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Statistical fluctuations in a saturation laser model with correlated noises*

Chen Shi-Bo(陈世波) and Mei Dong-Cheng(梅冬成)†

Department of Physics, Yunnan University, Kunming 650091*, China*

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We study the effects of correlations between quantum and pump noises on fluctuations of the laser intensity in a saturation laser model. An approximative Fokker–Planck equation and analytic expressions of the steady-state probability distribution function (SPD) of the laser system are derived. Based on the SPD, the normalized mean, the normalized variance, and the normalized skewness of the steady-state laser intensity are calculated numerically. The results indicate that (i) the correlation strength λ of correlated noises always enhances the fluctuation of laser intensity; (ii) the correlation time τ of correlated noises strengthens the fluctuation of laser intensity for the below-threshold case but τ weakens it for the above-threshold case.

Keywords: saturation laser model, correlated noises, fluctuation phenomena

PACC: 0540, 0250

1. Introduction

The studies of the statistical properties of laser systems have revealed that the consideration of both quantum and pump noises simultaneously existing in laser systems is of importance to deeply understand statistical fluctuations of the laser system.[1−11] Saturation effects on the behaviour of the laser do show their significance.[12−16] Zhu[16] discussed the saturation effects in a laser system with quantum and pump white noises. Cao *et al*[17,18] analysed the effects of saturation in the transient process of a dye laser with quantum and pump noises. Noise forces that are present simultaneously in laser systems were usually treated as uncorrelated random variables in most of the previous studies. However, noises in some stochastic process may have a common origin and can be correlated,[19,20] as they do in laser systems.[21] An adequate account of the statistical properties of a single-mode laser system was given in Ref.[22], with the correlations between pump and quantum noise taken into consideration. Also Madureira *et al* have pointed out that there are some situations where strong external noises can lead to some changes in the internal structure of a system, and hence the internal noise and external noise should not be independent.[23] Presently, the effects of correlations between quantum and pump noises on statistical fluctuation of a single-mode laser system have attracted much attention.[21,24−29] In Ref.[21], Zhu investigated the steady-state properties of the cubic model of the single-mode laser with correlations between an quantum noise and a pump noise. Cao *et al*[24] studied the phase lock induced by correlations between an quantum noise and a pump noise for the case of zero correlation time in a white-gain-noise model. Long *et al*[25] discussed the phase lock induced by correlations between an quantum noise and a pump noise in a cubic model. Liang *et al* [26] employed the cubic model to discuss the steady-state properties of the single-mode laser with a multiple-coloured noise and an additive coloured noise that are cross-correlated. Xie *et al*[27] investigated the effects of correlated noises on the intensity correlation fluctuation of the laser system. Chen *et al*[28] discussed intensity correlation time of a gain-noise model driven by two coloured noises with coloured cross-correlation. Cheng *et al*[29] studied the time evolution of intensity correlation function in the cubic model, driven by both coloured pump noise and quantum noise with cross-correlation between its real and imaginary parts. Most of the laser models with correlations between an quantum noise and a pump noise described in the previous analyses[21,25−27] contain only cubic nonlinearities or consider only the case of zero correlation time between an quantum noise and a pump noise.[24] The saturation effect of the laser system with corre-

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†Corresponding author. E-mail: kmdcmei@public.km.yn.cn

lations between an quantum noise and a pump noise for the case of nonzero correlation time between the quantum noise and the pump noise has not been fully understood yet.

In this paper, we investigate the effects of correlations between the quantum and the pump noises for the case of nonzero correlation time in the saturation model of a single-mode laser. In Section 2 the analytic expressions of steady-state distribution function are derived. In Section 3 the mean, variance, and skewness of laser intensity are discussed by numerical calculation.

2. Steady-state distribution function

The laser model, with the saturation effects taken into account, follows the Langevin equation[16]

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -KE + \frac{F_1 E}{1 + A \mid E \mid^2 / F_1} + p(t)E + q(t), \quad (1)$$

where the electric field E is complex and dimensionless and K is the cavity decay rate for the electric field and $F_1 = a_0 + K$ is the gain parameter, a_0 and A are real-valued and denote the net gain and self-saturation coefficients respectively. The random variables $p(t)$ and $q(t)$ are complex and present the pump and quantum noises. Applying the polar coordinate transformation $E = r\mathrm{e}^{\mathrm{i}\phi}$ to Eq.(1), one can obtain two equations of field amplitude r and phase ϕ. We consider only the intensity fluctuation of laser and let the laser intensity $I = r^2$. Thus Eq.(1) can be rewritten as the Langevin equation of laser intensity I

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2F_1 I}{1 + AI/F_1} + D + 2Ip(t) + 2\sqrt{I}q(t). \quad (2)$$

The statistical properties of the noise term are characterized by their first and second moments:

$$\langle q(t) \rangle = \langle p(t) \rangle = 0, \quad (3)$$

$$\langle q(t)q(t') \rangle = 2D\delta(t - t'), \quad (4)$$

$$\langle p(t)p(t') \rangle = 2Q\delta(t - t'), \quad (5)$$

and

$$\langle q(t)p(t') \rangle = \langle p(t)q(t') \rangle = \frac{\lambda\sqrt{DQ}}{\tau} \exp[-|t - t'|/\tau] \longrightarrow 2\lambda\sqrt{DQ}\delta(t - t') \text{ as } \tau \to 0, \quad (6)$$

where D and Q are the quantum and pump noise strengths respectively. The parameter λ measures the strength of correlations between $q(t)$ and $p(t)$. $\lambda = 0$ means that the two components are uncorrelated, and then the problem reduces to the case discussed in Ref.[16]. Equation (2) has two stable states

$$I_{\mathrm{s}1} = 0 \text{ and } I_{\mathrm{s}2} = \frac{F_1}{A}\left(\frac{F_1}{K} - 1\right). \quad (7)$$

By virtue of the Novikov theorem,[30] Fox's approach[31] and the ansatz of Hänggi *et al*,[32] the approximate Fokker–Planck equation (AFPE) corresponding to Eq.(2) with Eqs.(3), (4), (5) and (6) is obtained[33,34]

$$\frac{\partial P(I,t)}{\partial t} = -\frac{\partial}{\partial I}[A(I)P(I,t)] + \frac{\partial^2}{\partial I^2}[B(I)P(I,t)], \quad (8)$$

where

$$A(I) = -2KI + \frac{2F_1 I}{1 + AI/F_1} + D + (D + 3\lambda'\sqrt{DQ}I^{1/2} + 2QI), \quad (9)$$

$$B(I) = 4(DI + 2\lambda'\sqrt{DQ}I^{3/2} + QI^2), \quad (10)$$

with

$$\lambda' = \frac{\lambda}{1 + 2\tau K(1 - K/F_1)}. \quad (11)$$

Note that this AFPE holds under the condition $1 + 2\tau K(1 - K/F_1) > 0$. When the laser operates above a threshold value of $a_0 > 0$ $(F_1 = a_0 + K)$ i.e. $F_1 > K$, the AFPE holds for all the values of τ. When the laser operates below a threshold value of $a_0 < 0$, i.e. $F_1 < K$, the AFPE holds only for the case of $\tau < F_1/[2K(K - F_1)]$. We must point out that there is not any effect of correlation time τ on the laser system when the laser operates at a threshold value of $a_0 = 0$. From Eq.(11), we can see that $\lambda' = \lambda$ when $F_1 = K$.

The steady-state probability distribution (SPD) function $P_{\mathrm{st}}(I)$ can be obtained directly from Eq.(8) together with Eqs.(9) and (10), and is given by

$$P_{\rm st}(I) = N\left(\frac{Q}{2}I+\lambda'\sqrt{DQ}I^{1/2}+\frac{D}{2}\right)^{\beta_1}\left(\frac{A}{F_1}I+1\right)^{\beta_2} \times\exp\left[\beta_3\times\arctan\frac{QI^{1/2}+\lambda'\sqrt{DQ}}{\sqrt{DQ[1-\lambda'^2]}}+\beta_4\times\arctan\left(I^{1/2}\sqrt{\frac{A}{F_1}}\right)\right] \tag{12}$$

for $0\le\lambda<1$, and

$$P_{\rm st}(I) = N_0\left(\frac{Q}{2}I+\lambda'\sqrt{DQ}I^{1/2}+\frac{D}{2}\right)^{\beta_1}\left(\frac{A}{F_1}I+1\right)^{\beta_2} \times\exp\left[\frac{-2\alpha}{QI+\lambda'\sqrt{DQ}}+\beta_4\times\arctan\left(I^{1/2}\sqrt{\frac{A}{F_1}}\right)\right] \tag{13}$$

for $\lambda=1$, where

$$\beta_1=-\frac{K}{Q}-\frac{F_1^2(AD-F_1Q)}{[(AD-F_1Q)^2+4\lambda'^2DQF_1A]}-1,$$

$$\beta_2=\frac{F_1^2(AD-F_1Q)}{[(AD-F_1Q)^2+4\lambda'^2DQF_1A]},$$

$$\beta_3=\frac{\lambda'}{\sqrt{1-\lambda'^2}}\left[\frac{2K}{Q}-\frac{2F_1^2(AD+F_1Q)}{(AD-F_1Q)^2+4\lambda'^2DQF_1A}-1\right],$$

$$\beta_4=\frac{4\lambda'F_1^{\frac{5}{2}}\sqrt{DQA}}{(AD-F_1Q)^2+4\lambda'^2DQF_1A},$$

and

$$\alpha=\lambda'\sqrt{DQ}\left[\frac{K}{Q}-\frac{F_1^2(AD+F_1Q)}{(AD-F_1Q)^2+4\lambda'^2DQF_1A}-\frac{1}{2}\right],$$

N and N_0 are the normalization constants for Eqs.(12) and (13), respectively, and are determined by

$$\int_0^\infty P_{\rm st}(I){\rm d}I=1.$$

The expectation values of the nth power of the laser intensity I are given by

$$\langle I^n\rangle=\int_0^\infty I^nP_{\rm st}(I){\rm d}I. \tag{14}$$

So the normalized mean, the normalized variance, and the normalized skewness of the laser intensity can be given by the following forms, respectively:

$$\langle I\rangle=\int_0^\infty IP_{\rm st}(I){\rm d}I, \tag{15}$$

$$\lambda_2(0)=\frac{\langle I^2\rangle}{\langle I\rangle^2}-1, \tag{16}$$

and

$$\lambda_3(0)=\frac{\langle I^3\rangle}{\langle I\rangle^3}-3\lambda_2(0)-1. \tag{17}$$

When $\lambda=0$, Eq.(12) is back to Eq.(16) for the case discussed in Ref.[16]. We can study the statistical properties of the laser model by the numerical computation from Eqs.(15), (16) and (17).

3. Discussion and conclusions

To see the effects of correlation between the quantum and the pump noises in a laser system, we investigate the results of the SPD, the normalized mean, normalized variance, and normalized skewness of the laser intensity from Eqs.(15)–(17) by numerical calculation.

The steady-state distribution $P_{\rm st}(I)$ is plotted as a function of laser intensity I for different values of λ and τ as shown in Figs.1, 2.

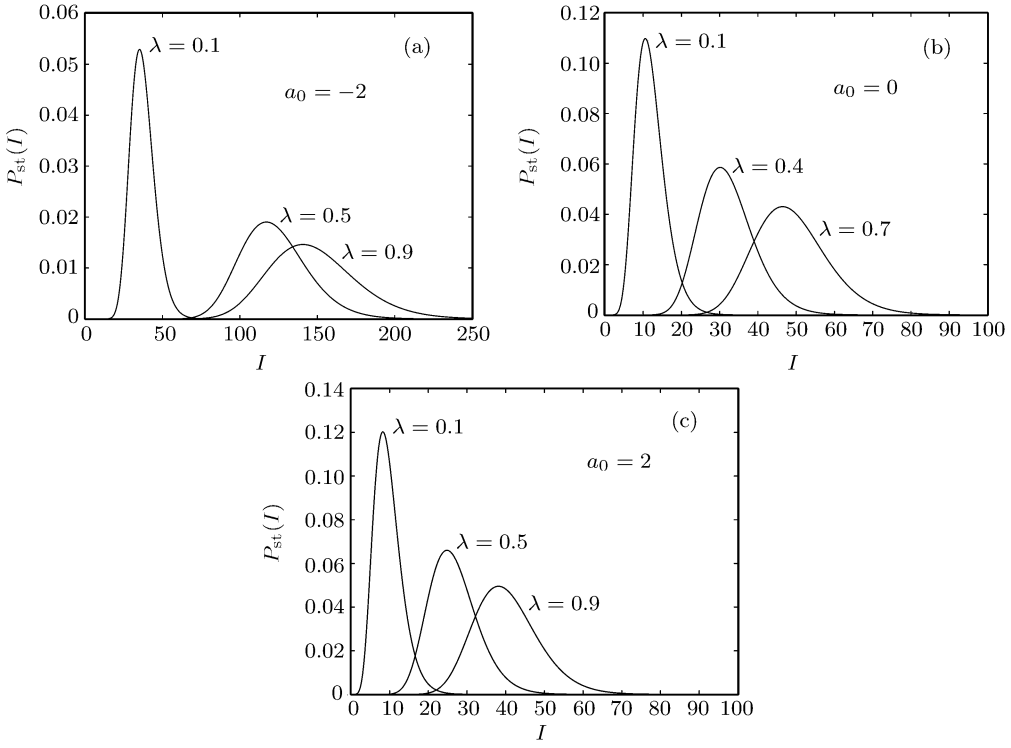


Fig.1. The steady-state probability distribution function $P_{\rm st}(I)$ vs laser intensity I for different values of λ. (a) $A = 1$, $D = 2$, $Q = 1.38$, $\tau = 0.2$, $K = 150$ and $a_0 = -2$. (b) $A = 1$, $D = 2$, $Q = 1.38$, $\tau = 0.2$, $K = 150$ and $a_0 = 0$. (c) $A = 1$, $D = 2$, $Q = 1.38$, $\tau = 0.2$, $K = 150$ and $a_0 = 2$.

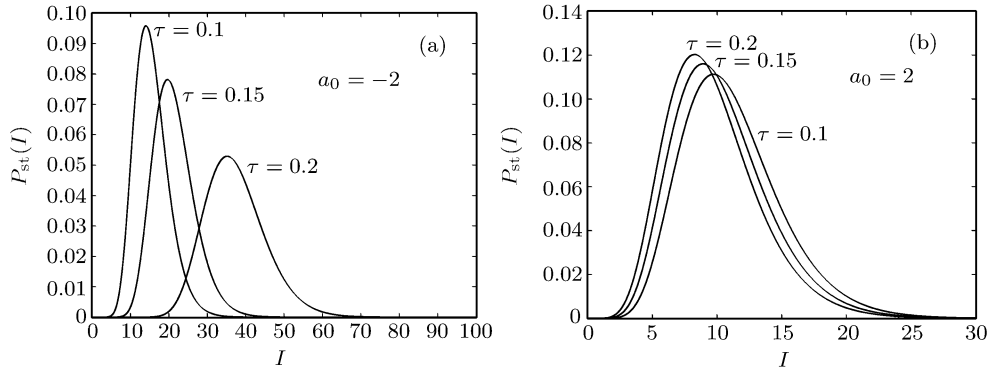


Fig.2. The steady-state probability distribution function $P_{\rm st}(I)$ vs laser intensity I for different values of τ. (a) $A = 1$, $D = 2$, $Q = 1.38$, $\lambda = 0.1$, $K = 150$ and $a_0 = -2$. (b) $A = 1$, $D = 2$, $Q = 1.38$, $\lambda = 0.1$, $K = 150$ and $a_0 = 2$.

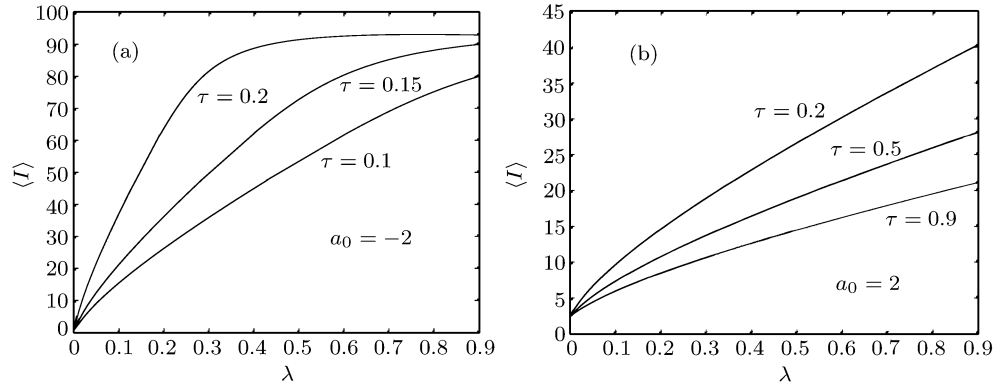


Fig.3. The normalized mean intensity $\langle I \rangle$ as a function of the correlation intensity λ for different values of τ. (a) $A = 1$, $D = 2$, $Q = 1.38$, $K = 150$ and $a_0 = -2$. (b) $A = 1$, $D = 2$, $Q = 1.38$, $K = 150$ and $a_0 = 2$.

From Fig.1, we can see that the maximum of $P_{\rm st}(I)$ moves to the right (i.e. moves to larger values of I) with increasing λ for the cases of $a_0 < 0, a_0 = 0$ and $a_0 > 0$. Namely, the strength λ of correlated noises always enhances the fluctuation of laser intensity.

Figure 2(a) shows that the maximum of $P_{\rm st}(I)$ moves to the right with increasing τ for the case of $a_0 < 0$, however, the maximum of $P_{\rm st}(I)$ moves to left with increasing τ for the case of $a_0 > 0$ as shown in Fig.2(b). Namely, the correlation time τ of correlated noises enhances the fluctuation of laser intensity for the below-threshold case but weakens it for the above-threshold case.

The normalized mean $< I >$, the normalized variance λ_2, and normalized skewness λ_3 as a function of λ for different values of τ are plotted as shown in Figs.3–5.

Figures 3(a) and 3(b) show that $< I >$ increases

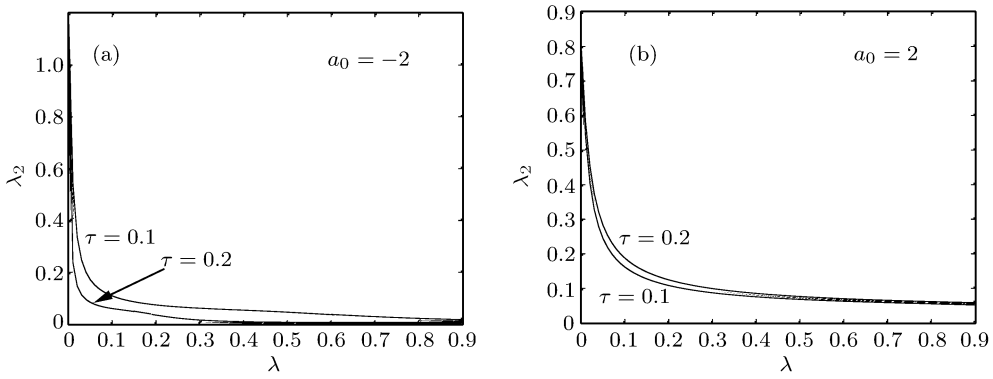


Fig.4. The normalized variance λ_2 as a function of the correlation intensity λ for different values of τ. (a) $A = 1$, $D = 4$, $Q = 2$, $K = 150$ and $a_0 = -2$. (b) $A = 1$, $D = 4$, $Q = 2$, $K = 150$ and $a_0 = 2$.

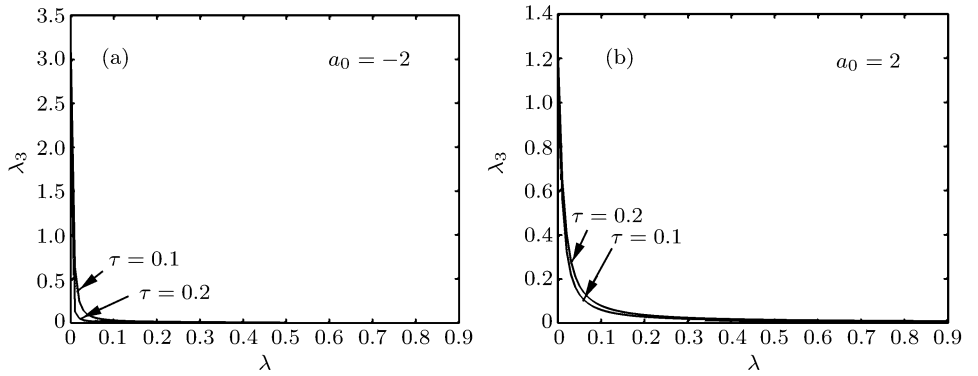


Fig.5. The normalized skewness λ_3 as a function of the correlation intensity λ for different values of τ. (a) $A = 1$, $D = 4$, $Q = 2$, $K = 150$ and $a_0 = -2$. (b)$A = 1$, $D = 4$, $Q = 2$, $K = 150$ and $a_0 = 2$.

monotonically with increasing λ for the below- and above-threshold cases and that $< I >$ increases with increasing τ for the below-threshold case but decreases with increasing τ for the above-threshold case.

Figures 4(a) and 4(b) show that λ_2 decreases monotonously with increasing λ for the below- and above-threshold cases, and that λ_2 decreases with increasing τ for the below-threshold case but $< I >$ increases with increasing τ for the above-threshold case.

Figures 5(a) and 5(b) show that λ_3 decreases monotonically with increasing λ for the below- and above-threshold cases and that λ_3 decreases with increasing τ for the below-threshold case but $< I >$ increases with increasing τ for the above-threshold case.

In summary, in a saturation laser system, the intensity λ of correlated noises always enhances the fluctuation of laser intensity. The correlation time τ of correlated noises strengthens the fluctuation of laser intensity for the below-threshold case but weakens it for the above-threshold case.

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