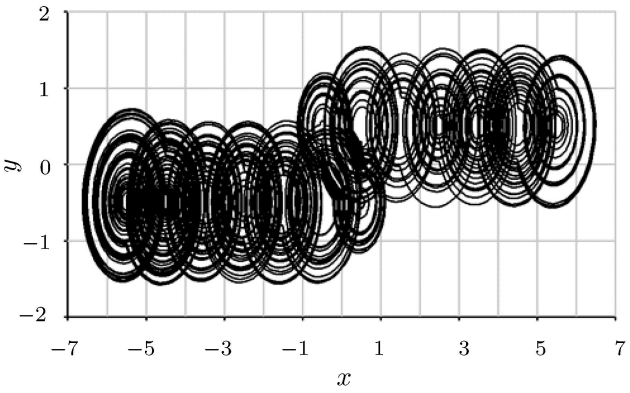


Fig.8. The 14-scroll chaotic attractors on EWB.

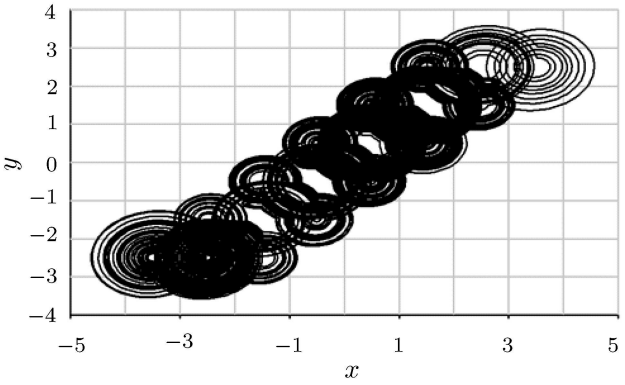


Fig.10. The 18-scroll chaotic attractors on EWB.

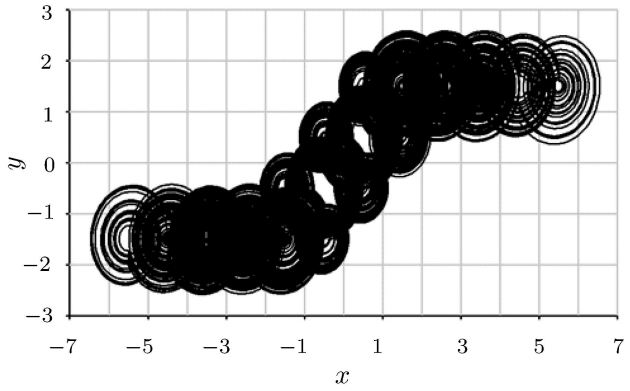


Fig.9. The 18-scroll chaotic attractors on EWB.

5. Conclusion

In summary, this new multiple-scroll chaotic generator is very simple, but the different number of scroll chaotic attractors and different arrangement of scroll chaotic attractors with the same number of scrolls can be obtained through modifying the natural numbers n and m. The results of simulation and the circuit experiment on EWB are uniform. It is hoped that chaos applications to engineering problems would benefit from the results of this paper.

References

[1] Suykens J A K and Vandewalle J 1991 *IEE Proceedings* G **138** 595

[2] Suykens J A K and Vandewalle J 1993 *IEEE Trans. Circuits Syst.* I **40** 861

[3] Suykens J A K and Vandewalle J 1993 *Proceedings of the International Symposium on Nonlinear Theory and its Applications* 829

[4] Yalcin M E, Ozoguz S, Suykens J A K and Vandewalle J 2001 *Electron. Lett.* **37** 147

[5] Yalcin M E, Suykens J A K, Vandewalle J and Ozoguz S 2002 *Int. J. Bifur. Chaos* **12** 23

[6] Lü J H, Chen G R, Yu X H and Leung H 2004 *IEEE Trans. Circuits Syst.* I **51** 2476

[7] Yu S M, Lü J H, Leung H and Chen G R 2005 *IEEE Trans. Circuits Syst.* I **52** 1459

[8] Eguchi K, Inoue T and Tsuneda A 1999 *IEICE Transaction Fundamentals* E **82** A 965

[9] Yalcin M E, Suykens J A K and Vandewalle J 1999 *Proceeding of the International Workshop on Nonlinear Dynamics of Electronic Systems* 257

[10] Tang K S, Man K F and Chen G R 2001 *Proceedings of the IEEE International Symposium on Circuits and Systems* 787

[11] Yalcin M E, Suykens J A K and Vandewalle J 2004 *IEEE Trans. Circuits Systems* I **51** 1395

[12] Tang K S, Zhong G Q, Chen G R and Man K F 2001 *IEEE Trans. Circuits Syst.* I **48** 1369

[13] Yu S M, Lin Q H and Qiu S S 2003 *Acta Phys. Sin* **52** 25 (in Chinese)

[14] Yu S M 2005 *Acta Phys. Sin.* **54** 1500 (in Chinese)

[15] Yu S M, Ma Z G, Qiu S S, Peng S G and Lin Q H 2004 *Chin. Phys.* **13** 317