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Immunization for scale-free networks by random walker*

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Based on the random walk and the intentional random walk, we propose two types of immunization strategies which require only local connectivity information. On several typical scale-free networks, we demonstrate that these strategies can lead to the eradication of the epidemic by immunizing a small fraction of the nodes in the networks. Particularly, the immunization strategy based on the intentional random walk is extremely efficient for the assortatively mixed networks.

Keywords: immunization strategy, scale-free network, random walk, intentional random walk

PACC: 0250, 0590

1. Introduction

Complex networks provide an efficient description for a great variety of natural and artificial systems,[1−3] such as social contacts,[4,5] the Internet,[6,7] metabolic and protein networks.[8,9] Recently, it has been observed that these systems share some global topological properties. In particular, the small-world and scale-free (SF) features, appear as common properties of many real-world networks. Thanks to the paradigms of the Watts–Strogatz (WS)[10] and Barabási–Albert (BA) models,[11] transparent pictures for the microscopic origins of the leading features are presented. With the ever-increasing knowledge of topological features of networks, a better foundation for the understanding of dynamical complexity of realistic systems is gradually established, which is favourable for many applications, e.g., Internet attacks[12] and searchability,[13] synchronization of networks,[14] the epidemic spreading,[15−20] and so on.

An extensive study for epidemic dynamics has uncovered some interesting characteristics of complex networks. In particular, a somewhat unexpected result derived from SF networks shows that the epidemic threshold is absent. This indicates that such kind of network is prone to the spreading and persistence of infections.[15−18] This feature, observed in several epidemiological models, is due to large connectivity fluctuations in the SF networks.[15−18] Because of the fact that SF networks serve as models covering a wide range of realistic systems, especially the social contacts and the Internet, it is undoubtedly of particular importance to find efficient immunization strategies to cope with the weakness. Immunization is to permanently remove the immunized individuals from networks. In previous studies, three types of typical immunization strategies, namely the uniform, proportional, and targeted ones, have been examined.[21] However, it has been commented that these strategies are quite difficult to be applied in practice. In order to stop the epidemics, uniform immunization needs to immunize a very large fraction of nodes in the SF networks.[21] Obviously, it is not economical. In the cases of proportional and targeted immunization, an epidemic can be prevented by immunizing just a small amount of the nodes with high connectivity but good knowledge of degrees of each node in the networks is required. Such global knowledge about networks often proves to be hard to obtain. From this point of view, Cohen *et al*[22] proposed an efficient strategy for immunization (called as acquaintance immunization), requiring no knowledge of the nodes' degrees or any other global information. Owing to the immediate practical and economical implications, further studies involving the other local immunization strategies is still of considerable significance.

Recently, a few local strategies have been developed in the studies of the complex networks.[14,22,23] Particularly, the random walk, as a fundamental dynamic process,[24] is widely applied in building the scale-free network model,[25] searching in power-law

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networks,[14] as well as studying the structural properties of complex networks.[26,27] In this paper, by means of the random walk and the intentional random walk, we propose two immunization strategies which are expected to successfully prevent the epidemics.

In our strategies, we first choose a node in the network at random, from which a walker begins his tour. The nodes visited by the walker, rather than the originally chosen nodes, are assumed to be successfully immunized. The probability that a walker can walk from the current node i to a certain nearest neighbour j, is uniform for the case of the random walk, while for the case of the intentional random walk, it can be expressed as

$$\prod_{i\to j} = \frac{k_j}{\sum\limits_{l\in\Lambda_i} k_l}, \tag{1}$$

where k_j is the order of a node j, and Λ_i denotes the neighbourhood of node i. Therefore, in the case of the intentional random walk, we have applied a reasonable assumption that each node has a local knowledge about its nearest neighbours' degrees. By varying the steps of walk, the fraction of immunized nodes f can be modulated. Obviously, the strategies are local ones in which the global knowledge of a network is unnecessary.

This paper is organized as follows. In Section 2, we introduce the standard susceptible–infected–susceptible (SIS) epidemiological model.[28] In Section 3, we apply the present immunization strategies to several typical SF networks, and compare our results with those of the uniform, targeted and acquaintance immunization strategies. Finally, Section 4 is a brief summary.

2. The SIS model

Several models have been proposed for examining the behaviours of epidemic dynamics on the SF networks.[15−20] Here, we focus on the SIS model, where a node in the network representing an individual can be in two discrete states: susceptible (healthy) or infected. Connections between individuals along which the infection can spread are represented by links. In each time step, a susceptible node is infected with probability ν if one or more of its neighbours are in an infected state. Meanwhile, the infected nodes recover and turn into susceptible states with probability δ. Therefore, the effective spreading rate for the epidemic is defined as $\lambda = \nu/\delta$.[28] Clearly, nodes change from susceptible state to infected one, and then back to susceptible one to experience a stochastic cycle by repeatedly contracting the infection because of lack of self-acquired permanent immunization. Traditionally, the SIS model is studied on the basis of regular lattices and random graphs. The most significant result is the general prediction of a nonzero epidemic threshold λ_c. If the value of λ is above the threshold, i.e., $\lambda \geq \lambda_c$, the epidemic spreads and becomes persistent. If $\lambda < \lambda_c$, the system falls, in a finite time, in a healthy state with no infected individuals.

3. Immunization for scale-free networks by random walker

For the immunization strategies, a main consideration is the cost effectiveness: i.e., how many nodes are needed to be immunized to obtain healthy state of the network.[21] In the presence of immunization, we study the cost effectiveness by looking at the infection's prevalence in the stationary regime (endemic state) as a function of the immunization f. Initially we infect half of the susceptible nodes, and iterate rules of the SIS model with parallel updating. After a transient regime the system reaches a steady state, characterized by a constant average density of infected nodes: i.e., the infection's prevalence ρ_f which is the time average of the fraction of infected nodes. In addition, the reduced prevalence is defined as the ratio ρ_f/ρ_0, where ρ_0 is the infection's prevalence without immunization.[21] A critical fraction f_c of immunization, above which the prevalence in the steady state is null, is referred to as the immunization threshold.

Many realistic complex systems possess, along with the SF property, some more detailed topological features which vary strongly for different systems.[29−31] Due to the lack of a single model encompassing the topological features of complex networks, we consider a few SF networked structures, aiming to test the present immunization strategies.

3.1. Immunization for the Barabási–Albert network

As a paradigmatic example of SF networks, we consider the BA model which has been universally used as a general structure to study various dynamical processes taking place in complex networks.[32] This model introduces two simple mechanisms that are be-

lieved to be common in reality: growth and preferential attachment, to construct the SF networks.[11] We follow the algorithm devised in Ref.[11]: Starting with a small number m_0 of nodes; and then at every time step a new node is added, with m links that are connected to an old node i with k_i links according to the probability $k_i/\sum_j k_j$. After iterating this scheme a sufficient large number of times, we obtain a network composed of N nodes with connectivity distribution $P(k) \propto k^{-3}$ and average connectivity $\langle k \rangle = 2\,m$. In this section, we take $m_0 = m = 3$ and $N = 10^5$.

Numerical simulations for the epidemic spreading on the BA model have been performed to estimate the efficiencies of the present immunization strategies. For comparison the uniform immunization strategy and the targeted one are also considered. Figure 1 shows that in the case of the uniform immunization, the reduced prevalence decays slowly with increasing immunization fraction f, and it would be null until almost all nodes of the network are successfully immunized. It is shown that an immunization threshold disappears, in other words, the epidemic can prevail in SF networks even after massive uniform immunization. In contrast, in the cases of both the present immunization strategies, the targeted and the acquaintance ones, a drastic variation in the prevalence behaviours is found (see Figs.1 and 2). In particular, the reduced prevalence ρ_f/ρ_0 suffers a very sharp drop and exhibits the onset of an immunization threshold above which no endemic state is possible.

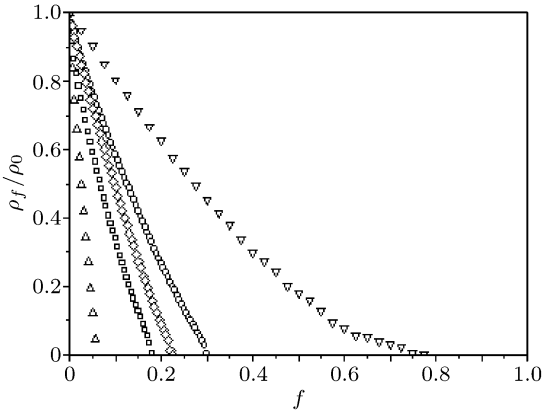


Fig.1. Reduced prevalence ρ_f/ρ_0 from computer simulations of the SIS model in the BA networks with the uniform immunization (∇), the targeted immunization (Δ) and the acquaintance immunization ($\diamond$), as well as the immunization based on the random walk ($\circ$) and intentional random walk ($\Box$), at a fixed spreading rate $\lambda = 0.25$. The prevalence is computed by averaging over at least 100 different starting configurations, performed on at least 10 different realizations of the networks. The network size $N = 10^5$ and $\langle k \rangle = 6$.

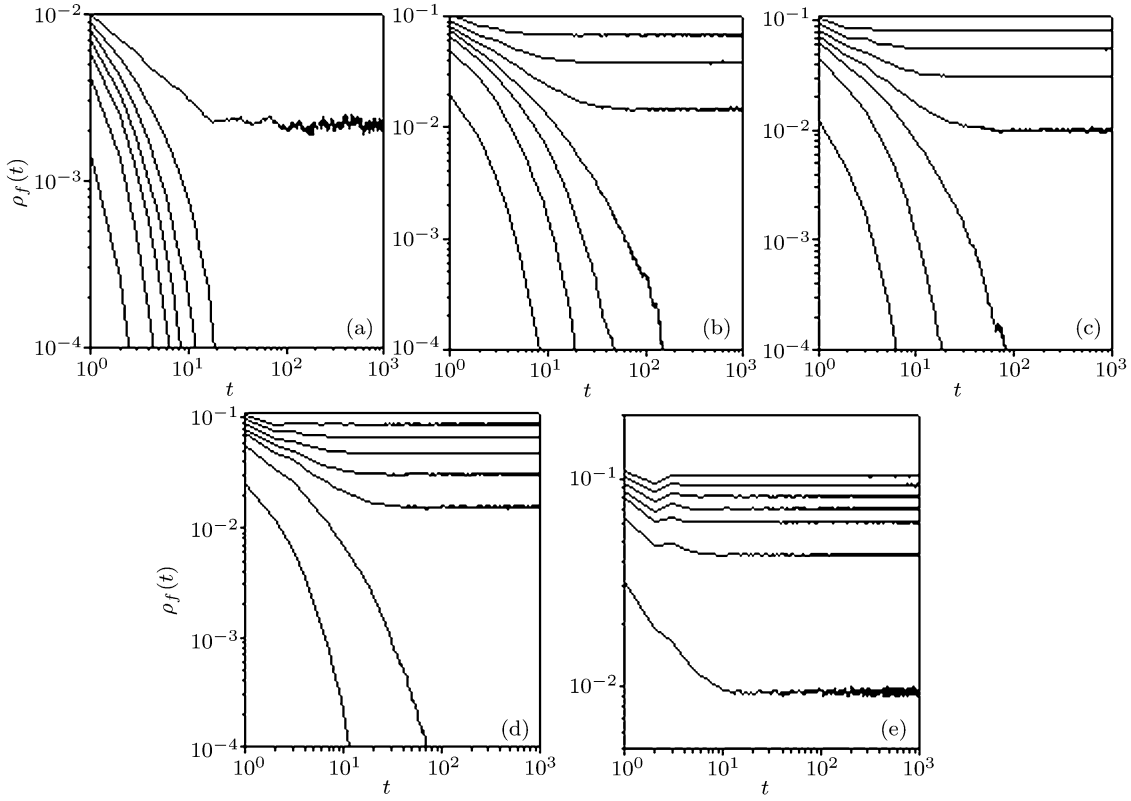


Fig.2. Typical plots of $\rho_f(t)$ as a function of time for SIS model in BA networks with different immunization schemes: (a) the targeted immunization; (b) the immunization based on the intentional random walk; (c) the acquaintance immunization; (d) the immunization based on the random walk; and (e) the uniform immunization. From top to bottom in each plot: $f = 0.05$, 0.1, 0.15, 0.2, 0.25, 0.35 and 0.6. The network's parameters are the same as in Fig.1.

In Fig.1, by using a linear extrapolation from the largest values of f, one can obtain the immunization thresholds for these strategies. Apparently, they are remarkably lower than that of the uniform strategy. When $\lambda = 0.25$, the threshold $f_c = 0.06$ for the targeted immunization, which is in good agreement with the analytic solution $f_c = \exp[-2/m\lambda]$;[21] $f_c = 0.3$ for the immunization based on the random walk, and $f_c = 0.18$ for the immunization based on the intentional random walk; $f_c = 0.225$ for the acquaintance immunization. Also, in Fig.2, the temporal evolution of the prevalence ρ_f confirms these results. For the values of f well below the immunization threshold f_c the prevalence ρ_f always approaches a nonzero plateau value, whereas for f above f_c the epidemic dies out exponentially. These results indicate that on the BA networks, the present immunization strategies could lead to the eradication of the epidemics by immunizing only a small fraction of the nodes. In addition, it is not unexpected that the reduced prevalence for the immunization based on the intentional random walk decays more rapidly than the one based on the random walk and exhibits a very low immunization threshold. This implies that the application of local connectivity information might greatly improve the efficiency of immunization.

It should be pointed out that the present immunization based on the random walks might be reduced to the acquaintance immunization by expanding it to the case of an ensemble of random walkers. However, the present immunization seems to be more favourable for implementation in practice. For instance, in the Internet, it is very difficult to dispense anti-virus software to beforehand chosen computers, while it would become convenient by applying the scheme of the random walks. In addition, the immunization based on the intentional random walks is actually a modified version of that based on the random walks. This strategy, although needs the local connectivity information, improves the efficiency of immunization. Thus, to find a balance between the requirement of information and the efficiency of immunization would be of considerable significance in further studies.

3.2. Immunization for the assortatively and disassortatively mixed networks

Many realistic networks are recently found to exhibit assortative or disassortative mixing patterns of degree which also are referred to as the degree correlations.[29] For instance, many social networks exhibit that the nodes with high connectivity (hubs) connect more preferably to the hubs, which are called the assortatively mixed networks. On the other hand, many technological and biological networks show disassortative mixing: i.e., the hubs are preferably connected to the nodes with small degree. In this case, the BA model that has been used as a model of the structures of many realistic networks is incomplete due to the absence of these mixing patterns.[29] Therefore, testing the present immunization strategies on the assortatively and disassortatively mixed SF topologies would be a better approach to the realistic situation than that on the BA model would be.

Complex networks with the assortative or disassortative mixing can be constructed by applying a Metropolis dynamics to a random graph with the desired degree distribution.[29] Especially, we here regard the BA model as an initial graph to ensure that all networks possess same power-law degree distribution. For a detailed algorithm for generating these networks see Ref.[29]. In every realization of the networks, we obtain that the assortatively and disassortatively mixed networks after 5×10^5 Monte Carlo steps, and the assortativity coefficients averaged over 10 different realizations of the networks are equal to 0.05786 and -0.04405, respectively. On the two types of networks, the behaviours of both the reduced prevalence ρ_f/ρ_0 and the temporal evolution of the prevalence ρ_f are plotted in Figs.3 and 4, respectively.

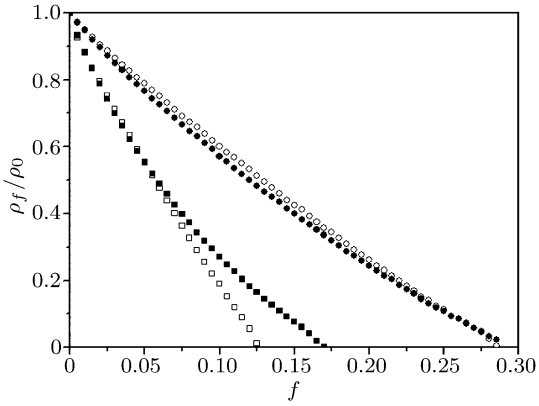


Fig.3. Reduced prevalence ρ_f/ρ_0 from computer simulations by using the SIS model in the disassortatively (full symbols) and assortatively (empty symbols) mixed networks with the immunization based on the random walk (circles) and intentional random walk (squares), at a fixed spreading rate $\lambda = 0.25$. A linear extrapolation from the largest values of f yields the estimate of the thresholds $f_c = 0.125$ in the assortatively mixed networks, $f_c = 0.16$ in disassortatively mixed networks with the immunization based on the intentional random walk, and $\lambda_c = 0.285$ in both the assortatively and disassortatively mixed networks with the immunization based on the random walk. The network's parameters are the same as in Fig.1.

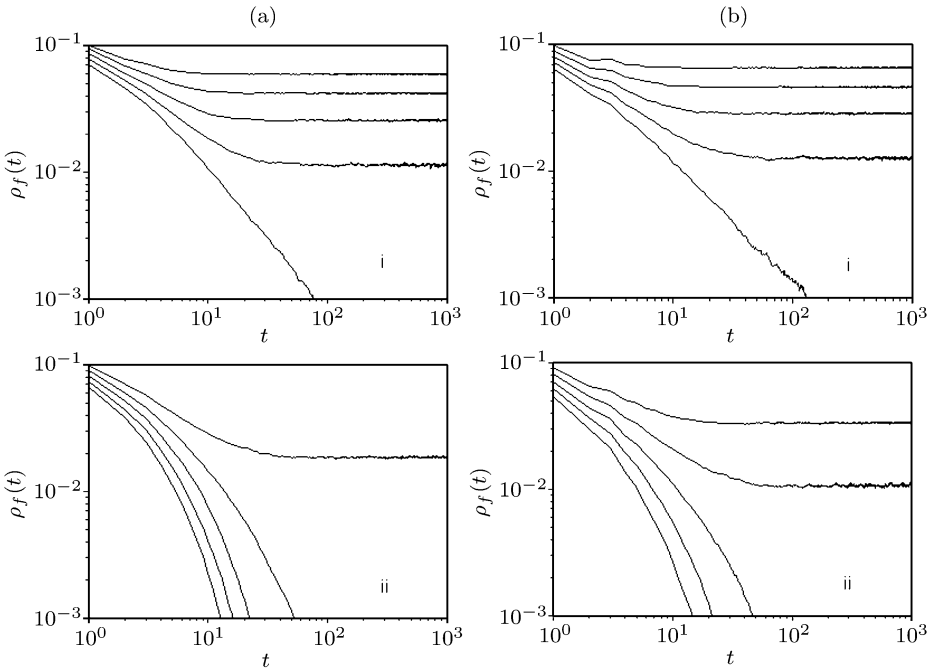


Fig.4. Typical plots of $\rho_f(t)$ as a function of time for SIS model in (a) the assortatively mixed networks and (b) the disassortatively mixed networks with the immunization based on (i) the random walk and (ii) the intentional random walk. From top to bottom in each plot: $f = 0.1$, 0.15, 0.2, 0.25 and 0.3. The network's parameters are the same as in Fig.1.

On the two types of SF networks, the strategy based on the intentional random walk is more effective than that on the BA model. Particularly, the immunization threshold is remarkably reduced on the assortatively mixed networks. Intuitively, the influence of the mixing patterns on the present immunization strategies has two aspects. On the one hand, previous studies have shown that the assortatively mixed networks are more robust to the removal of the hubs,[29] so that the present immunization strategies, which are naturally prone to immunization for the hubs, seem to be somewhat ineffective, while the disassortatively mixed networks are more vulnerable to the removal of their hubs;[29] and thus, these strategies should be more effective for such networks. On the other hand, owing to the different mixing patterns, a random walker starting at a certain hub arrives at other hubs faster on the assortatively mixed networks than on the disassortatively mixed networks. From this point of view, the present strategies, especially, the strategy based on the intentional random walk, should be more effective on the assortatively mixed networks than on the disassortatively mixed ones. Apparently, the improvement of the efficiencies of these immunization strategies results from the competition between the two aspects.

4. Immunization for the Internet

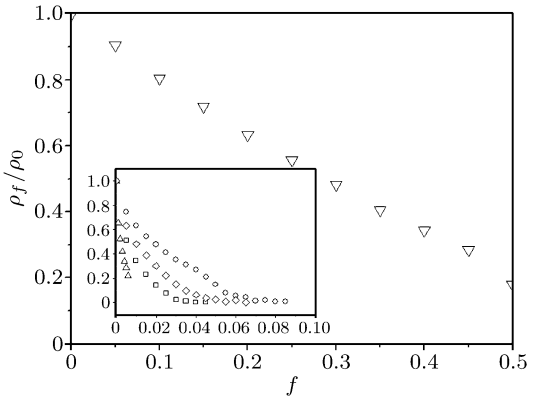


Fig.5. Reduced prevalence ρ_f/ρ_0 from computer simulations of the SIS model in a portion of a real Internet map with the uniform immunization (∇), the targeted immunization (Δ) and the acquaintance immunization ($\diamond$), as well as the immunization based on the random walk ($\circ$) and intentional random walk ($\square$) at a fixed spreading rate $\lambda = 0.25$.

Finally, we apply the present immunization strategies to a real map of the Internet and investigate the spreading of computer viruses to examine these strategies for practical application. The real map of the Internet considered here, provided by the National

Laboratory for Applied Network Research and available at the web site http://www.moat.nlanr.net/Routing/rawdata/, contains 6474 nodes and 12572 links, corresponding to an average connectivity $\langle k \rangle = 3.88$. The connectivity distribution is scale-free, with a connectivity exponent $r = 2.2$. In the simulations, the spreading rate λ is still fixed at 0.25. Figure 5 illustrates that, the uniform immunization does not allow any notable reduction of the infection prevalence when increasing the immunity f, while the strategies, based on the random walk and the intentional random walk, drastically removes the occurrence of endemic states even at very low value of the immunization fraction. This confirms that the present strategies are especially effective for the Internet. It is expected that in the Internet, these strategies could be applied in practice, by which only a small portion of the routers or Internet domains are secured by a random walker who puts out anti-virus software to whomever it visits.

5. Summary

We have proposed two types of immunization strategies that require only local connectivity information of each node. By comparing the efficiencies of different immunization strategies on several different SF networks, it is found that the present strategies are efficient. Especially, the strategy based on the intentional random walk is unexpectedly efficient for the assortatively mixed networks, and allows for a particular low threshold of immunization, despite that such networks are more prone to the spreading and persistence of infections.

References

[1] Dorogovtsev S N and Mendes J F F 2002 *Adv. Phys.* **51** 1079

[2] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47

[3] Newman M E J 2003 *SIAM Rev.* **45** 167

[4] Wasserman S and Faust K 1994 *Social networks analysis* (Cambridge: Cambridge University Press)

[5] Liljeros F, Edling C R, Amaral L A N, Stanley H E and Ãberg Y 2001 *Nature* **411** 907

[6] Faloutsos M, Faloutsos P and Faloutsos C 1999 *Comput. Commun. Rev.* **29** 251

[7] Caldarelli G, Marchetti R and Pietronero L 2000 *Europhys. Lett.* **52** 386

[8] Jeong H, Tombor B, Albert R, Oltvai Z N and Barabási A L 2000 *Nature* **407** 651

[9] Maslov S and Sneppen K 2002 *Science* **296** 910

[10] Watts D J and Strogatz S H 1998 *Nature* **393** 440

[11] Barabási A L and Albert R 1999 *Science* **286** 509

[12] Albert R, Jeong H and Barabási A L 2000 *Nature* **406** 378

[13] Watts D J, Dodds P S and Newman M E J 2002 *Science* **296** 1302

[14] Li Z and Shi S J 2004 *Chin. Phys.* **13** 996

[15] Pastor-Satorras R and Vespignani A 2001 *Phys. Rev. Lett.* **86** 3200
Pastor-Satorras R and Vespignani A 2001 *Phys. Rev.* E **63** 066117

[16] Li Y, Liu Y, Shan X M, Ren Y, Jiao J and Qiu B 2005 *Chin. Phys.* **11** 2153

[17] Boguñá M, Pastor-Satorras R and Vespignani A 2003 *Phys. Rev. Lett.* **90** 028701
Boguñá M and Pastor-Satorras R 2002 *Phys. Rev.* E **66** 047104

[18] Moreno Y, Pastor-Satorras R and Vespignani A 2002 *Eur. Phys. J.* B **26** 521

[19] Barthélemy M, Barrat A, Pastor-Satorras R and Vespignani A 2004 *Phys. Rev. Lett.* **92** 178701

[20] Yan G, Zhou T, Wang J, Fu Z Q and Wang B H 2005 *Chin. Phys. Lett.* **22** 510

[21] Pastor-Satorras R and Vespignani A 2002 *Phys. Rev.* E **65** 036104

[22] Cohen R, Erez K, ben Avraham D and Havlin S 2003 *Phys. Rev. Lett.* **91** 247901
Madar N, Kalisky T, Cohen R, ben-Avraham D and Havlin S 2004 *Eur. Phys. J.* B **38** 269

[23] Guimerà R, Díaz-Guilera A, Vega-Redondo F, Cabrales A and Arenas A 2002 *Phys. Rev. Lett.* **89** 248701

[24] Hughes R D 1995 *Random Walks: Random Walks and Random Environments* (Oxford: Clarendon) 1

[25] Saramäki J and Kaski K 2004 *Physica* A **341** 80

[26] Tadić B 2003 *Modeling Complex Systems*, *AIP Conference Proceedings* **661** 24

[27] Noh J D and Rieger H 2004 *Phys. Rev. Lett.* **92** 118701

[28] Diekmann O and Heesterbeek J 2000 *Mathematical Epidemiology of Infectious Diseases: Model Building, Analysis and Interpretation* (New York: Wiley)

[29] Newman M E J 2002 *Phys. Rev. Lett.* **89** 208701
Huang Z X, Wang X R and Zhu H 2004 *Chin. Phys.* **13** 0273

[30] Girvan M and Newman M E J 2002 *Proc. Natl. Acad. Sci.* **99** 7821

[31] Milo R, Shen-Orr S, Itzkovitz S, Kashtan N, Chklovskii D and Alon U 2002 *Science* **289** 824

[32] Dorogovtsev S N 2001 *Nature* **410** 268