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Preparation of entangled atomic states through simultaneous nonresonant atom–field interaction*

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A scheme is proposed for generating a three-atom maximal entanglement W state. It is based on the simultaneous nonresonant interaction of atoms with a single-mode cavity field. Our scheme is insensitive to the cavity field, so the cavity field in our scheme can be initially in thermal states.

Keywords: entangled W state, nonresonant interaction

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1. Introduction

Entanglement is one of the striking features of quantum mechanics. It is not only fundamental in demonstrating quantum nonlocality, but also useful in quantum information theory, such as quantum cryptographic conference[1] and quantum secrete sharing.[2] In recent years, with the development of quantum information, much attention has been paid to entanglement.[3−6] Greenberger *et al*[7] have studied maximally entangled states of three or more particles, i.e. the Greenberger-Horne-Zeilinger (GHZ) states, which can provide possibilities to test quantum mechanics against local hidden theory without inequality.[8] Multi-photon entanglement has been observed.[9] Entangled states for three or more massive particles have been experimentally realized in microwave cavity quantum electrodynamics (QED)[10] and trapped ion systems.[11]

On the other hand, Dür *et al*[12] have shown that there are two inequivalent classes of tripartite entanglement states: the GHZ class and the W class, under stochastic local operation and classical communication. The W state especially has some interesting properties. For example, it retains bipartite entanglement when any one of the three qubits is traced out, which is argued as an important property in those situations where any one of the three particles decides not to cooperate with the others. W states can be used to test quantum nonlocality without inequality.[13] They are also useful in quantum information processing.[14] Many schemes have been proposed for generating W states for three or more atoms in cavity QED and ion trap systems.[15−19] In Ref.[15], Guo *et al* let three or more atoms interact simultaneously with a vacuum cavity dispersively to prepare W states.

In this paper, we propose an alternative scheme for generating W states, which is based on the simultaneous nonresonant interaction of atoms with a single-mode cavity field. In comparison with the previous scheme,[15] our scheme allows the cavity to be initially in thermal state and exchange energy with the environment. Following the scheme by Zheng and Guo,[20] a two-atom entangled state has been realized experimentally in cavity QED.[21] We believe that our scheme could also be realized in practice.

2. Generation of entangled W states

Consider a simultaneous interaction of three identical two-level atoms with a single-mode cavity field. The Hamiltonian can be written as

$$H = \omega a^{+}a + \sum_{j=1}^{3} \omega_0 S_{jZ} + \sum_{j=1}^{3} g\left(aS_j^{+} + a^{+}S_j^{-}\right), \tag{1}$$

$$H_0 = \omega a^{+}a + \sum_{j=1}^{3} \omega_0 S_{jZ}, \tag{2}$$

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$$H_i = \sum_{j=1}^{3} g\left(aS_j^+ + a^+ S_j^-\right), \tag{3}$$

where $S_{jZ} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ $(j = 1, 2, 3)$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity field frequency, ω_0 is the atomic transition frequency, ω is the cavity frequency, and g is the atom-cavity coupling strength. In the case $\Delta = \omega_0 - \omega \gg g\sqrt{\bar{n}+1}$, with $\bar{n}$ being the mean photon number of the cavity field, there is no energy exchange between the atomic system and the cavity. The energy conversing transition are between $|e_j g_k n\rangle$ and $|g_j e_k n\rangle$. The Rabi frequency λ for the transitions between these states, mediated by $|g_j g_k n+1\rangle$ and $|e_j e_k n-1\rangle$, is given by[22]

$$\lambda = \frac{\langle e_j g_k n|H_i|g_j g_k n+1\rangle\langle g_j g_k n+1|H_i|g_j e_k n\rangle}{\Delta} + \frac{\langle e_j g_k n|H_i|e_j e_k n-1\rangle\langle e_j e_k n-1|H_i|g_j e_k n\rangle}{-\Delta} = \frac{g^2}{\Delta}. \tag{4}$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity field. Then the effective Hamiltonian is given by

$$H_{\rm e} = \lambda \sum_{j=1}^{3}\left(|e_j\rangle\langle e_j|aa^+ - |g_j\rangle\langle g_j|a^+a\right) + \lambda \sum_{j,k}^{3}\left(S_j^+ S_k^- + S_j^- S_k^+\right), \quad (j \neq k). \tag{5}$$

The first and second terms on the right-hand side in the above expression describe the photon-number dependent Start shifts, the third and the fourth terms describe the dipole coupling between the jth and kth atoms induced by the cavity mode. We can rewrite the effective Hamiltonian as

$$H_{\rm e} = H_{\rm e1} + H_{\rm e2}, \tag{6}$$

where

$$H_{\rm e1} = \lambda \sum_{j=1}^{3}(|e_j\rangle\langle e_j|aa^+ - |g_j\rangle\langle g_j|a^+a), \tag{7}$$

$$H_{\rm e2} = \lambda \sum_{j,k}^{3}\left(S_j^+ S_k^- + S_j^- S_k^+\right) \quad (j \neq k). \tag{8}$$

Now we define the number of excited atoms as

$$N_{\rm e} = \sum_{j=1}^{3} |e_j\rangle\langle e_j|. \tag{9}$$

Since $[H_{\rm e}, a^+a] = 0$ and $[H_{\rm e}, N_{\rm e}] = 0$, the number of photons and the number of excited atoms remain constant during the interaction between the atoms and the cavity. For simplicity we first suppose that the cavity field is initially in the Fock state $|n\rangle$. If the system is initially in the state $|\psi(0)\rangle = |e_1 g_2 g_3 n\rangle$, we can define a basis for this system as follows:

$$|\phi_1\rangle = |e_1 g_2 g_3 n\rangle, \tag{10}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|g_1 e_2 g_3 n\rangle + |g_1 g_2 e_3 n\rangle). \tag{11}$$

As $[H_{\rm e1}, H_{\rm e2}] = 0$,the evolution operator is written as $\mathrm{e}^{-\mathrm{i}(H_{\rm e1}+H_{\rm e2})t} = \mathrm{e}^{-\mathrm{i}H_{\rm e1}t}\mathrm{e}^{-\mathrm{i}H_{\rm e2}t}$. We first obtain the evolution determined by $H_{\rm e2}$,

$$|\psi'(t)\rangle = \frac{1}{3}\left[(\mathrm{e}^{-\mathrm{i}2\lambda t} + 2\mathrm{e}^{\mathrm{i}\lambda t})|e_1 g_2 g_3 n\rangle + (\mathrm{e}^{-\mathrm{i}2\lambda t} - \mathrm{e}^{-\mathrm{i}\lambda t})(|g_1 e_2 g_3 n\rangle + |g_1 g_2 e_3 n\rangle)\right). \tag{12}$$

Then the evolution of the system is given by

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_{\rm e1}t}|\psi'(t)\rangle = \frac{1}{3}\mathrm{e}^{\mathrm{i}n\lambda t}\left[(\mathrm{e}^{-\mathrm{i}3\lambda t} + 2)|e_1 g_2 g_3 n\rangle + (\mathrm{e}_1^{-\mathrm{i}3\lambda t} - 1)(|g_1 e_2 g_3 n\rangle + |g_1 g_2 e_3 n\rangle)\right]. \tag{13}$$

Assume that the interaction time t is chosen such that $\lambda t = 2\pi/9$,the state of the system is

$$|\psi(t)\rangle = -\frac{\mathrm{e}^{\mathrm{i}(n\lambda t+\pi/6)}}{\sqrt{3}}\left(\mathrm{e}^{\mathrm{i}2\pi/3}|e_1 g_2 g_3\rangle + |g_1 e_2 g_3\rangle + |g_1 g_2 e_3\rangle\right)|n\rangle. \tag{14}$$

The expression (14) shows that the three atoms and the cavity field are in a disentangled state. Discard the common phase factor, and the three atoms will be in the entangled W state. Obviously, the time evolution of the system is independent of the photon number of the cavity field. That is to say, our scheme allows the cavity field to be in an arbitrary Fock state. We know that a thermal state is not a pure state, but its density matrix can be written as $\rho = \sum_n P_n |n\rangle\langle n|$, where $|n\rangle$ is the Fock state and P_n is the probability of finding the cavity field in the state $|n\rangle$. So we can draw the conclusion that our scheme allows the cavity field to be in a thermal state. From the above discussion, we can arrive at another conclusion: that the thermal state of the cavity field is allowed to change. So the cavity field can exchange energy with the environment in our scheme. It would be easier to carry out our scheme than the scheme in Ref.[15].

We can also use our scheme to generate a four-atom maximal entanglement W state with a successful probability of unit.

3. Summary

In summary, we have proposed a scheme for generating a three-atom maximally entangled W state. The scheme is based on the simultaneous nonresonant interaction of atoms with a single-mode cavity field. Its distinct advantage is that it allows the cavity field to be in a thermal state and exchange energy with the environment. we believe that our scheme might be realized in practice.

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