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A new representation and probabilistic teleportation of an arbitrary and unknown N-particle state

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A new representation of an arbitrary and unknown N-particle state is presented at first. As an application, a scheme for teleporting an arbitrary and unknown N-particle state is proposed when N pairs of two-particle non-maximally entangled states are utilized as quantum channels. After Alice (sender) makes Bell-state measurement on her particles, Bob (recipient) introduces an auxiliary particle and carries out appropriate unitary transformation on his particle and the auxiliary particle depending on classical information from Alice. Then, von Neumann measurement that confirms whether the teleportation succeeds or not is performed by Bob on the auxiliary particle. In order to complete the teleportation, another N-1 times operations need to be performed which are similar to the above ones. It can be successfully realized with a certain probability which is determined by the product of the smaller coefficients of non-maximally entangled pairs. All possible unitary transformations are given in detail.

Keywords: arbitrary and unknown N-particle state, probabilistic teleportation, entanglement

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1. Introduction

Quantum teleportation is a communication protocol for transmitting the state of a quantum system from some place to other place without transmitting the system itself. In 1993, Bennett *et al*[1] first showed that quantum entanglement can be used to teleport an unknown quantum state via an Einstein–Podolsky–Rosen (EPR) pair with the help of some classical information. Teleportation of the polarized photon and a single coherent mode of the radiation field was also realized experimentally by using parametric down-conversion.[2,3] Since then, quantum teleportation has always been an interesting topic due to its important applications in quantum computation and quantum communication. A number of feasible schemes were presented.[4−28] In these schemes, the particles in the state to be teleported increases to two particles or three particles.[4,5] The quantum channels are generalized to non-maximally entangled state in some schemes.[6−21]

In recent years, some teleportation schemes of multi-particle unknown state have been discussed,[22−27] especially for N-particle state.[24−27] But in these schemes, the representation of an N-particle state that one wants to teleport is complicated and troublesome. In this paper, a new representation of an arbitrary and unknown N-particle state is put forward. As an application, teleportation scheme of an unknown N-particle state and the corresponding successful probability are investigated.

Alice (sender) uses N pairs of non-maximally two-particle entangled states as quantum channels shared with Bob (recipient). After Alice makes Bell-state measurement on her particles, she informs Bob of her measurement result via a classical channel. Depending on Alice's classical information, Bob introduces an auxiliary particle and performs an appropriate unitary transformation on his particle and the auxiliary particle. And then, Bob performs von Neumann measurement to confirm whether the teleportation succeeds or not on the auxiliary particle. After they perform another N-1 times operations which are similar to the above one, the whole teleportation is realized. And the probability of the successful teleportation is proved to be $2^N \prod_{i=1}^{N} |C_i|^2$.

The rest parts of this paper are organized in the following way. In Section 2, we propose a new representation of an arbitrary and unknown N-particle state. As its application, the scheme of teleportation is put forward in Section 3. The discussions and conclusions are given in Section 4.

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2. A new representation of an arbitrary and unknown N-particle state

In some papers about teleportation schemes,[4−27] a complete basis expansion of multi-particle Hilbert space is adopted as the representation of an unknown state to be teleported, the coefficients of the front of basis vector are Greek letters or Latin letters with subscript. For example, an arbitrary and unknown two-particle state can be expressed as

$$|\Psi\rangle_{12} = (x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle)_{12}, \quad (1)$$

where the coefficients x_i are complex numbers and satisfy normalization condition: $|x_0|^2 + |x_1|^2 + |x_2|^2 + |x_3|^2 = 1, \{|00\rangle_{12}, |01\rangle_{12}, |10\rangle_{12}, |11\rangle_{12},\}$ are the basis vectors of two-particle four-dimensional Hilbert space. With the particles of the state to be teleported increasing, the representation gets complicated and troublesome. For instance, an arbitrary and unknown N-particle state with unknown coefficients $x_i (i = 0, 1, 2, 3, \ldots, 2^N - 1)$ can be written in the following way.[24−26]

$$\begin{aligned} |\Psi\rangle_{12\ldots N} = \; & x_0 \underbrace{|00\ldots00\rangle}_{N} + x_1 \underbrace{|00\ldots01\rangle}_{N} + \cdots \\ & + x_a \underbrace{|0\ldots0_i\ldots1_j\ldots1\rangle}_{N} + \cdots \\ & + x_{2^N-1} \underbrace{|11\ldots11\rangle}_{N} \end{aligned} \quad (2)$$

where the coefficients x_i are complex numbers and x_i satisfy normalization condition, i.e., $|x_0|^2 + |x_i|^2 + \cdots + |x_{2^N-1}|^2 = 1$. The authors need to pay much attention to the representation, but the possibility of clerical error is still big, and it is also complicated and troublesome to understand for the readers. The notion of multi-level particle state is presented in Ref.[28]. Here we put forward a new representation which can be used to denote the arbitrary and unknown N-particle. That is,

$$|\Psi_0\rangle = \alpha_i|\Psi_i\rangle|0\rangle_i + \beta_i|\Psi'_i\rangle|1\rangle_i. \quad (3)$$

$|\Psi_0\rangle$ is a normalized wavefunction, where $|0\rangle_i$ and $|1\rangle_i$ are denoted as the spin eigenstate of the particle i which is selected randomly. The quantum state of the rest particles in the entangled state can be described as $|\Psi_i\rangle$ and $|\Psi'_i\rangle$, which are also normalized wavefunctions. Due to wavefunction normalized condition

$$\langle\Psi_i|\Psi_i\rangle = 1, \langle\Psi'_i|\Psi'_i\rangle = 1, \langle\Psi_0|\Psi_0\rangle = 1. \quad (4)$$

Consequently,

$$\begin{aligned} \langle\Psi_0|\Psi_0\rangle &= |\alpha_i|^2\langle\Psi_i|\Psi_i\rangle + |\beta_i|^2\langle\Psi'_i|\Psi'_i\rangle \\ &= |\alpha_i|^2 + |\beta_i|^2 \\ &= 1. \end{aligned} \quad (5)$$

We can conclude from Eqs.(4) and (5):

(1) When$|\Psi_0\rangle$, $|\Psi_i\rangle$ and $|\Psi'_i\rangle$ are normalized wavefunctions, the coefficients α_i and β_i in Eq.(3) naturally satisfy $|\alpha_i|^2 + |\beta_i|^2 = 1$;

(2) If $|\Psi_i\rangle$ and $|\Psi'_i\rangle$ are normalized wavefunctions and $|\alpha_i|^2 + |\beta_i|^2 = 1$, $|\Psi_0\rangle$ satisfies normalized condition.

3. Probabilistic teleportation of an arbitrary and unknown N-particle state

The EPR state is used as quantum channel in the original scheme proposed by Bennett *et al.*[1] Since the influence of environment exists, the entanglement degree of two-particle entangled state is not always maximal. The faithful teleportation cannot be implemented when quantum channel is non-maximally entangled state. The investigation indicates that if non-maximally entangled state is used as quantum channel, the teleportation would be successfully realized with a certain probability not equal to one.[6−25]

In order to realize the teleportation of the state $|\Psi_0\rangle$, Alice sets up the quantum channels $|\Psi\rangle'$ between her and Bob, which are composed of N pairs of non-maximally two-particle entangled states

$$|\Psi\rangle' = \prod_{j=1}^{N}(B_j|00\rangle + C_j|11\rangle)_{b_j c_j}, \quad (6)$$

where the coefficients B_j and C_j are real, which satisfy normalized condition $|B_j|^2 + |C_j|^2 = 1$. Generally speaking, all the coefficients are real and different $|B_j| \neq |C_j|$. Without loss of generality, it is assumed that $|C_j|$ is smaller than $|B_j|$.

Particle b_j and particle c_j belong to Alice and Bob respectively. The state of the system composed of arbitrary and unknown N-particle state and the

quantum channels is denoted as

$$\begin{aligned}|\Psi\rangle = & |\Psi_0\rangle|\Psi\rangle' \\ = & \prod_{j=1}^{N}(B_j|00\rangle + C_j|11\rangle)_{b_j c_j} \\ & \otimes(\alpha_i|\Psi_i\rangle|0\rangle_i + \beta_i|\Psi'_i\rangle|1\rangle_i). \end{aligned} \tag{7}$$

Equation (7) can be rewritten in the following form:

$$\begin{aligned}|\Psi\rangle = & |\Psi_0\rangle|\Psi\rangle' = \prod_{j=1}^{N}{}'(B_j|00\rangle + C_j|11\rangle)_{b_j c_j} \\ & \otimes[(\alpha_i|\Psi_i\rangle|0\rangle_i + \beta_i|\Psi'_i\rangle|1\rangle_i)(B_i|00\rangle \\ & + C_i|11\rangle)_{b_i c_i}]. \end{aligned} \tag{8}$$

where $\prod_{j=1}^{N}{}'$ means that j cannot be equal to i. One may express the state of the system with Bell-state of particle i and particle b_i,

$$\begin{aligned}|\Psi\rangle = & |\Psi_0\rangle|\Psi\rangle' = \prod_{j=1}^{N}{}'(B_j|00\rangle + C_j|11\rangle)_{b_j c_j} \otimes \frac{1}{\sqrt{2}}\left[|\Phi^+\rangle_{ib_i}(\alpha_i B_i|\Psi_i\rangle|0\rangle_{c_i} + \beta_i C_i|\Psi'_i\rangle|1\rangle_{c_i})\right. \\ & + |\Phi^-\rangle_{ib_i}(\alpha_i B_i|\Psi_i\rangle|0\rangle_{c_i} - \beta_i C_i|\Psi'_i\rangle|1\rangle_{c_i}) + |\Psi^+\rangle_{ib_i}(\alpha_i C_i|\Psi_i\rangle|1\rangle_{c_i} + \beta_i B_i|\Psi'_i\rangle|0\rangle_{c_i}) \\ & \left. + |\Psi^-\rangle_{ib_i}(\alpha_i C_i|\Psi_i\rangle|1\rangle_{c_i} - \beta_i B_i|\Psi'_i\rangle|0\rangle_{c_i})\right]. \end{aligned} \tag{9}$$

The four Bell-state of particle (i, b_i) are given as

$$\begin{aligned}|\Phi^{\pm}\rangle_{ib_i} &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ib_i}, \\ |\Psi^{\pm}\rangle_{ib_i} &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ib_i}. \end{aligned} \tag{10}$$

In order to fulfil the teleportation, Alice makes a joint Bell-state measurement on her particle i and particle b_i. Then, all possible measurement results are $|\Phi^{\pm}\rangle_{ib_i}$ or $|\Psi^{\pm}\rangle_{ib_i}$ with a certain probability which is given in Table 1. For example, if Alice's measurement result is $|\Phi^{\pm}\rangle_{ib_i}$, the state of the rest N-1 particles of unknown N-particle state and particle c_i will collapse into

$$\frac{1}{\sqrt{2}}(\alpha_i B_i|\Psi_i\rangle|0\rangle_{c_i} + \beta_i C_i|\Psi'_i\rangle|1\rangle_{c_i}). \tag{11}$$

After that, she informs Bob of her measurement result via a classical channel. Bob introduces an auxiliary particle a_i with initial state $|0\rangle_{a_i}$. At this time, the state composed of the rest N-1 particles of the unknown state and particles (c_i, a_i) is described as

$$\begin{aligned}&\frac{1}{\sqrt{2}}(\alpha_i B_i|\Psi_i\rangle|0\rangle_{c_i} + \beta_i C_i|\Psi'_i\rangle|1\rangle_{c_i})|0\rangle_{a_i} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha_i B_i|\Psi_i\rangle \\ \beta_i C_i|\Psi'_i\rangle \\ 0 \\ 0\end{pmatrix}. \end{aligned} \tag{12}$$

The basis of the matrix which is adopted in Eq.(12) is $\{|00\rangle_{c_i a_i}, |10\rangle_{c_i a_i}, |01\rangle_{c_i a_i}, |11\rangle_{c_i a_i}\}$. Bob performs a unitary transformation U_i on particles c_i and auxiliary particle a_i depending on Alice's measurement result.

$$U_i = \begin{pmatrix} \frac{C_i}{B_i} & 0 & \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 & -\frac{C_i}{B_i} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{13}$$

After Bob's unitary transformation, the state composed of the rest N-1 particles of the unknown state and particles (c_i, a_i) are transformed to the following state:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}\alpha_i B_i|\Psi_i\rangle \\ \beta_i C_i|\Psi'_i\rangle \\ 0 \\ 0\end{pmatrix} \xrightarrow{U_i} \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha_i C_i|\Psi_i\rangle \\ \beta_i C_i|\Psi'_i\rangle \\ \alpha_i\sqrt{B_i^2 - C_i^2}|\Psi_i\rangle \\ 0\end{pmatrix}, \tag{14}$$

that is,

$$\begin{aligned}&\frac{1}{\sqrt{2}}(\alpha_i C_i|\Psi_i\rangle|00\rangle_{c_i a_i} + \beta_i C_i|\Psi'_i\rangle|10\rangle_{c_i a_i} \\ &+\alpha_i\sqrt{B_i^2 - C_i^2}|\Psi_i\rangle|01\rangle_{c_i a_i}). \end{aligned} \tag{15}$$

Table 1. Alice's Bell-state measurement results and appropriate unitary transformation of Bob.

Bell-state measurement results of Alice	$\|\Phi^+\rangle_{ib_i}$	$\|\Phi^-\rangle_{ib_i}$
Unitary transformation of Bob	$\begin{pmatrix} \frac{C_i}{B_i} & 0 & \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 & -\frac{C_i}{B_i} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} \frac{C_i}{B_i} & 0 & \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 \\ 0 & -1 & 0 & 0 \\ \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 & -\frac{C_i}{B_i} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Resultant probability	$\frac{1}{2}\|C_i\|^2$	$\frac{1}{2}\|C_i\|^2$
Bell-state measurement results of Alice	$\|\Psi^+\rangle_{ib_i}$	$\|\Psi^-\rangle_{ib_i}$
Unitary transformation of Bob	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{C_i}{B_i} & 0 & \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 \\ 0 & 0 & 0 & -1 \\ \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 & -\frac{C_i}{B_i} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{C_i}{B_i} & 0 & \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 \\ 0 & 0 & 0 & -1 \\ \sqrt{1-\frac{C_i^2}{B_i^2}} & 0 & \frac{C_i}{B_i} & 0 \end{pmatrix}$
Resultant probability	$\frac{1}{2}\|C_i\|^2$	$\frac{1}{2}\|C_i\|^2$

Bob carries out von Neumann measurement on the auxiliary particle a_i. If the result is $|1\rangle_{a_i}$, the teleportation fails. The teleportation succeeds with a certain probability $\frac{1}{2}|C_i|^2$ when the result is $|0\rangle_{a_i}$.

It can be seen that the operations and process are similar to $|\Phi^+\rangle_{ib_i}$ when Alice's result is $|\Phi^-\rangle_{ib_i}$ or $|\Psi^\pm\rangle_{ib_i}$. The corresponding probabilities and appropriate unitary transformations of these four cases are given in Table 1. After the process, the information that particle i carries has been transferred to particle c_i, and the successful probability is $4\times\frac{1}{2}|C_i|^2 = 2|C_i|^2$. The above process is only $1/N$ of the whole teleportation, the rest processes are similar to the above one.

From the above analysis, we can see that the total probability of successful teleportation is $2^N\prod_{i=1}^{N}|C_i|^2$, because the choice of particle i is random. If $|B_i| = |C_i| = \frac{1}{\sqrt{2}}$, i.e. for the quantum channels composed of N pairs of EPR states, the probability of the whole teleportation is equal to 1.[4,5,7,27] That is to say, quantum channels composed of N pairs of EPR states are the special form of $|\Psi\rangle' = \prod_{j=1}^{N}(B_j|00\rangle + C_j|11\rangle)_{b_jc_j}$.

By the way, the discussions of teleportation schemes about quantum channels composed of the product states which are the arbitrary combination of $(B_j|00\rangle \pm C_j|11\rangle)_{b_jc_j}$ and $(B_j|01\rangle \pm C_j|10\rangle)_{b_jc_j}$ are same to the above one and the probability of successful teleportation is also $2^N\prod_{i=1}^{N}|C_i|^2$.

4. Discussions and conclusions

Compared with the previous representations in other papers, the present one has the following advantages. First, the present representation is simpler in shape and written more easily than other representations. Due to the advantage, the author may avoid some clerical errors which happen easily in other representations. Second, it is easily to understand the representation itself. Third, it can clearly illuminate the scheme presented in this paper. Finally, the calculation of probability of an successful teleportation is convenient due to the condition $|\alpha_i|^2 + |\beta_i|^2 = 1$.

In contrast to the previous teleportation schemes,[26] the present one is more easily realized due to the fact that the unitary transformation of the two-qubit is more easily to prepare and operate than multi-qubit in the experiment processing.

The successful probability of the scheme goes to zero when the quality of quantum channel is not good and the particles of the state to be teleported are many. Hence, for the sake of improving the success probability, we should improve the quantum channels to make them in the maximally entangled state to the best of our abilities.

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