Vol 15 No 12, December 2006
1009-1963/2006/15(12)/2840-07

Chinese Physics

© 2006 Chin. Phys. Soc.
and IOP Publishing Ltd

Teleportation of atomic entangled states with a thermal cavity*

Zheng Xiao-Juan(郑小娟)[a)b)], Fang Mao-Fa(方卯发)[a)†],
Cai Jian-Wu(蔡建武)[a)], and Liao Xiang-Ping(廖湘萍)[a)]

[a)] *College of Physics and Information Science, Hunan Normal University, Changsha* 410081, *China*

[b)] *School of Physics Science and Technology, Central South University, Changsha* 410083, *China*

(Received 20 March 2006; revised manuscript received 10 July 2006)

We propose a most simple and experimentally feasible scheme for teleporting unknown atomic entangled states in driven cavity quantum electrodynamics (QED). In our scheme, the joint Bell-state measurement (BSM) is not required, and the successful probability can reach 1.0. Furthermore, the scheme is insensitive to the cavity decay and the thermal field.

Keywords: multipartite entanglement, quantum teleportation, thermal cavity, GHZ states

PACC: 0365, 4250

1. Introduction

Quantum teleportation,[1,2] first proposed by Bennett *et al*,[3] is one of the most important applications of quantum entanglement. In original contributions,[3,4] the joint Bell state measurement (BSM) is a key step in quantum teleportation. Hitherto, we still cannot discriminate the Bell states totally and conclusively.[5] In addition, the joint BSM is very complex for experimental realization,[6] and thus is a 'bottle-neck' for the further development of quantum teleportation. To overcome the difficulty, several schemes have been proposed to teleport unknown atomic states in cavity QED. Zheng[7] suggested a more feasible teleportation scheme for an unknown single atomic state using resonant optical cavity without BSM, and its successful teleportation probability could reach 0.25. But the scheme is subjected to the effects of both the cavity decay and the thermal field. Then, Ye *et al*[8] proposed another scheme, also without BSM, to teleport an unknown single atomic state and an unknown atomic entangled state, where the effect of cavity decay was eliminated and the successful probability of teleportation was 0.5. But the effect of the thermal field was still present. A scheme for teleporting an unknown single atomic state proposed by Yang *et al*[9] has eliminated the effects of both the thermal field and the cavity decay, and its successful probability of teleportation is 1. Jin *et al*[10] have proposed a scheme with the same advantages as those of the scheme of Yang[9] to teleport an unknown two-atom entangled state, but before the atoms 1, 2, and 3 that Alice possesses are detected, Alice must perform a rotation operation on the state of atom 1, i.e. Alice must switch on another classical field that acts on atom 1, otherwise quantum teleportation fails. Xue *et al*[11] also proposed a scheme with the same advantages as those of the scheme of Yang[9] to teleport an unknown bipartite entanglement by entanglement swapping with separate measurements in cavity QED and in addition they generalized their scheme to teleport the multipartite entanglement. In Xue's scheme, to teleport an n-atom entangled state, Alice needs to realize the joint BSM on n atom pairs, but the BSM is complex for experimental realization.[6] Therefore Alice investigates separate atomic measures to achieve the goal of joint BSM by using n cavities, i.e. Alice sends each atom pair that is in the Bell state to the cavity, separately, to distinguish Bell states. For the teleportation of multipartite entanglement, this scheme is very complex, and it is not easy to realize experimentally. This is because Xue *et al* need n Einstein, Podolsky and Rosen (EPR) pairs (Alice possesses $2n$ atoms and Bob possesses n atoms) in order to teleport n atoms entanglement successfully, thus the times of atom-cavity interactions in the whole teleportation process at least are $2n$ in total.

Advances in ion cooling and trapping have opened

*Project supported by the National Natural Science Foundation of China (Grant No 10374025).

†Corresponding author. E-mail: mffang@hunnu.edu.cn

new prospects of teleporting an unknown ionic internal state. It is proposed that [12] an effective quantum teleportation scheme of longer distance for unknown ionic internal state via trapped ions without the joint BSM and also with the successful probability being able to reach 1.

In this paper, combining the advantages of the schemes in Refs.[9–11], we propose a simple and efficient scheme for teleporting unknown atomic entangled states. Like the schemes in Refs.[9–11], our scheme has eliminated the effects of thermal field and cavity decay, and its successful probability of teleportation is 1. However, our scheme is most simple and easy to realize experimentally. Different from the scheme in Ref.[10], our scheme does not require another classical field. Unlike the scheme in Ref.[11], our scheme needs only $2n+1$ atoms, and the times of atom-cavity interactions in the whole teleportation process are 2 in spite of n. Our scheme can be generalized to teleport an N-atom entangled GHZ class state. In a word, our scheme is very simple and easy to realize experimentally.

2. The model

We consider N identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. The interaction between the atoms and the cavity can be described as follows:[13]

$$H=\omega_0\sum_{j=1}^{N}S_{z,j}+\omega_a a^+a+\sum_{j=1}^{N}\left[g\left(a^+S_j^-+aS_j^+\right)+\Omega\left(S_j^-e^{\mathrm{i}\omega t}+S_j^+e^{-\mathrm{i}\omega t}\right)\right], \tag{1}$$

where ω_0, ω_a and ω are the atomic transition frequency, the cavity-field frequency and the frequency of the laser field, respectively, $S_{z,j}=\frac{1}{2}(|e\rangle_j\langle e|-|g\rangle_j\langle g|)$, $S_j^+=|e\rangle_j\langle g|$, $S_j^-=|g\rangle_j\langle e|$, with $|e\rangle_j$ and $|g\rangle_j$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators of the cavity field, g is the coupling constant between the atoms and the cavity field, and Ω is the Rabi frequency of the classical field. We consider the case $\omega_0=\omega$, and $2\Omega\gg\delta\gg g/2$(δ is the detuning between the atomic transition frequency and the cavity-mode frequency). In the interaction picture, the evolution operator of the system is[13]

$$U(t)=\mathrm{e}^{-\mathrm{i}H_0t}\mathrm{e}^{-\mathrm{i}H_et}, \tag{2}$$

where $H_0=\sum_{j=1}^{N}\Omega\left(S_j^-+S_j^+\right)$, H_e is the effective Hamiltonian[13]

$$H_e=\lambda\left[\frac{1}{2}\sum_{j=1}^{N}(|e\rangle_j\langle e|+|g\rangle_j\langle g|)+\sum_{j,k=1}^{N}\left(S_j^+S_k^++S_j^+S_k^-\right)+\mathrm{H.c.}\right],\quad j\neq k, \tag{3}$$

where $\lambda=g^2/(2\delta)$. From the form of the effective Hamiltonian, we conclude that the interaction Hamiltonian is independent of the photon number of the cavity field. In addition, there is no exchange of energy between the atoms and the cavity mode. So the effects of both the cavity decay and the thermal field are eliminated.

If three atoms 3, 4 and 5 that are initially in the state $|ege\rangle_{345}$, or $|gee\rangle_{345}$, or $|geg\rangle_{345}$, or $|egg\rangle_{345}$ are simultaneously sent into the cavity and, at the same time, they are driven by a classical field, then the state of three atoms will evolve into

$$\begin{aligned}|ege\rangle_{345}\longrightarrow\ & \mathrm{e}^{-\frac{5\lambda t\mathrm{i}}{2}}\{\cos 2\lambda t(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\\ & -\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\ & +\frac{\mathrm{i}}{2}\sin 2\lambda t[(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\end{aligned}$$

$$
\begin{aligned}
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)]\},
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
|gee\rangle_{345}\longrightarrow\ &\mathrm{e}^{-\frac{5\lambda t\mathrm{i}}{2}}\{\cos 2\lambda t(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&+\frac{\mathrm{i}}{2}\sin 2\lambda t[(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)]\},
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
|geg\rangle_{345}\longrightarrow\ &\mathrm{e}^{-\frac{5\lambda t\mathrm{i}}{2}}\{\cos 2\lambda t(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&+\frac{\mathrm{i}}{2}\sin 2\lambda t[(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)]\},
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
|egg\rangle_{345}\longrightarrow\ &\mathrm{e}^{-\frac{5\lambda t\mathrm{i}}{2}}\{\cos 2\lambda t(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&+\frac{\mathrm{i}}{2}\sin 2\lambda t[(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&-(\cos\Omega t|e\rangle_3-\mathrm{i}\sin\Omega t|g\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|e\rangle_4\\
&-\mathrm{i}\sin\Omega t|g\rangle_4)(\cos\Omega t|g\rangle_5-\mathrm{i}\sin\Omega t|e\rangle_5)\\
&-(\cos\Omega t|g\rangle_3-\mathrm{i}\sin\Omega t|e\rangle_3)(\cos\Omega t|g\rangle_4\\
&-\mathrm{i}\sin\Omega t|e\rangle_4)(\cos\Omega t|e\rangle_5-\mathrm{i}\sin\Omega t|g\rangle_5)]\}.
\end{aligned}
\tag{7}
$$

3. Teleportation of bipartite entanglement

Suppose that the sender Alice and the receiver Bob possess five atoms in total. The atoms 1, 2 and 3 belong to Alice and the other two atoms 4 and 5 belong to Bob. The atomic entangled state that Alice wants to teleport to Bob is written as

$$|\psi\rangle_1 = a|ge\rangle_{12} + b|eg\rangle_{12}, \tag{8}$$

where a and b are unknown coefficients, and $|g\rangle$ and $|e\rangle$ are the ground and the excited states of the atom, respectively. Atoms 3, 4 and 5 are initially in the $|ege\rangle_{345}$. To generate the quantum channel, the atoms 3, 4 and 5 are sent into a single-mode cavity. At the same time, they are driven by a classical field. After interaction time t, the state of the subsystem, including atoms 3, 4 and 5, evolves according to Eq.(2) into Eq.(4). We can choose $\lambda t = \pi/4$ and realize the condition $\Omega t = 3\pi/4$ by modulating the driving field appropriately. Then the state of the subsystem evolves into

$$|\psi\rangle_{345} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{7\pi}{8}\mathrm{i}}(\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|ege\rangle_{345} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|geg\rangle_{345}). \tag{9}$$

This is a Greenberger–Horne–Zeilinger (GHZ) state. Thus we have generated the quantum channel.

The initial state of the whole system composed of atoms (1, 2) and the quantum channel is written as

$$|\psi\rangle_{12345} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{7\pi}{8}\mathrm{i}}(a|ge\rangle_{12} + b|eg\rangle_{12}) \otimes (\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|ege\rangle_{345} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|geg\rangle_{345}). \tag{10}$$

Next, Alice sends atoms 1, 2 and 3 into a single-mode cavity. At the same time, she drives the three atoms by using a classical field. The state of the subsystem, including atoms 1, 2 and 3, evolves according to Eq.(2) as well. According to Eqs.(4)–(7), we choose interaction time t satisfying the condition $\lambda t = \pi/4$ and $\Omega t = 3\pi/4$, then the state $|\psi\rangle_{12345}$ evolves into

$$\begin{aligned}|\psi\rangle_{12345} \longrightarrow\ & \frac{1}{2}\mathrm{e}^{-\frac{7\pi}{4}\mathrm{i}}[a\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}(\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|gee\rangle_{123} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|egg\rangle_{123})|ge\rangle_{45} \\ & -a\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|geg\rangle_{123} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|ege\rangle_{123})|eg\rangle_{45} \\ & +b\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}(\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|ege\rangle_{123} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|geg\rangle_{123})|ge\rangle_{45} \\ & -b\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|egg\rangle_{123} - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}}|gee\rangle_{123})|eg\rangle_{45}].\end{aligned} \tag{11}$$

Eq.(11) can be written in another form as follows:

$$\begin{aligned}|\psi\rangle_{12345} \longrightarrow\ & \frac{1}{2}\mathrm{e}^{-\frac{7\pi}{4}\mathrm{i}}[-\mathrm{i}(a|ge\rangle_{45} - b|eg\rangle_{45})|gee\rangle_{123} - (a|ge\rangle_{45} + b|eg\rangle_{45})|egg\rangle_{123} \\ & -(b|ge\rangle_{45} + a|eg\rangle_{45})|geg\rangle_{123} - \mathrm{i}(b|ge\rangle_{45} - a|eg\rangle_{45})|ege\rangle_{123}].\end{aligned} \tag{12}$$

Then Alice detects the atoms 1, 2 and 3. If the result is $|egg\rangle_{123}$, the state of the atoms 4 and 5 will be

$$|\psi\rangle_{45} = \frac{1}{2}\mathrm{e}^{-\frac{7\pi}{4}\mathrm{i}}(a|ge\rangle_{45} + b|eg\rangle_{45}). \tag{13}$$

So the teleportation succeeds with probability 1/4. If the measurement result is $|gee\rangle_{123}$, or $|geg\rangle_{123}$, or $|ege\rangle_{123}$, Bob can reconstruct the original state by corresponding unitary transformation according to Table 1, the teleportation also can succeeds with probability 1/4. Thus the probability of successful teleportation reaches 1.

Table 1. The results of the teleportation scheme. M denotes the measurement result on atoms 1, 2 and 3. Operation denotes the Bob's required operating. σ_{ij} is a Pauli operation σ_j on atom i.

M	$\|\psi\rangle_{45}$	Operation
$\|gee\rangle_{123}$	$a\|ge\rangle_{45} - b\|eg\rangle_{45}$	$I_4 \otimes \sigma_{5z}$
$\|egg\rangle_{123}$	$a\|ge\rangle_{45} + b\|eg\rangle_{45}$	$I_4 \otimes I_5$
$\|geg\rangle_{123}$	$b\|ge\rangle_{45} + a\|eg\rangle_{45}$	$\sigma_{4x} \otimes \sigma_{5x}$
$\|ege\rangle_{123}$	$b\|ge\rangle_{45} - a\|eg\rangle_{45}$	$\sigma_{4x} \otimes \sigma_{5y}$

4. Teleportation of multipartite entanglement

We also note that the scheme in Section 3 can directly be generalized to teleport the multipartite entanglement. In this section, we consider the generalization of our scheme.

Suppose that Alice possesses atoms 1, 2, ..., n, and $2n+1$, and Bob possesses atoms $n+1$, ..., $2n$. The atomic state to be teleported is

$$|\phi\rangle_A = a|g_1 \dots g_n\rangle + b|e_1 \dots e_n\rangle, \tag{14}$$

where a and b are unknown coefficients. Atoms $n+1$, ..., $2n$, $2n+1$ are initially in the state $|g_{n+1} \dots g_{2n} e_{2n+1}\rangle$. To generate the quantum channel, atoms $n+1$, ..., $2n$ and $2n+1$ are sent into a cavity. At the same time, they are driven by a classical field. Thus the state of this subsystem evolves according to Eq.(2). For the case where n is even we choose interaction time t satisfying the condition $\lambda t = \pi/4$ and $\Omega t = 3\pi/4$, then the state $|g_{n+1} \dots g_{2n} e_{2n+1}\rangle_{\mathrm{e}}$ evolves into

$$\begin{aligned} |g_{n+1} \dots g_{2n} e_{2n+1}\rangle_{\mathrm{e}} \longrightarrow \frac{1}{\sqrt{2}} \mathrm{e}^{-\frac{7\pi}{8}\mathrm{i}} \Big[& \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_{n+1} \dots g_{2n} e_{2n+1}\rangle \\ & - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} |\mathrm{e}_{n+1} \dots e_{2n} g_{2n+1}\rangle \Big]. \end{aligned} \tag{15}$$

For the case where n is odd we choose interaction time t satisfying the condition $\lambda t = \pi/4$ and $\Omega t = \pi$, then the state $|g_{n+1} \dots g_{2n} e_{2n+1}\rangle_{\mathrm{o}}$ evolves into

$$|g_{n+1} \dots g_{2n} e_{2n+1}\rangle_{\mathrm{o}} \longrightarrow \frac{1}{\sqrt{2}} \left[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_{n+1} \dots g_{2n} e_{2n+1}\rangle + \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n}{2}} |e_{n+1} \dots e_{2n} g_{2n+1}\rangle \right]. \tag{16}$$

Thus the initial state of the whole system composed of atoms (1, 2, ..., n) and the quantum channel is written as

$$\begin{aligned} |\psi\rangle_{\mathrm{e}} = \frac{1}{\sqrt{2}} \mathrm{e}^{-\frac{7\pi}{8}\mathrm{i}} \Big[& a \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_1 \dots g_n e_{2n+1}\rangle |g_{n+1} \dots g_{2n}\rangle \\ & - a \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} |g_1 \dots g_n g_{2n+1}\rangle |e_{n+1} \dots e_{2n}\rangle \\ & + b \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |e_1 \dots e_n e_{2n+1}\rangle |g_{n+1} \dots g_{2n}\rangle \\ & - b \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} |e_1 \dots e_n g_{2n+1}\rangle |e_{n+1} \dots e_{2n}\rangle \Big], \end{aligned} \tag{17}$$

$$\begin{aligned} |\psi\rangle_{\mathrm{o}} = \frac{1}{\sqrt{2}} \Big[& a \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_1 \dots g_n e_{2n+1}\rangle |g_{n+1} \dots g_{2n}\rangle \\ & + a \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n}{2}} |g_1 \dots g_n g_{2n+1}\rangle |e_{n+1} \dots e_{2n}\rangle \\ & + b \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |e_1 \dots e_n e_{2n+1}\rangle |g_{n+1} \dots g_{2n}\rangle \\ & + b \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n}{2}} |e_1 \dots e_n g_{2n+1}\rangle |e_{n+1} \dots e_{2n}\rangle \Big]. \end{aligned} \tag{18}$$

Then Alice sends atoms 1, 2, ..., n, and $2n+1$ into a cavity. At the same time, these atoms are driven by a classical field. The state $|\psi\rangle$ evolves according to Eq.(2). we choose interaction time t according to the above assumption. Thus the evolvement of the state $|\psi\rangle$ is written as

$$\begin{aligned} |\psi\rangle_{\mathrm{e}} \longrightarrow \frac{1}{2} \mathrm{e}^{-\frac{7\pi}{4}\mathrm{i}} \Big\{ & a \mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} \Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_1 \dots g_n e_{2n+1}\rangle - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} |e_1 \dots e_n g_{2n+1}\rangle \Big] |g_{n+1} \dots g_{2n}\rangle \\ & - a \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} \Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}} |g_1 \dots g_n g_{2n+1}\rangle - \mathrm{e}^{\frac{\pi}{4}\mathrm{i}} (-1)^{\frac{n+1}{2}} |e_1 \dots e_n e_{2n+1}\rangle \Big] |e_{n+1} \dots e_{2n}\rangle \end{aligned}$$

$$+b\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|e_1\ldots e_n e_{2n+1}\rangle-\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n+1}{2}}|g_1\ldots g_n g_{2n+1}\rangle\Big]|g_{n+1}\ldots g_{2n}\rangle$$
$$-b\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n+1}{2}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|e_1\ldots e_n g_{2n+1}\rangle-\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n+1}{2}}|g_1\ldots g_n e_{2n+1}\rangle\Big]|e_{n+1}\ldots e_{2n}\rangle\Big\}, \tag{19}$$

$$\begin{aligned}|\psi\rangle_{\mathrm{o}}\longrightarrow\ &\frac{1}{2}\Big\{a\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|g_1\ldots g_n e_{2n+1}\rangle+\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}|e_1\ldots e_n g_{2n+1}\rangle\Big]|g_{n+1}\ldots g_{2n}\rangle\\
&+a\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|g_1\ldots g_n g_{2n+1}\rangle+\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}|e_1\ldots e_n e_{2n+1}\rangle\Big]|e_{n+1}\ldots e_{2n}\rangle\\
&+b\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|e_1\ldots e_n e_{2n+1}\rangle+\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}|g_1\ldots g_n g_{2n+1}\rangle\Big]|g_{n+1}\ldots g_{2n}\rangle\\
&+b\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}\Big[\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}|e_1\ldots e_n g_{2n+1}\rangle+\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}(-1)^{\frac{n}{2}}|g_1\ldots g_n e_{2n+1}\rangle\Big]|e_{n+1}\ldots e_{2n}\rangle\Big\}.\end{aligned} \tag{20}$$

Eqs.(19) and (20) can be written in another form as follows:

$$\begin{aligned}|\psi\rangle_{\mathrm{e}}\longrightarrow\ &\frac{1}{2}\mathrm{e}^{-\frac{7\pi}{4}\mathrm{i}}\Big[-i(a|g_{n+1}\ldots g_{2n}\rangle+b|e_{n+1}\ldots e_{2n}\rangle)|g_1\ldots g_n e_{2n+1}\rangle\\
&-(-1)^{\frac{n+1}{2}}(a|g_{n+1}\ldots g_{2n}\rangle+b|e_{n+1}\ldots e_{2n}\rangle)|e_1\ldots e_n g_{2n+1}\rangle\\
&-(-1)^{\frac{n+1}{2}}(b|g_{n+1}\ldots g_{2n}\rangle+a|e_{n+1}\ldots e_{2n}\rangle)|g_1\ldots g_n g_{2n+1}\rangle\\
&-\mathrm{i}(b|g_{n+1}\ldots g_{2n}\rangle+a|e_{n+1}\ldots e_{2n}\rangle)|e_1\ldots e_n e_{2n+1}\rangle\Big],\end{aligned} \tag{21}$$

$$\begin{aligned}|\psi\rangle_{\mathrm{o}}\longrightarrow\ &\frac{1}{2}\Big[-\mathrm{i}(a|g_{n+1}\ldots g_{2n}\rangle+b|e_{n+1}\ldots e_{2n}\rangle)|g_1\ldots g_n e_{2n+1}\rangle\\
&+(-1)^{\frac{n}{2}}(a|g_{n+1}\ldots g_{2n}\rangle+b|e_{n+1}\ldots e_{2n}\rangle)|e_1\ldots e_n g_{2n+1}\rangle\\
&+(-1)^{\frac{n}{2}}(b|g_{n+1}\ldots g_{2n}\rangle+a|e_{n+1}\ldots e_{2n}\rangle)|g_1\ldots g_n g_{2n+1}\rangle\\
&-\mathrm{i}(b|g_{n+1}\ldots g_{2n}\rangle+a|e_{n+1}\ldots e_{2n}\rangle)|e_1\ldots e_n e_{2n+1}\rangle\Big].\end{aligned} \tag{22}$$

Then Alice detects the atom $2n+1$ and any of atoms 1, 2, ..., n. If the result is $|g_i e_{2n+1}\rangle$, or $|e_i g_{2n+1}\rangle (i = 1, 2, \ldots, n)$, the state of the atoms $n+1$, ..., $2n$ will become

$$|\phi\rangle_{\mathrm{B}} = a|g_{n+1}\ldots g_{2n}\rangle + b|e_{n+1}\ldots e_{2n}\rangle. \tag{23}$$

So the teleportation succeeds with probability $\frac{1}{2}$. If the measurement result is $|g_i g_{2n+1}\rangle$, or $|e_i e_{2n+1}\rangle$, Bob can reconstruct the original state by corresponding unitary transformation according to Table 2, the teleportation also can succeed with probability $\frac{1}{2}$. Thus the probability of successful teleportation reaches 1.

Table 2. The results of the teleportation scheme. M denotes the measurement result on atoms i and $2n+1$. Operation denotes the Bob required operating. σ_{ij} is a Pauli operation σ_j on atom i.

M	$\|\phi\rangle_{\mathrm{B}}$	Operation
$\|g_i e_{2n+1}\rangle$ or $\|e_i g_{2n+1}\rangle (i = 1, 2, \ldots, n)$	$a\|g_{n+1}\ldots g_{2n}\rangle + b\|e_{n+1}\ldots e_{2n}\rangle$	$I_n \otimes \ldots \otimes I_{2n}$
$\|g_i g_{2n+1}\rangle$ or $\|e_i e_{2n+1}\rangle (i = 1, 2, \ldots, n)$	$b\|g_{n+1}\ldots g_{2n}\rangle + a\|e_{n+1}\ldots e_{2n}\rangle$	$\sigma_{n,x} \otimes \ldots \otimes \sigma_{2n,x}$

5. Conclusions

We have presented a most simple and efficient scheme for teleporting an unknown two-atom entangled state, and generalized the scheme to teleport a multi-atom entangled state. In our scheme, the quantum channel is a three-atom or multi-atom entangled Greenberger–Horne–Zeilinger state.[14] Like the schemes in Refs.[9–11], our scheme has eliminated the effects of the thermal field and the cavity decay, and the successful probability of teleportation is 1. But our scheme is more simple and more feasible to realize experimentally.

References

[1] Zheng Y Z, Gu Y J, Wu G C and Guo G C 2003 *Chin. Phys.* **12** 1070

[2] Cai X H 2003 *Chin. Phys.* **12** 1066
Cao H J, Guo Y Q, Song H S 2006 *Chin. Phys.* **15** 915

[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[4] Liu J M and Guo G C 2002 *Chin. Phys. Lett.* **19** 456
Lu H and Guo G C 2000 *Phys. Lett.* A **276** 209
Dai H Y, Li C Z and Chen P X 2003 *Chin. Phys. Lett.* **20** 1196
Shi B S and Tomita A 2002 *Phys. Lett.* A **296** 161

[5] Zheng S B and Gan G C 2001 *Phys. Rev.* A **63** 044302

[6] Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737

[7] Zheng S B 2004 *Phys. Rev.* A **69** 064302

[8] Ye L and Guo G C 2004 *Phys. Rev.* A **70** 054303

[9] Yang M and Cao Z L Preprint quant-ph/0411195

[10] Jin L H, Jin X R and Zhang S 2005 *Phys. Rev.* A **72** 024305

[11] Xue Z Y, Yang M, Yi Y M and Cao Z L Preprint quant-ph/0508040

[12] Zheng X J, Fang M F, Cai J W and Liao X P 2006 *Chin. Phys.* **15** 492

[13] Zheng S B 2003 *Phys. Rev.* A **68** 035801

[14] Greenberger D M, Horne M A, and Zeilinger A 1989 in *Bell's Theorem*, *Quantum Theory*, *and Conceptions of the Universe*, edited by M. Kafatos (Dordrecht: Kluwer)
Greenberger D M, Horne M A, Shimony A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131